[SQUEAKING] [RUSTLING] [CLICKING]

JACK HARE: Welcome to our second lecture. Today, we are going to be talking about magnetic probes. These are surprisingly simple devices. But you can also measure a surprising amount about your plasma from these. So we'll start by considering just a simple loop of wire.

So let's imagine we have a loop of wire like this. It's got some area A. And it's got some magnetic field pointing out of the page, B, like this. And we can imagine that this is a time-varying magnetic field, B of time here, OK?

And then we go and grab one of Maxwell's equations. We'll take the curl of E is equal to minus the time derivative of B here. So I'm writing that as B-dot. We can take this equation and integrate over this area here, integrate up the curl of E over this surface, the dotting with the S and doing the same thing on the other side, integrating up the magnetic field dotted with S, like that.

And then we can use one of our nice vector calculus identities and convert this from being a surface integral over the area into a line integral around the curve which bounds this surface here. And that will allow us to write this instead as the integral of E dot dL.

And we recognize that if we're integrating an electric field around a path, this is just going to end up being a voltage. So it'll be a voltage difference between these two points-- this point here and this point here. We're not going to measure it right here. We'll attach these with cables to some digitizer or oscilloscope out here.

So there'll be a potential difference around this loop. We'll call that potential difference V. And that's simply going to be equal to the integral of B-dot dotted with dS here. We've already said that the area of this surface is simply A. So we'll just write that as A. And then we'll write this as B-dot here.

And if you want to increase the voltage you get for a given time-changing magnetic field, you can put more loops on this. So your probe could have one loop, or it could have two loops, or it could have n loops, like this. And we would simply multiply this by n at the end there, where n is just the number of loops we have. So that's how you make this more sensitive.

And so because of this dependency on B-dot here, these are called B-dots, very inventive. OK. So this is a very simple diagnostic. It senses magnetic fields. Now, I've made a few approximations or assumptions while I've been deriving this. Can someone give me a reasonable condition on the magnetic field, particularly thinking about its spatial variation, that enables me to derive this simple equation? Yeah?

STUDENT: [INAUDIBLE]?

JACK HARE: Yeah, OK. And so we might have some length scale over which this magnetic field varies. So we call this LB. And we could write this as B over grad B. And you'll excuse my sloppy notation here. I've sort of dropped the fact that these are vectors here.

And we have some length scale associated with the size of the probe. So we can call this LP. And that's going to go like square root of A. And so we want to make sure the magnetic field doesn't change much over this length scale here. So we might demand that LP over LP is much, much less than 1 here.

So effectively, your magnetic field is only changing in time. It's not changing spatially. Of course, if it is changing spatially you will still measure something. You'll still get a voltage out. It's just that your interpretation won't be quite as straightforward as this. If you know something about how the magnetic field varies spatially over your loop, then that's OK. But if you don't, it's going to make your interpretation very, very tricky. OK, any questions on this? Yeah?

STUDENT: [INAUDIBLE]?

JACK HARE: OK. So these are two characteristic length scales that we're comparing here. LP is a characteristic length scale of our probe. It's just going to go as a square root of the area. And LB is the characteristic length scale over which the magnetic field varies. If you look at this dimensionally, you see that it's going to have dimensions of length.

And so we're just saying that our magnetic field length scale has to be much larger than our probe length scale such that the magnetic field is basically constant across the probe. Anyone else? Anyone from Columbia? OK, good stuff.

So of course, now, you've got your voltage out. And you've digitized it with your oscilloscope. Or at least if you haven't digitized it, it's coming out these little cables here. And of course, you need to do something with it to get B-dot. You need to integrate it, right? So you have got an equation V equals A B-dot n. And you want an equation that looks like B-dot equals V over An. And so therefore, B is going to be the integral with respect to time of B over An like this from time 0 up to whatever time T you're at at that point.

OK. And there are a few ways to do this. Back in the day, you might want to try and do this automatically because doing this numerically may be computationally expensive. You can do this passively. You could have some sort of RC type filter. Those of you who know your analog electronics would recognize something like this, where we've got our B-dot loop at the end there.

And we've got some resistance and some capacitance. And this forms a low pass filter which, depending on the frequency of your signal will passively integrate it up. So that's very simple to build. If you're trying to put 1,000 B-dot probes around your tokamak, you might think that this is a nice solution.

If you want something a little bit more fancy, you can do this with an op amp. So we can put an op amp in here. And we can put in something where we have a resistor and a capacitor for each of the inputs. And this is nicer because, of course, it's going to buffer your signal. As well, it makes it easier to do the impedance matching. So you might want to consider an active system like this.

So just to be clear, this is passive. It has no electrical components that require power. And this is an active system. And of course, the third way you can do this, especially these days, is simply take this signal and do it digitally. So we can just sum up V over An times delta T, where delta T is whatever your time base is.

Now, you have to be a little bit careful here because there are other things which can induce a voltage. Oh, look, it's still there. There are other things which can induce a voltage on your probe other than just the magnetic field. But you only measure the voltage at the end. So you don't really know if something else has come and interfered. But there are some tricks to try and get around that.

So let's have a chat about some other voltages that might crop up on your probe that you're not aware of. You could, for example, have some stray capacitively coupled voltage on here. So you may end up having a system where you have a loop which has a voltage on it, a B-dot, as you expect, but there is also some other capacitively coupled signal, like noise.

So your signal would just look like this. But on top of it, you've got some other noisy signal that corrupts it. And as you start integrating this up, it's going to make a bit of a mess of your integration. If this is truly just noise, if it's randomly distributed, the integration is obviously going to get rid of that. It's very nice for smoothing things out.

But if this has some structure-- for example, it's related to potential inside your plasma-- then this is going to screw up your measurements. So the way we get around this is we perform a differential measurement. We have two B-dots. One of the B-dots is wound in one direction, like this. And the other B-dot, which we place as close as possible, is wound in the other direction, like this.

So the sense in which we evaluate this dot dL integral is in opposite directions. So for probe 1 and probe 2, we get different voltages out. For probe 1, we're going to get a voltage that looks like V1 equals A B-dot plus this capacitively coupled voltage.

But for 2, the sense in which the L goes is opposite. So we get V2 equals minus A B-dot plus some capacitively coupled voltage. And so we can do a trick that we like doing in experimental physics, which is to make a differential measurement. And we can take the difference of these.

We can take the difference of V1 minus V2. And you'll see that these capacitively coupled voltages, which are the same on both loops, will cancel out. But we will amplify up the signal that we actually want. And so we'll end up with 2 A B-dot, like this. So this is, as I said, a differential measurement. And whenever possible, if you're trying to get rid of noise, you want to try and do things like differential measurements.

So if you're asking where does this capacity coupled signal come from, if you just imagine having your B-dot probe like this and there's some plasma floating over here, this plasma may be at some potential B plasma, like this. And your probe may be attached near ground. So it's floating at close to V equals 0.

But you can see effectively there's going to be some capacitive coupling. If I drew a circuit diagram, it'll be as if there was some capacitor between these two. And that would be enough to induce some small fraction VC on here, which is going to be much, much less than the plasma potential.

But the plasma potential could be a kilovolt or so if you're dealing with electron volt temperature plasmas. And so this VC can still be large enough that it will throw off your measurements. This is why it's worth doing differential measurements if you can. Any questions on that? Yes?

STUDENT: What exactly [INAUDIBLE]?

JACK HARE: Yeah, that's a good point. Yes, it is.

STUDENT: That ever [INAUDIBLE].

JACK HARE: Yeah, we'll talk about that more in a minute because if you've got very slowly changing magnetic fields where this is an issue, like on a superconducting tokamak, B-dots aren't really going to do any good because your signal is going to be absolutely tiny because B-dot will be very, very small. So your voltage will be very, very small. So you probably want to rely on other sensors to do that instead.

Right, this really should read like delta B. This is the change in magnetic field. So you can't measure a DC magnetic field with these. Yeah, and there are lots of problems when you do this practically with drift. So if your oscilloscope isn't perfectly calibrated and it's got some slight offset to it, that's going to be integrated up as a gently ramping up magnetic field, which is another good reason to do differential measurements because you can get rid of those offsets as well. Any other questions? Yeah?

STUDENT: [INAUDIBLE]?

JACK HARE: Oh, well done. Yes, that's a good point. I should have mentioned it. I've assumed here that the magnetic field is perpendicular when I drew this diagram. It's parallel to the normal of this surface here. So that's where we have B-dot DS.

Of course, the magnetic field could be at some arbitrary angle to it. We're only going to pick up the component which is normal to the surface. So this is insensitive to magnetic fields, for example, pointing in this direction. This is actually quite important in the tokamak where-- or in other devices, where you may not actually know what the magnetic field is.

We know what the vacuum magnetic field is. But once there's current flowing in the plasma, we may not know which component we're picking up. So you may also want to have not, for example, just two loops for this measurement, but three axis B-dot probes as well. And so on laboratory astrophysics devices like LEDD and MRX, they have these arrays of probes with three different axes, so they can measure the full vector magnetic field. Yeah?

STUDENT: [INAUDIBLE]?

JACK HARE: Yeah, so the idea here is this could be due to anything. This is due to some high voltage being present somewhere in your system. So it could be a capacitor bank discharging. Or it could be a killer electron volt plasma. But the fact is that any conductor and any other conductor have a capacitance between them just always. There is an ability to induce some potential on one conductor from the other.

And so all I'm saying is if you have some stray potential, it will couple with the same sign to these two probes. But because they're oppositely wound, we'll be able to cancel out that same sign potential there. That's the other reason why you want the probes as close together as possible because if you put them further apart, this VC may be different on the two probes. And then they won't cancel perfectly. So ideally, you'd have two loops perfectly counter wound lying directly on top of each other, yeah. Any questions from Colombia? I see two hands. Maybe Nigel first?

STUDENT: Hi, could you remember to repeat questions when they're asked in person?

JACK HARE: Yeah, thank you. I forgot about that. Yeah, was that Matthew's question as well? Or is it a different question?

STUDENT: No, I have a separate question. Nigel, did you have anything else?

STUDENT: No.

STUDENT: OK, so do you have to worry about inductive coupling between two oppositely wound up probes that are very closely spaced?

JACK HARE: The brief answer is you'd have to worry about that if the current flowing through the probes was sufficient to create a magnetic field that was comparable to the magnetic field, you're trying to measure. So in general, it's not. Yeah, it's a good question. But say you've got a volt or so that you're measuring. And you're measuring it over a 50 ohm terminator, which is what we normally do for the matching with 50 ohm cables. That's going to be a pretty small current. So the magnetic field you're inducing is pretty small. It's a good question.

STUDENT: OK. All right, thanks.

JACK HARE: Yeah, another question?

STUDENT: [INAUDIBLE]?

JACK HARE: Right, if you integrate a zero mean Gaussian noise, then you're right. You will get rid of VC. But it doesn't have to be a zero mean Gaussian noise. This VC, in fact, will probably represent something like-- say you're doing a discharge in a plasma. The voltage will rise and fall in a similar way to the current in a magnetic field.

So you might actually have a system where, let's say, this is your signal B-dot, like this. And then your signal is doing something like-- let me draw it-- like this. So this is a capacitor bank ramping up and discharging again. And so you can see, if you average that out, you're not going to get rid of it. So this is for non-random one, yeah.

STUDENT: I felt like [INAUDIBLE]

JACK HARE: Oh, I guess you're right, yeah. It's going to be relatively small depending on how big your signal to noise ratio is. But you're right, it's like a random walk. So you're going to diffuse away from where you start, yeah. Yeah, I guess, as with all measurements, you want to have a good signal to noise to avoid that.

STUDENT: [INAUDIBLE]?

JACK HARE: It depends. You should design as a diagnostic designer a system which gives you sufficient signal to noise, which might mean increasing the area of your loop. Or it might mean increasing the number of terms in order to fulfill that.

OK, I'm going to move on. So just to say, you can take these two signals here and you could feed them in automatically to an op amp similar to this. And you can make an op amp circuit that does subtraction. And then you could feed that into an op amp circuit which does integration.

The trouble is, unless you inspect these two signals visually, you may find that one of them is more affected by the capacitive coupling than the other. This is a problem we have in my research, where one of your signals might look really nice, like this. And the other signal just goes like that.

And if you feed these automatically into an op amp without digitizing them and just accept whatever comes out the other end, then you're not going to get a decent answer from this. This data is junk. You want your signals to look as close to opposite as possible. So probably you want to digitize them separately.

Anyway, we're going to be focusing at the moment on B-dots which are placed outside of the plasma. I will talk about B-dots placed inside of the plasma because that's something that I like to do in my research. But for most of the time, if we're dealing with things like tokamaks, magnetic confinement fusion devices, you don't want to stick things inside there. They're just going to melt. So we'll be sticking with external probes.

OK. There was a question earlier about measuring when B-dot is small, when we've got almost steady state magnetic fields. So let's have a quick chat about that. So when we have B-dot goes to 0, we'll call these steady states. Of course, they don't have to be absolutely not time changing. They just have to be time changing slowly enough that we can't measure the voltage easily.

So this would give V on our B-dot also going to 0. So at this point, you probably want to resort to a different device. And these are often called Hall effect sensors. And another name for a Hall effect sensor is a Gauss meter. So the way these work is we have a block of a special semiconductor.

So this is a terrible drawing of a cuboid. There we go. There's a magnetic field that's threading through this that we want to sense. We allow a current to flow through it in this direction, from this face to this face back here.

And because we have charged particles moving in a magnetic field, there's a J cross B force which separates the charges. And so on one side, we build up a positive charge. And on the other side, we build up a negative charge. And if we attach our oscilloscope here, we can measure that potential difference. So there's going to be an electric field in this direction.

And by knowing the properties of our semiconductor and measuring this electric field, we can infer what the magnetic field is here. Again, there's a J cross B force that separates the charges in these two directions. So this works pretty well. It doesn't work very fast. So you can't use these replace B-dots. If you want to have a very high frequency signal, this doesn't work.

And the other problem is that the plasma is a very harsh environment, especially if you're going to a nuclear environment, like the next generation of tokamaks. And that causes degradation of this semiconductor. And then you have to work out how you're going to calibrate these things in situ. So you can't just pop them out because they're hot, radioactively hot.

So these sorts of devices are very good. But we really need to have rad-hardened versions of them. And as far as I know, from at least the last time I looked, I don't think that we have a solution to this problem at the moment. But I may be wrong.

Another technique you might want to use is Faraday rotation or the Faraday effect. We'll actually talk about this a bit more later on in the context of Faraday rotation inside plasmas. But in fact, the Faraday effect was first discovered by Faherty, who didn't know anything about plasmas. And he was using, well, actually, all sorts of things.

If you go back to Faraday's notebook, it's kind of remarkable. He built one of the first large magnets. He also had an ability to detect the polarization of light. And he did what any reasonable experimental physicist would do. He started putting things inside the magnet and seeing if they rotated the polarization of light.

And so he's got in his notebook milk. Milk does. Beef. Beef doesn't because the light didn't go through. But fair on him for trying. It's great. So yeah, there's all sorts of cool things. Now, these days, we don't use beef inside most experiments. What we use is a special type of glass which has-- it's usually called verdet glass because it's got something called a verdet constant. This is just probably some French guy's name.

So if you've got some magnetic fields in this direction and you pass some linearly polarized light-- so we'll start off with a polarization, for example, in this up-down direction. When the light comes out, that polarization is going to be rotated. And the angle that it's rotated by, beta, is equal to V, which is the verdet constant-- don't get this confused with voltages. This is just a constant that is a property of this glass-- and then times by the magnetic field times by the length of this lump of glass here.

OK. And so this means that if you put a little chunk of this glass inside a fiber optic, this is extremely convenient because you can just put that fiber optic somewhere. You don't have to worry about mirrors for this light because it's all trapped inside the fiber. And we have lots of really great technology and fibers for looking at polarization because the telecommunications industry uses it for multiplexing.

And so you can sense this polarization. And that means that you can then infer what the magnetic field is because you know V. You know L. You measure theta. And therefore, you get B. Once again, this is B in the direction of propagation. So same with the B-dots, it's got that limitation. You don't get a vector out.

And the other problem is that, again, is rad hardening. And actually not just rads, as in like neutrons and things like that, though that will definitely alter the structure here and therefore change the verdet constant, but even things like X-rays can cause blank inside this glass. So people have tried to use this on the Z-machine at Sandia, which is the world's most energetic X-ray source. And they don't work very well because the X-rays get inside the fibers. And they blank things out.

So this is another possibility. But once again, it's something we need a lot more technology development on before we can use it in the harsh environments we expect in fusion devices. So any questions on the Hall effect, gas meters, or Faraday effect? Yes? And I'll try to remember to repeat your question.

STUDENT: [INAUDIBLE]?

JACK HARE: OK. So the question was, why can't the Hall effect be used for fast changes? Simply because there is some drift velocity associated with these carriers. And we can't make them drift fast enough. So if you have a very rapidly changing V, they'll all be sloshing to the right. And then by the time they've got there, they'll be sloshing back to the left. And they'll never make it back.

So I don't know off the top of my head what the time scale is. But it may be in Hutchinson's book. Or you may be able to look it up. But my impression is that this is more for milliseconds, second kind of time scales, where B-dots can do much faster.

This can be very fast. Faraday effect sensing can be as fast as your digitizer. So one can get a 50ghz digitizer these days, if you have a lot of money. So you can send very, very fast. And yeah?

STUDENT: [INAUDIBLE]?

JACK HARE: No, I don't know what Spark is using. So there's a good question. They presumably have a plan. I haven't seen like a diagnostics paper for Spark yet. I keep nagging Spark people to do something. But I haven't seen it yet. So it'd be cool to see it. Other questions from Columbia?

STUDENT: I have one. For the Faraday effect, how do you account for you have to modulo 2 pi with the angles. You take it across multiple-- do you measure the angle at multiple stages?

JACK HARE: Oh man, we're going to be talking a lot about ways to disambiguate modulo 2 pi stuff when we get into promontory. So I'll just say any technique you can use there, you can do here. So there's some really cool stuff you can do. Yeah, we can do like temporary heterodyne polarization measurements, things like that.

So yeah, it's doable. This simple method here is flawed, like you pointed out. But there are other things we can do, which can get around that ambiguity. Yeah?

STUDENT: Do perpendicular components of the magnetic field affect the Faraday effect sensor?

JACK HARE: The question is, do perpendicular components of the magnetic field affect the Faraday effect sensor? I don't think so. There is an effect in plasmas. It's called the Cotton-Mouton effect, which can have an effect. But I don't know what happens in verdet glass. It's kind of like a solid state kind of thing, which I'm less familiar with. So I've never heard people talking about that as being a problem. Good question, though, yeah. Another question here?

STUDENT: Does the verdet constant depends on the wavelength of light?

JACK HARE: Yes, the verdet constant depends on the wavelength of light, maybe not very strongly. And you would know what wavelength you were using. So it wouldn't be a big problem. So for telecoms, we use 1,550 nanometers because that's what's being developed. So if you're going to do a fiber diagnostic, it's probably going to be at 1,550, unless you want to spend a lot of money on custom [INAUDIBLE]. All right, let's keep moving. OK. And now, a related diagnostic, where should we go?

So a related diagnostic, another firm favorite of mine is something called a Rogowski coil, a rogowki coil, depending on what you think [INAUDIBLE]. So a Rogowski coil measures the enclosed current. Enclosed by what, you ask. Enclosed by the coil. So I will draw on in a second. And you'll see what I'm talking about.

So a Rogowski coil looks like-- let's see what the best way to draw this is. We have, a little bit like a B-dot, a cable coming up. Instead of just going around in a loop like this, we have something that spirals around in a helix.

And then for reasons I won't go into right now but you may feel free to ponder, we tend to not take the cable down here. But instead, we wind it back all the way around inside, like this. The answer to this is in Hutchinson's book. But I'm not going to go in right now.

And the current that's enclosed-- for example, we can consider that we have some sort of current carrying rod, like this, that goes through this Rogowski coil. So we've set our Rogowski coil around some sort of conductor. Obviously, that current has to close. And it will close outside the conductor, like that.

OK, so let's have a look at the geometry here, just a simpler diagram. We've got current. We've got some sort of surface like this. And we've got all these little loops. And these little loops-- hey, hey-- have an area. There's a current here. So there's going to be a magnetic field through it, like that.

There's going to be two dL's. There's going to be the dL around this little loop. But there's also going to be the dL prime around this loop, like that. So we're actually going to end up with a double integral. We're going to end up with having a flux that is going through this loop. This magnetic flux is going to be equal to the number of turns per unit length because we're going to have multiple of these little surfaces all the way around like this.

We're going to be integrating along L prime, which is the circumference of this Rogowski. And we're going to be integrating over A, which is the area of one of these little loops here. We're going to be integrating B dot dL-- actually, that's not quite right. B dot dS again dL, like that.

Now, the key insight here is that this flux is going to be due entirely to the current that's going inside here. We know from Ampere's law that the integral of the magnetic field around some path, dL, which is this one here, using dL prime, is going to simply be mu I enclosed.

So this magnetic field B here, at least on average around this whole circle, is going to be proportional just to mu, which is going to be the permeability of whatever material we're using, times the enclosed current. And the voltage that we get out, down here, which is the thing we're going for, is simply going to be the time rate of change of the total flux through this circle circuit. And so we're going to end up with something that looks like nA mu I enclosed dotted.

So once again, we've got a voltage, which is proportional to the time derivative of something. It's also proportional to an area. Instead of having a total number of turns n, we now have a number of turns per unit length. So this is turns per unit length.

This is not the area of the whole thing. It's the area of one of these little terms here. So this could be, for example, pi A squared if the radius of one of these little loops is A, like that. And the only reason I wrote this as mu as opposed to mu 0 is because it's really mu 0 mu R.

And you want to be a little bit careful here because, for some materials, if you get very strong magnetic fields, they have saturation of mu R, things like steels and stuff like that. You want to avoid those because this mu R is going to start changing depending on the strength of your signal.

And so you're going to have not a nice linear relationship between voltage and current, but a non-linear relationship. So the only reason I put this in here is just to say that you shouldn't use steel for these things if you're going up to high magnetic fields.

The nice thing about this is it doesn't matter where the current is inside here. Remember, Ampere's law just talks about enclosed currents. I could split this up and have two conductors, like this. I could just have one conductor over all the way on one side. I would still get the same signal out of this.

That's nice because it means I don't really need to know the exact location of all the conductors inside here. That means that I don't have to position this thing quite so precisely, which is extremely useful. So this I enclosed here means that the conductor position, or should we say current channel, position is independent of this.

OK. And you can go and wind one of your own of these. You can strip back a BNC cable and get some magnet wire and wrap it all the way back around, which is what Thomas Varnish has been doing for [INAUDIBLE] experiments. Or you can go online and you can get something from Pearson. So everyone buys Pearson's. These nice, green loops. And they have a passive integrator built into them, like we discussed before. And they're labeled in volts per amp.

So if you just want something off the shelf and you've got lots of money, you can buy something like that. So these are absolute workhorses not just for plasma physics. These are just if you want to measure current going through something. So any questions on these before we keep moving and show how to use them in plasma systems? Mm-hmm?

STUDENT: [INAUDIBLE]?

JACK HARE: Yeah. Once again, yes, you're right. It only measures the current normal to this big surface here. So I guess I glossed over that. Yeah, you're quite right.

STUDENT: [INAUDIBLE]?

JACK HARE: It's going to happen somewhere in Ampere's law. And I think when we talk about I enclosed here, we are-- huh, Interesting. Maybe it doesn't matter because that doesn't look like we care particularly about the angle of the conductor here. I don't know the answer off the top of my head. If someone else does know the answer, shout out. Otherwise, I'll have a think about it and get back to you.

STUDENT: [INAUDIBLE].

JACK HARE: OK, so I've skipped it in that line. Yeah, I think you're right. So OK, I think it does matter what the orientation of your conductor is. Yeah, cool. Thank you. Other questions? Yeah?

STUDENT: [INAUDIBLE]?

JACK HARE: No, you can't use this for constant current still because the signal V is proportional to I dot. So you'd have to use another current sensor if you want to do that. And these suffer from similar problems to your B-dots, where you can pick up stray voltages on them. So you might want to use an oppositely wound pair of these side by side and do differential measurements.

There's also a continuous version of this, where you replace this helix with just a groove in a plate of metal. And I'll leave it to you to think about how that works because it's actually a little bit mind-bending. But it does still work. What I want to do now is talk about how to use these in the context of plasma physics. So any other questions before we move on? Columbia? All right, looks like we're good.

Okey doke, let's have a think about something like a tokamak. I'm going to draw the cross-section of a tokamak like this. It's going to have a plasma inside it. And in that plasma, there's going to be some current. This is going to be the toroidal current, I phi, which loops around the torus.

This toroidal current is being driven inductively by a transformer. So this is an iron core. And we have a primary winding. And the tokamak plasma forms a secondary winding. So we're driving some current using this ohmic transformer.

We can drive a constant current with this. That will require us to keep ramping up the voltage on this transformer. But that's allowed. And what we'll do is we'll put-- around one part of our plasma, will put our Rogowski coil. So here comes Rogowski coil, loop de loop de loop, back wound. And we measure V goes as I phi dot.

OK, so we can measure that current. I also want to draw this same system now looking from above because it makes it easier for what the next point is. Say so, this is now our torus. We've got our plasma, like this. Hopefully, it's not that unstable. And we've got some toroidal current, I phi, going around it.

We've got some transformer, as already discussed, like that. We've put in our Rogowski coil, like that. And we're also going to put in another loop. And that loop is going to be our voltage loop. So this loop simply looks like this. And it measures V in the toroidal direction.

The reason it does this is because this transformer is inducing some toroidal potential. It has to. That's what's driving this toroidal magnetic field. This is equal to V toroidal over the plasma resistance, whatever that means really in this context.

Now, ideally, we would measure this by sticking something inside the plasma. But actually, we don't need to because it's not the plasma that's inducing V toroidal. It's this whacking great transformer. So we can put our loop, say, above the plasma and just measure it here.

It doesn't matter where the break is. It just matters that when we measure difference between these two, as you traverse a circuit around here, you're going to pick up some V toroidal, which is the same V toroidal, as I said, that's driving the pattern here. It will become clear why we're trying to measure this in a moment. It's actually very much related to this plasma resistivity here. So this is an important quantity that you might want to measure.

OK. So let's set this up and have a go at measuring the plasma resistivity using just two loops of wire Rogowski and V phi. OK, so we consider a plasma. It has a volume V. And it's got a surface-- what do I call it-- partial V like that. OK, and our loops are outside of here. So both the ROG and the V phi are outside this surface.

OK, good. Now, let's think about energy balance in this system-- so the amount of energy in the system, how it's changing in time, and how we're injecting energy into the system. So we're going to have a few different terms here. We're going to have an ohmic heating term, E dot J. So that's going to be related to the current inside this system and the electric field, which is related to the voltage.

So we can say that this is going to be-- straight away, we can see, this is going to be something with I phi. And this is going to be something to do with V phi. There's also going to be a term related to the change in total magnetic energy inside the system. So the magnetic energy is B squared over 2 mu 0. And we want to have the time rate of change of that to make it a power.

I'm going to write that slightly differently to make it clearer what's going on here. We want the time rate of change of the magnetic energy density, like this. And all of this is integrated over this volume, dV. And this is going to be equal to-- this change in internal energy of the system is going to be equal the energy that we're injecting into the system. We're injecting the energy into the system through this transformer. And this is going to be equal to 1 upon mu 0 the integral through this surface partial V of E cross B integrated over the surface.

So this is the Poynting flux from electromagnetism. This is ohmic heating. And this is effectively the inductive power, the energy we have to spend to change the magnetic field. And these are balanced because we're going to be discussing a system in steady state. So we're not going to allow the energy to change without us putting some power into it. So I'm not sure I need that assumption, but there we go. OK, questions on this setup before we try and work out some of these terms? Yeah?

STUDENT: You said it's [INAUDIBLE]

JACK HARE: Yeah.

STUDENT: [INAUDIBLE]?

JACK HARE: Yeah, so imagine we're microwaving a lump of chicken. This left-hand side here is the temperature, the thermal energy density of the chicken. So we're integrating over the chicken, OK? The right-hand side is the Poynting flux, which is the microwaves penetrating the surface of the chicken, OK? And so we're setting those two equal to each other.

So this is the internal energy of the plasma. And this is the power that we're putting into the plasma. So this is not the internal energy. This is the change in internal energy as a function of time. That's an ohmic heating rate. That's a power. This is the time derivative of energy. So it's also a power. And this is a power integrated with [INAUDIBLE]. Yeah?

STUDENT: [INAUDIBLE]?

JACK HARE: If we counterwound the B phi, we would measure nothing.

STUDENT: [INAUDIBLE]?

JACK HARE: Yes, you could have two oppositely wound loops in B phi. Yeah, sure. And yeah, any questions from Colombia? Yeah, Grant?

STUDENT: [INAUDIBLE]?

JACK HARE: We are not including radiation because we don't want to be here all day. Yes, OK, cool, but good question. Yes, there should be radiation. Where would radiation be? Left or right-hand side?

STUDENT: [INAUDIBLE]?

JACK HARE: Yeah, so if we included radiation, it would also be a Poynting flux. It'd be a Poynting flux going outwards instead. And we'd see that because we carefully examine E cross B. And we would see what was going out that way. It's in there. It's in there. I'm really not going to consider it, as you'll see in the next slide. Other questions, yes?

STUDENT: [INAUDIBLE]?

JACK HARE: Absolutely. So you can get current here from other effects. We're considering a very simple model of it. I mean, the bootstrap current you only get if there's some other current driven in the system. You have to have some other current, gradients of that current drive the bootstrap current. So you need to have something else.

We could also have non-inductive current drive and all sorts of clever things. But none of those have really been demonstrated. So we're sticking with this. This also is a very simple model, which allows us to make some progress. Yeah? There was another hand up over there? No? Any other questions? We're going to try and make this slightly more comprehensible in a moment. I've already given you a hint about where some of these terms are going to come from.

STUDENT: Hey, quick question. How did we measure the loop voltage? The toroidal voltage?

JACK HARE: We measured the toroidal voltage in a really dumb way, which is we simply just put a loop of wire around the tokamak above the plasma. And the same loop voltage, which is induced inside the plasma, will also be induced inside this loop of wire.

STUDENT: OK.

JACK HARE: And so we are just using the wire as a proxy bit of plasma because we can conveniently stick electrodes into it, which we can't do with our plasma itself. And so we just have this loop here. That's what measures loop voltage for us. Cool.

OK. So we now have a look at taking apart some of these terms here. The Poynting term-- in reality, this looks like electromagnetic radiation going in. So we could try and work it all out with E and B properly. But we can also just use some circuit theory and see what's going on there.

So a circuit theory would tell us that the power is equal to I times V. And so we're going to decompose that. It's going to be V phi I phi-- so these are both in the toroidal direction. And there could be a poloidal component of this, V theta, I theta. There's unlikely to be a radial component of this. And that's because it's pretty hard to arrange for currents to go radially outwards because you violate the divergence theorem. So let's get rid of that.

Now, it also turns out that, in something that's in steady state-- and so this could be a tokamak or a stellarator, or some other device like that-- this V theta would only come about if we had a change in the toroidal magnetic field in time. Because we're in steady state, our toroidal magnetic field is constant. We've set it with, say, our big superconducting magnets or something like that. That doesn't happen.

So that's 0. And this whole term is 0. So as you might have predicted from the start, the energy that we're pushing in just from a circuit model is related to the voltage that we're dropping around the loop and the current that we're driving here. So this is just treating this as some ohmic resistor, which is dissipating some energy. There's nothing particularly plasma physics inside here. This all makes sense so far?

OK. So then we can write power is going to be equal to still E dot J, which is the ohmic power still. And then we're going to set that equal to B phi I phi. So this is the Poynting flux. I'll just label that ohmic as well.

And then I'm going to take this inductive term to the other side. So I'm going to put it as a minus sign here. And I'm going to write it as a partial derivative not of the integral of the magnetic energy density over the volume, but in terms, again, of circuit quantity. So this is a 1/2 L I phi squared, like that. So I'm running out of space. So where shall I write this?

So this L here is the inductance. And it's a circuit property that probably many of you are familiar with. We're going to define our inductance here as 1 over mu 0 I phi squared integral of B theta squared dV, so integration of the volume.

And if you look at that closely, you realize all I've done is something incredibly tautological. So I've just defined L to include all of this stuff that we still don't know. But this now looks a lot more familiar because we've got L. That contains everything we don't know. But I phi is stuff that we do know because we're measuring it with a recalcitrant. So don't worry about it. I haven't done anything. This is just sleight of hand at the moment. But we will resolve it. What we're going to resolve is try and work out what's going on with L here. Perhaps we can just get rid of it.

OK. I just want to point out the exact value of L doesn't just depend on the current that's flowing inside here. It depends on the distribution of current. And so if I have, for example, a tokamak where the current density as a function of radius looks like this, that has a very different inductance from one where the current density looks like that. And so that L is a measure, in some sense, of the geometry of your system. It actually doesn't care about the strength of the current. It just cares about the distribution of the current.

OK. And the other thing I want to say here is that the definition of L we're using here is sometimes referred to as the energy inductance. There's another version, which is called the flux inductance. And that one is defined as L is equal to the magnetic flux times the current.

These definitions are often similar. They often give similar results. But actually, there are some really subtle differences. And if you want to go plunge into Jackson or some other equivalent electromagnetism textbook, you can have a look at it. But the purposes of this, if you've ever seen this definition of inductance, it's not the one we're using here today. We're using this definition inductance, which is the total magnetic energy squared [INAUDIBLE] total magnetic energy over the current squared instead.

OK. Questions on this before we move on? Oh, we're running out of time, I see. Okey doke.

Working with this equation here. OK. So we want to ask ourselves, what is the time derivative of a 1/2 L I phi squared, like that. Well, we can chain rule this. We'll get L I phi I phi dot. We'll get plus 1/2 I phi squared L dot.

OK. So there's actually two terms here. We can have a change in the magnetic energy because this, again, just still represents the stored magnetic energy. It can change because the amount of current flowing through the system changes. That kind of makes sense. We know if we change the current, we're going to change the magnetic field. So we'll change the magnetic energy density.

But we'll also get a change if we change L. So I gave you this example earlier, that L is related to the current distribution. So say, for example, you have a sawtooth crash or something like that inside your tokamak that redistributes current. That current redistribution is going to drive a change in L. And so that will change the magnetic energy.

And so that magnetic energy will have to be paid for by, for example, the Poynting flux. So they'll have to be some balance here. But for our purposes, we're going to set this all equal to 0. We're going to set it equal to 0 because we're saying that this is 0 because we're in steady state.

And slightly more weaker, we're going to just sort of say, yeah, this is roughly 0 compared to the other things. And the reason we're doing that is we can't set it to exactly 0. Otherwise, we won't be able to sense it with our Rogowski. And we will have completely failed in this task. But we're going to say that it's small enough that our Rogowski, which is super sensitive, can get it. But it's not big enough to actually cause a change to our overall picture here. So we're going to be able to get rid of this term entirely.

So this means that the power balance for our plasma is just going to be external heating is equal to the external-- the externally injected power is going to go purely into ohmic heating our plasma. And so that means that the power coming in is going to be equal to the integral over the volume of the current density over the local conductivity of our plasma, sigma, integrated over the volume here. This is just a rearrangement where we say that J is equal to sigma E, like that. So we replace J dot E with J squared over sigma.

And you will remember that, of course, in general, sigma is a tensor, especially in plasmas. It can be a tensor, which is very much not a multiple of the identity matrix. But here, we're just going to treat it as a scalar. And there can be some subtleties involved in this technique if you think about the tensor nature of this instead.

OK. So now, we've got this squared here. And again, J is just going to be equal to I phi over pi A squared. I probably forgot to mention somewhere that our tokamak has a minor radius of A. But we use this a lot. So this is- - actually, sorry, I'm being slightly vague here.

So this is like the average J here that we're going to work with. And there's going to be some subtlety between going through J squared averaged over the volume to just J average of the volume squared. I'm going to sweep that under the rug. We're just going to do the second one, yeah.

And so that means we end up with an-- let's see here. We also have a volume of our system, which is pi A squared dotted with 2 pi capital R. That's the volume of a torus with minor radius and major radius R.

OK, good. So that means we end up by being able to write our conductivity volume averaged conductivity as equal to I phi squared over the power times by-- this is going to be 2 pi R over pi A squared here. And this I phi squared over the power is just going to be equal to I phi over D phi, like this, still times by 2 R over A squared.

So we can now measure the conductivity of our plasma because we have measured this using our Rogowski. We've measured this using our flux loop. And I sincerely hope we know the size of our tokamak. Otherwise, we've kind of lost the stuff.

So this is a pretty neat result. And the reason it's neat is because of what theory tells us about this scalar conductivity. So theory tells us that the conductivity sigma is something like 2 times 10 to the 4 TE the 3/2 over z, which is we'll talk about it in a moment, but a bit like the atomic charge, but not quite here because I'm going to give it a little z sigma to show it's not quite the z you've been expecting-- and then times by the Coulomb logarithm, that way up.

And this is in units of 1 over ohm meters. And I think, to get this coefficient out front, this is in units of electron volts. So if you look inside the NRL Plasma Formulary, you'll get this nice equation. So this is to do with the Spitzer resistive, which you saw in part 1.

And so there's some kind of nice things in here. We've now measured sigma. The Coulomb logarithm is a very slowly changing function of the plasma parameters because it's in a logarithm. And for something like a tokamak, it's about 10. So you can use 10. You don't really care that much.

This is interesting. So z, or z sigma, is always going to be greater than the true z of your plasma. It's a little bit like z effective, which you may have come across in-- yeah, we did it in the fusion class. It's great. It's a little bit like z effective, in that it also takes into account the fact that you have some impurities inside your plasma. And they increase the number of electrons available. But it's also not z effective.

And if you want to understand what it is, you have to go do some detailed calculations where you take into account what those extra electrons from impurities are doing. However, if you have a perfectly pure plasma, like a pure hydrogen, or deuterium, or tritium, then you do end up with z sigma equals z equals 1. So let's say you've done a really good job and you have got the very nice and pure plasma. You could probably set that equal to 1 or at least 1-ish. Let's not worry about it too much.

So that means, by measuring sigma-- by measuring sigma from your Rogowski and your flux loop and the geometry of your tokamak, you can get out the plasma temperature, which is not a bad measurement for two loops of wire.

The volume average plasma temperature admittedly, which temperatures are going to be dominated by. It's going to be dominated by the cold plasma at the edge or the hot plasma at the core. [INAUDIBLE]?

STUDENT:

JACK HARE: Maybe. Any advances on that?

STUDENT: [INAUDIBLE].

JACK HARE: Yeah, so the hot plasma is much more conductive. We aren't including inductance here. So inductance would force the plasma to the outside because it's going to want to reside on the skin of the conductor, if we've got a non-steady state system.

But in steady state, the inductance doesn't matter. The current has time to soak through the entire conductor. So this would be a pretty good measure of where most of the current is flowing, which is in the core of the plasma. So that's actually pretty cool because this is a really hard thing to measure. I mean, the temperature is something that people invest a huge amount of money in doing.

Now, this isn't a great measurement. These loops are not exactly the most accurate things. And of course, you've got a T to the 3/2, right? And T to the 3/2 means that any small error in sigma is going to be amplified a bit when we measure the temperature. But it is still remarkably easy to do and MC of plasma.

So this, again, applies for any plasma. Let's see. It needs to be steady state. We've derived it with toroidal symmetry. But I don't see any reason why you couldn't try and redo this for like a z-pinch plasma. I think we probably have made some assumptions. I don't think we've made that many assumptions in terms of the axisymmetry of the plasma because the current has to be constant around it. So even if it's a stellarator, you still need one measurement with a Rogowski to get the current through the plasma. So yeah, questions on this? Yeah?

STUDENT: [INAUDIBLE]?

JACK HARE: Yeah, so always when we say things like steady state and things like that, what we should say-- it's like saying large or something. We should come up with a dimensionless parameter which characterizes that. So what dimensionless parameter should we have to enable us to use the steady state assumption, which was dropping this term and actually also this term as well?

We're talking about steady state. We're talking about times. So there should be some sort of timescale that we're measuring. So there should be like an inductive timescale here where we could form a timescale out of this.

STUDENT: [INAUDIBLE].

JACK HARE: Yeah, rate of change of current or the current over the rate of change of the current will give you some timescale. So that would be a good timescale. But that needs to be small. That needs to be much, much less-- not much less than 1 because that has units of time. It needs to be smaller than the pulse length.

So the current must change on the times-- I got it the wrong way around. The current must change on a time scale which is long compared to the current pulse. So it changes slowly, right? So if the current changes sufficiently slowly, you'll be able to use these equations.

And if the change in current is, say, on the order of 10 to the minus 2, then you'll have errors on the order of 10 to the minus 2 or something in here because I think this is mostly linear. And that will go slightly wrong when you try and work out the temperature from sigma because it's been linear up to this point. And then we'll have some slight non-linearity.

But you get the idea. So you with all of these things, whenever we make these assumptions, these assumptions can be justified. There'll be some dimensionless parameter that allows us to justify that assumption. Yeah, other questions? Anything from Colombia?

STUDENT: [INAUDIBLE]?

JACK HARE: Well, we're averaging-- really, what we're averaging is this quantity here, J dot E. So E is going to be practically uniform throughout the entire thing because we're driving this flux of the transformer. Or we're driving this potential as a transformer. That's not going to change very much. I'm not worried about that.

J I agree is peaked in general. For your typical tokamak plasma, it's going to be peaked. So I think it's possible that we are biased towards the hotter temperatures because of that. And that's probably where it comes in, as I was saying, that they are the ones carrying the current. And so we're biased towards measuring their temperature.

So yeah, it's also interesting, like what does it mean to measure an average temperature? Because if you have a small bit of plasma that's very hot but very low density, it has very little thermal energy density. And if you have a large amount of plasma that's cold, if you mix those together, like mixing hot water with cold water, you don't just average the temperature.

You don't say, this is boiling, this is freezing, the average is 50. You would weight it by the number of particles you have. And so yeah, I think all of this is kind of slightly hand-wavy. And again, I think Hutchinson goes into a bit more detail and says that, given the accuracy that you can make these measurements with, some of the subtleties of the theory are not that important because your noise is larger than the inadequacy of your models.

But it's a good question. If you really wanted to do this to measure temperature on like ITER, you might want to spend a little bit more time thinking about. Yeah, cool.

STUDENT: [INAUDIBLE]?

JACK HARE: Yes, absolutely. Yeah, so this is really just a steady state section, yeah. Cool. Yeah. OK. We've still got a little time. So we're going to keep going. Let me clear some space.

So another use of this in MTF-- here, we have a system in which we create a well inside our magnetic pressure, which our plasma sits inside. So if I have a diagram like this, where this is the minor radius of our machine-- actually, just some spatial coordinates. I don't think I've drawn this correctly here.

But we have some magnetic field pressure profile, like this. And we've got a little dip inside it. And that little dip is designed such that the thermal pressure of our plasma, E equals NT, sits inside it. And so the total pressure is constant here. So this means that the system is in pressure balance. It's in equilibrium.

OK. The neat thing about that is, because the magnetic field is the thing that's allowing-- the magnetic field is the thing that's confining the pressure, by measuring the magnetic field in an MTF device, we can indirectly measure the pressure. And you'll remember that a pressure is a pretty key quantity for our fusion because we want to have something that goes like-- what's that nice propaganda formula? We've got like p squared-- anyway, whatever.

I can't remember how they write it on the board in NW-17. But the fusion power is going to go as the pressure squared. So we'd like to know what the pressure is because we want to know what our fusion power is. And we can do that by making very careful measurements of the magnetic field. And we're going to talk about that in detail now.

But we're going to get a little bit lost in the weeds. I'm not going to have a chance to finish it this lecture. So I just want you to focus on the fact that what we're trying to do is measure P, the pressure, from the magnetic field. So again, we're trying to get at some interesting quantities inside the plasma just using a few B-dot probes, which, again, is kind of cool.

OK. So we'll start by looking at this in terms of a force balance. So our steady state MHD equation is J cross B minus the gradient of pressure equals 0, like that. If we insert Maxwell's equation-- one of Maxwell's equations, ignoring the displacement current-- so curl of B equals mu 0 J-- into this equation, we end up with something that we can eventually rewrite in the form minus the gradient of pressure plus B squared over 2 mu 0 plus 1 over mu 0 B dot the gradient of B equals 0.

So this balance equation here-- again, it's equivalent to this one. We've just eliminated J-- has two sets of terms in it. These terms look like pressure gradients. The thermal pressure is definitely a pressure gradient. But the magnetic energy density also looks like a sort of magnetic pressure gradient. So these two terms look similar.

And this term here is the tension force. Or you might want to call it the curvature force because you can see from B dot grad B, there's going to be some term associated with the curvature of the magnetic field.

Now, this is a very hard equation to solve in general. So this is fully three dimensional. If you're trying to solve this in a fully three dimensional object, like a stellarator or even inside a tokamak where you haven't made any assumptions, this is very, very, very hard. So what we're going to do is solve this instead in a cylinder.

So we're going to take our tokamak. We're going to cut. And we're going to unfold it. And we're going to turn it into a long, thin cylinder, like this. You can equivalently attempt to do this by making the aspect ratio of this very, very large. But rather than that, we're just going to do it this way around here.

So in our standard tokamak toroidal geometry, we have the phi coordinates and the theta coordinate, like that, the toroidal and poloidal coordinates. And now, we're going to have the phi coordinates in this direction. So I may occasionally refer it to it as z instead because it's now cylindrical coordinates. And our theta coordinate is still going to wrap around our plasma, like that.

So we're going to look at solutions to this equation in this geometry because it's much simpler. We'll be trying to find ways where we can use our B-dot probes to measure pressure in this geometry. It is possible to use all of these techniques in a full toroidal geometry. You just have to go back and solve this equation instead, much more complicated. Any questions on this before we keep going?

STUDENT: [INAUDIBLE]?

JACK HARE: Yeah, so you would have to solve this to get the [INAUDIBLE]. Or I think in [INAUDIBLE] you still assume axisymmetry. So you can still have a reduced form of this. But it's still very complicated. Yeah, exactly. So again, just for demonstrating how this works intuitively, we're going to use this very simple geometry. But these techniques still apply for real tokamak geometries as well. So don't think that this is only relevant to z-pinches. This is very much relevant to other stuff.

I'm just going to make my z-pinch a little bit shorter so that I can fit in an equation underneath here because one of the things we're going to be interested in measuring is the magnetic field B theta here. And this magnetic field will take some arbitrary values. But as with any other function, we can decompose this. We can do a Fourier decomposition.

And so we're going to do a Fourier decomposition on this magnetic field. And then we're going to talk about different terms in that Fourier decomposition as if they were separate things. They're not separate. They're just our way of representing this magnetic field. But it does allow us intuitively to get at some of the physics term by term.

So if we write this down as a Fourier decomposition, we'll have a DC term, C0 over 2. The 2 here is just our convention when we're doing Fourier transforms. We've seen this before. And then we're going to have a sum over an index M from 1 to infinity. And we're going to have a term CM cosine of M theta plus SM sine of M theta, like this.

So these are different coefficients, which we don't know. But of course, we can find them by doing the Fourier transform of this or the Fourier decomposition of this. And we do that by doing things like-- I'm sure you've seen this before, but just to remind you, this is going to be 1 over pi the integral of 0 to 2 pi of B theta cosine of M theta B theta and similarly for the sine component.

OK. So we have our signal. We can decompose it into these modes. And then we can give these modes names, right? And so we're going to particularly consider the modes M equals 0, which is this term, and the M equals 1 mode here.

These are the lowest order modes in our system. There are modes all the way up to infinity. But these are the ones which are usually the most serious in terms of plasma instabilities. And they're also the ones that provide the most information. So once we've measured our poloidal magnetic field, we've done this decomposition, we can learn an awful lot by studying first the M equals 0 component of the magnetic field, and then the M equals 1 component. If you want to, you can keep extending this analysis all the way up to infinity. But generally, people tend to get bored before they get there. So cool.

In the homework, which has now uploaded, you are asked to sketch these modes. Does anyone want to tell me what shapes these two first modes give us? Yeah?

STUDENT: [INAUDIBLE]

JACK HARE: OK, M equals 0 is a nice circle, like that. So this is a symmetric magnetic field. It doesn't depend on the azimuthal or deployed line here, yeah. What about the M equals 1? Hm?

STUDENT: [INAUDIBLE].

JACK HARE: Oval. OK. Is it an oval like this? Or an oval like this?

STUDENT: [INAUDIBLE].

JACK HARE: Sorry?

STUDENT: [INAUDIBLE].

JACK HARE: Right, OK. It's here. So which oval is it?

STUDENT: [INAUDIBLE].

JACK HARE: It's a trick question. Come on.

STUDENT: [INAUDIBLE].

JACK HARE: It's neither.

STUDENT: [INAUDIBLE].

JACK HARE: Yeah, exactly. So that's the M equals 2. Cool. OK, so you're telling me the M equals 1 mode is shifted. So if I had my initial position here, it's now up like this. Or is it like this? Or is it like this? Which of these is it?

STUDENT: [INAUDIBLE].

JACK HARE: Yes. OK, so it depends on CM and SM. These are effectively telling you which direction it's going to be shifted in. For example, CM is probably in this direction. And SM is in this direction. So your relative contributions of these two is going to tell you how much your plasma has shifted.

So straight away, that is a useful measurement. This measurement straight away is going to tell you if your plasma is moving. Perhaps, if your plasma is moving, it's going to hit the wall. And you can do some active feedback and stop it from doing that. But it turns out we'll learn even more by looking at the M equals 1, mode even for plasmas which are relatively stable.

OK. We are coming up on the end of this session. So I don't want to start on the next section, which is going to be using M equals 0 to measure diamagnetism. But I am happy to take any questions for the next few minutes before we pause today. Any questions? Yes?

STUDENT: [INAUDIBLE]?

JACK HARE: We're totally neglecting-- are we keeping the tension term in there if we're [INAUDIBLE] tension? We neglect the toroidal tension. The only thing that remains is the poloidal tension. These magnetic field lines themselves have curvature, right? So we keep the poloidal tension. We get rid of the toroidal tension.

So this is the same. This is why I said, if you make this very, very large aspect ratio, so the curvature is very small, you also get a similar kind of picture. So that is something that is missing from this model that may or may not be important in a real system. OK, yeah, it's a good question.