[SQUEAKING]

[RUSTLING]

[CLICKING]

JACK HARE: So just a quick recap of what we covered on Tuesday, and then we will continue and look at electron cyclotron emission. So first of all, we had a look at the radiation transport equation. And we wrote this radiation transport equation in terms of the intensity of light-- spectral radiance, to be correct-- coming out of the plasma and how it changes along the path through the plasma.

So we've got a little plasma here and we've got some path going through it that we parameterize by s. The change in this spectral radiance is equal to a quantity j, the emissivity, minus alpha times the spectral radiance. Alpha is the opacity here. And it's convenient to rewrite this equation in terms of a dimensionless parameter tau by dividing through by alpha. And then we get this compact form of equation, di d tau is equal to j from the alpha minus i.

And what we've done is we've defined this parameter tau as being equal to the integral of alpha ds here. And again, this is an indefinite integral. It's only when we start putting limits on this that we know whereabouts we're integrating from and integrating to. And the other thing to say is that this equation is true for all frequencies. Maybe we'll write these in terms of cyclic frequency or rads per second or wavelength or energy, whatever it is.

Just some way that we're measuring the difference in the spectrum here. And this equation then can be solved separately for all the different frequencies in your system. In our assumption here, we're not allowing the frequencies to interact in any way. There's no second harmonic generation. There's no absorption and fluorescence and things like that. And this tau parameter is a nice dimensionless parameter, so of course the first thing we want to do is work out what it means for that dimensionless parameter to be small and what it means for that dimensionless parameter to be large.

In the case where the parameter is small, we find a plasma which we call optically thin, which means the radiation streams through it without being absorbed and there's minimal emission. In the opposite case where tau is greater than one, our plasma is thick and incident radiation is strongly absorbed and the plasma is also strongly emitting. And in fact, we found out that in this thick case, our plasma is going to be so strongly absorbing that it's going to approximate a black body.

The black body has a spectral radiance that looks like mu squared upon c squared times the temperature. So it doesn't depend on any particular properties of your material. It just depends on its temperature. This is the ratty jeans approximation. The full black body is more complicated, but most of the time we can use this ratty jeans approximation. And the neat thing about that is we found that there was a correspondence between this equation here and the optically thick limit and the black body equation, which is Kirchhoff's law.

And Kirchhoff's law lets us set the emissivity over the opacity equal to the frequency squared over the speed of light squared times the temperature. This effectively tells us that the emissivity and the opacity are linked together. And so we've calculated, one we automatically have a calculation of the other. And simultaneously, if we have a region with high emission at some frequency, that's also a region with high absorption as well. So these two quantities are linked.

OK. And then we talked about how to actually calculate j, and therefore alpha, so that we can use this radiation transport equation to see how plasma allows electromagnetic radiation to propagate through it. And we did a sort of pictorial representation and Tucker sent around a very good web demo that you can play with to play with that pictorial representation of y accelerating charges radiate. And we also wrote down a very long equation and then immediately dropped the near field term.

But the remaining term, the far field electromagnetic radiation from moving charge, was equal to something with a bunch of constants and a few new variables related to where we are with respect to the particle. But the key part of it was that we started getting these cross products here that have within them some properties of the particle itself, namely its velocity crossed with its acceleration. So it's clear that in order to estimate the radiation from a moving charge, we're going to have to know these two properties.

And so the recipe that we're going to follow is we're going to firstly calculate v and v dot. And we're going to do that with the equation of motion. And then secondly, now we're given the radiation from just a single charge. We then need to integrate this over the distribution function f of v v3v to get out the emission from all of the charges in the plasma together. And that is going to give us the emissivity at a single point. And then of course, because they're linked, also the opacity. So this is what we're going after. Any questions on that before we move on to an example of this? Yes?

STUDENT: Can you explain again how we got the near field in the electric field?

JACK HARE: Yeah. So the near field term-- which I haven't written down here. The near field term goes as 1 upon r squared whereas the far field term goes as 1 upon r. So when you calculate s is equal to e cross b-- [INAUDIBLE]. The magnetic field is going to have the same radial dependence, the same drop off in distance as the electric field. And so the Poynting vector is going to go as one upon r to the 4. And that means that if you go far enough away from the particle, that electric fields and therefore the radiation from it is going to drop off faster than r squared.

And so you're not going to see it from any reasonable distance away. In fact, the distance you need to be away is a few wavelengths of the radiation, and so that's usually quite short. So we don't have to worry about this. Effectively what your electric field looks like-- and I had a little chat with Sean about this after the class, so hopefully I remember it. The electric field that you as an observer would see at some function of distance has 1 over r component, which is the far field, a much more steeply dropping 1 over r to the 4 component, which is near field.

And so you don't, as an observer, get to see the difference between these two. They look the same. What you'll end up seeing is just some field that looks like this. The near field component won't show up at any sensible distance away from the plasma so we never need to-- we do not need to worry about it in plasma physics as far as I know. But maybe there is some special case you do need to think about it. We're going to drop it in these lectures. Yeah, Shawn?

STUDENT: It makes sense that we don't need the near field for trying to observe the radiation that's emitted, but if you're trying to couple the emitted radiation back to the plasma for the absorption, shouldn't the near field contribute to that because you do have particles that are near the emitting particle?

JACK HARE: OK, so to the benefit of people online, the question is, doesn't the near field perturb the plasma? I don't know. I could handwave and say something about the y lengths. The plasma is very good at shielding out electric fields. Right? And so maybe it will depend on the wavelength of your radiation compared to the Debye length of that.

But I'm just spitballing here. I don't know the answer to that. Yeah. Thank you. You're going to love my next-- I went to talk to Paul Bernoulli and I'm like, hey, Paul, is this right? And he's like, yeah, that sounds right. So you're going to love my next approximation and a little bit, so. OK. Any other questions? Anything online? OK. Let's keep going. So we're going to be looking at electron cyclotron emission or ECE.

This is a key diagnostic in magnetically confined flasks. This is a very, very important diagnostic, which is why we're going to spend a significant amount of time on it. This diagnostic works on the basis that we have magnetic field lines inside our plasma and we have particles, electrons, which are spiraling around those magnetic field lines. There is also ion cyclotron emission as well, which we are not going to cover here. I believe that's because it has a much harder time propagating out of the plasma.

We'll discuss a little bit about that later on. At the moment, we're just focusing on the electrons here. And so what we want to do, as we said before, is write down an equation of motion. And that equation of motion is going to be simply that the time rate of change of the momentum of our electron is going to be equal to the force on our electron. So this is ev cross B here. We're assuming in this formula that there are no electric fields.

So we're just dealing with the particles gyrating the magnetic fields. You've seen this before. You've solved this before. The one thing that Hutchinson puts inside here that we didn't have previously is this factor gamma. So this is the relativistic gamma here, and that is 1 minus v squared upon c squared minus 1/2. The gamma is always greater than 1.

And this is a factor that effectively in this case, you want to think about it as increasing the rest mass of our particle, and this is because our particles tend to be moving in a MCF plasma where the electrons are at 10 keV or 20 keV. That's a significant fraction of their rest mass, and so there will be some relativistic effects on top of this. So in this case here, we know that this-- because we've solved this equation before, we know that this electron is going to oscillate.

I guess I can be precise. I can say it's going to gyrate at a frequency omega c, the cyclotron frequency. And this is just ever so slightly different from our normal gyrofrequency because it's different by a factor of gamma here. So the gamma changes a little bit, and so this is just e0 over electron mass times my gamma. Again, if you just keep the gamma sticking with the electron mass you won't go too badly inside this.

Because the electron is gyrating at this frequency, it is reasonable to believe that there is an electric field that generates which is also gyrating at that frequency. So this means we expect to have radiation at this frequency as well. And that's true. We do have radiation at frequency. But the thing I've been struggling to find an intuitive explanation for-- I went to go talk to Dr. Bernoulli about this just before this class-- is that we don't just get radiation at this frequency.

When you go into the mathematics of it, you find out that you get radiation at integer multiples of that, at harmonics of that. So we also have this factor of m here where m is some natural number. Now mathematically, I can tell you where this comes from. It comes from, for example, Hutchinson's equation 5.2.7. What this is is there's an exponential of some horrific quantity and Hutchinson and everyone else says, well, fortunately there's a really great trigonometric identity that converts the exponential of this horrific quantity into a sum over an infinite number of Bessel functions.

And those Bessel functions are all oscillating at harmonics here, and so therefore, you've got all these harmonics. Right? So mathematically it all makes sense. No one is upset about that. The trouble is that it doesn't make any physical sense. As far as I can tell, there's no reason why electrons should have to care about Bessel functions any more than I do. So something is going on physically, and so I went to go talk to Paul and ask about it.

And the thing we came up with is whenever you have harmonics in a system, in some sort of simple physical system, it's usually due to some nonlinearity. So we need a nonlinearity in our system where the system acts back upon itself in order to do so. And what our best guess is that the electric field that you produce at the fundamental frequency here, omega C, is going to go back and couple back into the electron equation of motion. So effectively the electron is going to be forcing itself slightly out of phase, and that is going to give rise to these other harmonics here.

So this is sort of a way of trying to make the system nonlinear because, of course, this electric field depends on v dot and v, and that's the thing we're solving for over here. So we think that it's a nonlinearity. So we think that these harmonics are due to nonlinearity. We looked at Sticks. We looked at Chen. We looked at [INAUDIBLE] piece. We couldn't find anyone who was willing to tell us what the answer is. So if anyone has any insight into why it's the case and if it's better than just waving my hands and saying nonlinearity, then please go for it.

But the alternative is you just have to treat this as a mathematical exercise and churn through all the algebra. And I assure you if you go and have a look at this, you'll see that it really is an awful lot of algebra. So I'm trying to keep the mathematics out of this as much as possible. But we're going to do this properly, you find out-- and this is equation 5.2.16.

You're going to get a sense of peaks in your emission, and those peaks are going to be at frequencies omega n, which are equal to m omega c, and they're actually going to be modified for future relativity. They're going to be 1 minus beta squared to the 1/2-- I'm going to explain what beta is in a moment-- 1 minus beta parallel theta. I'm going to explain all of these terms in a moment here. So we've still got this m inside here. It's just that there's also-- m is the control number.

It's just that we also have some modification due to relativistic effects here. So what are these betas? Some of you have come across these before. This is absolutely not the plasma beta. It has nothing to do with the plasma beta. The beta is defined as b upon c. So it's the ratio of the particle's velocity to the speed of light. And we also have theta parallel, which is equal to b parallel upon c. That parallel is with respect to the magnetic field, c0 here. I don't think it comes up, but you can also define beta perpendicular if you want to as well. OK. There's something else I wanted to mention in all this. Hm?

STUDENT: [INAUDIBLE].

JACK HARE: What?

STUDENT: [INAUDIBLE].

JACK HARE: Oh. That was the one. Thank you. Yes, I forgot to build a diagram early on. So theta, in this case, is the same theta that we had when we're talking about x mode and o mode and all that sort of stuff. It's the angle between the observer and the magnetic field here. So this is the term that is largest when you're looking along the electron trajectory towards you and smallest when you're looking perpendicular. And so you can see straight away this is a signed quantity.

It's going to end up looking a little bit like a Doppler shift. So that is the Doppler shift that you were expecting to see. But it's relativistic Doppler shift here. So for a single particle, just some electron moving on a magnetic field, we would have an emissivity that looked like a series of these peaks. OK? And these are delta functions in the theory. So you just have these very sharp emission lines and they're evenly spaced.

STUDENT: Are heights the same?

JACK HARE: The heights are not the same. If you want to get the heights, you have to go look at all those nasty Bessel functions, which I'm hiding from you. But if you want the height, the argument of the n-th Bessel function. And the thing that goes into the Bessel function is some complicated factor of all these other things here. So again, I'm not going to explain it, but they do not have to have the same height.

I've drawn them a little bit like this. And if that particle is moving particularly fast, then all of these will be shifted in one way. If you're looking at it from a certain angle, we shift it another way. But they will all be shifted in the same way because this is just for a single particle. OK. Any questions on that before we find out what happens when we have more than one electron? [INAUDIBLE].

OK. So the next thing we do is we look at many particles. Many electrons. And for this, it's useful to split our three dimensional distribution function f or b the vector v3v into a distribution function in beta parallel and beta perpendicular. It would be like this. So this is looking at particles which are streaming along the magnetic wall. This is looking at the component of velocity along the magnetic field and the component of velocity perpendicular to magnetic field.

That's because this problem is symmetric. It doesn't matter how I rotate my axes around the magnetic field. The velocity overall is going to be the same. So I don't have to actually deal with three dimensions here. I've just turned it into a two dimensional thing. And if you do this integration, you'll find that your betas are different from different particles, right? Remember your Maxwellian. I just take a one dimensional slice through it, it has lots of different velocities available.

And so that means some particles are going to be going with a velocity, which changes the sign of this to be bigger than one and some part of this denominator to be bigger than one, some to make it smaller one. So we can see that we're going to have broadening of these peaks, and this is the Doppler broadening that we expect. So let's just draw a single peak. It was a delta function at m omega c. It doesn't matter which harmonic it is for our purposes.

This delta function is now going to be broadened out. So the delta function was always rather unscientific, unphysical, and so it's going to be broadened out by these natural broadening processes. And there's two of them. There's this top process and the bottom process here. I'll just write them out again down here. So we have a factor of 1 minus beta squared to the half over 1 minus beta parallel cosine theta. So let's start with the numerator-- the denominator here, the term on the bottom.

For some symmetric Maxwellian like distribution function, what does this term do to this line? How does it broaden it? Is it symmetric, asymmetric? Imagine we're looking at a fixed angle because we are. We don't have eyes all the way around the tokamak, so you can just fix theta to be some angle. Pick an angle where theta isn't zero or otherwise it gets very boring. What will this term 1 over beta-- 1 minus beta parallel do? Yeah?

STUDENT: It'll broaden it symmetrically.

JACK HARE: Broaden it symmetrically, right? This is going to look like-- say I look along the field line. Cos theta equals what? Theta equals zero. This is going to be 1 plus or minus b upon c. The plus or minus there is saying for every particle going along the field line towards me, there's another particle going away to the other side.

And this is also going to be very small because the particles are not-- we're not dealing with actually ultrarelativistic particles here. And so this is going to look 1 plus or minus delta, and that means that it's going to broaden this peak symmetrically. What about this term at the top? So you can call this the Doppler term. What does this term on the top do? Anyone online? Is it always bigger than 1, always less than 1, sometimes bigger than 1, sometimes less than 1? Yes?

STUDENT: It's always going to be less than 1 since theta squared is always positive.

JACK HARE: So this is always less than 1 because, as you say, theta squared is always positive. So it doesn't care about the sine of v. So it's going to be asymmetric here. So this is the relativistic mass term. And that will broaden the peak further, but it will only broaden it in one direction like that. So your peaks end up looking like this.

If you want to know which of these two terms is larger, then you're asking the question, when is beta cosine-- beta parallel cosine theta greater than theta squared? That's the case when cosine theta is greater than dt upon c, the thermal velocity of the electrons. And that's the case for beta less than but almost up to pi upon 2. For almost all angles, this Doppler term dominates.

STUDENT: [INAUDIBLE]

JACK HARE: Yes. You're right.

JACK HARE: Almost all theta apart from theta exactly equal to pi upon 2, the Doppler term dominates. So you'd see mostly symmetric lines with some small asymmetric shift. Apart from if you're exactly pi upon 2, which is a place you're often at when you're looking inside a tokamak. So this could happen quite a lot, in which case you're going to see a line that is very asymmetric indeed. OK. Questions on this? Yes?

STUDENT: What is the j in the single particle graph?

JACK HARE: That is the emissivity. The thing we're trying to calculate. Hutchinson writes it in a sort of differential form, which is nice for keeping track of the units. I'm just going to call it j. Sometimes there's some steradians and stuff lying around that you have to deal with. But roughly this corresponds to your-- this corresponds to your intuition of brightness. Right? This is how bright the plasma is with this magnetic field and this velocity for this single particle. And then of course, all of these lines would be broadened by these terms. OK. Yeah?

STUDENT: I'm a little confused in the picture with the asymmetric peak.

JACK HARE: Yes.

STUDENT: Right? The orange equation on the right is saying that slower particles will have a-- like less reduced. I'm just trying to think about the way that you drew the--

JACK HARE: The relativistic mass correction, or the--

STUDENT: Yeah, yeah.

JACK HARE: Well, actually this part, this is due to fast particles.

STUDENT: OK. Yeah.

JACK HARE: They're so fast, they're so heavy, their gyrofrequency has gone down.

STUDENT: Oh, OK.

JACK HARE: Yeah. Good question. Yeah. Yeah, you're right. That's really counterintuitive because actually for the Doppler shift, the particles down here are the fast ones. That's true in both cases. Yeah. It's the fast ones in both cases. Yeah. These ones up here are the fastest, but. Any questions online? OK.

So we use this in practice by having a very good idea of what the magnetic field profile is inside our plasma. And a good example of a plasma where we have a very good idea of magnetic field profile is a tokamak, or maybe something like a stellarator. Something that's very low beta. So I'll show you what we do in that situation. So this tends to be used in magnetic confinement fusion with beta.

That's the thermal pressure over the magnetic pressure-- I don't make the rules-- is much less than 1. Nothing to do with the other theta. And this condition effectively means that no matter what your plasma is doing, your magnetic field is pretty much the same. So you know your magnetic field profile. And a good example of this would be inside something like a tokamak where we have an axis of symmetry here and I have some cross-section of my tokamak like this, and maybe the magnetic field in this tokamak is going to be dropping off as 1 upon r.

That just comes from Ampere's law, the fact that you have current flowing up through the magnets in the middle. It can hardly be anything other than 1 upon r, and that's true. And so the interesting thing here is that different regions of the plasma have different magnetic fields. Maybe some of you can see where this is going.

So we can call these, for example, r3, r1, and r2. Make sense? r1. Three different radii here. And what we're going to do is we're going to observe this plasma with some microwave antenna, some sort of horn. That's going to collect the electron cyclotron emission, which is in this gigahertz range here, and that electron cyclotron emission is going to stream out in every direction. But one of the directions it's going to stream out into is our horn, so hopefully we can catch it.

And then what we'll do with the signal that comes into this is we will split it a load of different times. I'm just going to draw n times here into n separate channels. We'll have a little bandpass filter on each of these which selects out for different frequencies. This is bandpass. And then each of these will go on to some detector.

And that detector will measure the power as a function of time contained within this frequency band here. And so this is effectively a time resolved spectrometer kind of system. OK. Why is this helpful? It's because, as you said before, B of r is well known. We think of the intensity that's detected on our detector over here. We're going to have some emission that's coming from r1. r1 has the highest magnetic field, therefore, it has the highest frequencies.

So its first emission is going to be, for example, at this frequency here, which is omega 1, like that. This is the frequency corresponding to position r1 and it's the first harmonic of it. In the same way, r2 corresponds to the middle of the plasma. The magnetic field is lower, the frequency is lower. Omega 2 and r3 corresponds to the lowest magnetic field. There's no good reason for these to be the same in general, aside from [INAUDIBLE]. But zigs around a little bit, so it's clear that they don't have to be [INAUDIBLE].

But of course, as well as that, you're also going to get the harmonics. You're going to get a harmonic. This is why I should have drawn these much closer together. It would have made it much less clear at 2 omega 3 and at 2 omega 2 and at 2 omega 1. And then of course at the third harmonic as well, now you can see, oh, dear, these are beginning to overlap. This is getting a little bit tricky. So maybe I shouldn't have done it like this. But they will. Eventually they will overlap. With high enough harmonics, you're going to have different harmonics all interacting with each other. So these are the third harmonics up here.

But if you focus just down in this region, for example, every frequency corresponds to a specific magnetic field, which corresponds to a specific radial location. So if you're seeing emission at this frequency, you know that an emission is coming from this bit of the plasma. So this is extremely powerful because you have, as well as the spectral resolution and time resolution, you now have spatial resolution. OK. Questions on that? Yeah? Yes, question on that?

STUDENT: Hi there, professor. So the spatial-- you can get spatial resolution because of the decreasing r field, but is there not going to be any overlap with the earlier harmonics in those? Couldn't you be closer to 1, then they'll overlap and--

JACK HARE: Depending on where your tokamak sits out here that gives you the gradient in magnetic field-- and that gradient in magnetic field is going to determine the spread of frequencies that you have that you're observing from. If your frequencies are closer together because, for example, your tokamak is further out in the 1 over r field, it's a large aspect ratio of tokamak, if those frequencies are closer together, you'll simply have to have a better spectrometer.

There, of course, comes a point where all of the broadening that we talked about before is going to screw you over a little bit, and so that's going to fundamentally limit the spatial resolution you have. But broadly, you can take this signal and maybe deconvolve it and say, I've got some spatial resolution here. Yeah. Really, your spatial resolution is also going to be limited by the number of these detectors that you can field. So, like, people will do a 12 channel system, and then they'll effectively be able to resolve 12 different spatial locations across the plasma.

STUDENT: And the radiation field--

JACK HARE: You're breaking up quite badly. I don't know if it's your microphone or what, but I can't really hear you very well.

STUDENT: Can you hear me now?

JACK HARE: Slightly better. Yeah, go for it.

STUDENT: The detectors, the emissions, they're not happening isotropically, are they? Are they happening all over the tokamak, or?

JACK HARE: They're happening in every direction. They're not happening isotropically, and we haven't derived the pattern that they make. And it's in Hutchinson as well. And you haven't asked about the polarization yet, and they've got different polarizations too. So it's definitely not isotropic, but you should definitely imagine that these are going out in lots of different directions. And if you're one of these sorts of people who likes putting things on the high field side of your tokamak, then you could also measure this just as well from that side as well.

STUDENT: So you can put the detectors anywhere in the tokamak?

JACK HARE: You can put the detectors anywhere. We tend to-- yes. Yes. You can put the detectors anywhere. Obviously this line of sight works particularly well because you're looking along the gradient in magnetic field. If you look from above, it's a little bit more complicated. But there was a very good paper just published on TCV, the tokamak at UTFL, where they had vertical electron cyclotron emission. Apparently it helps with some issues as well. So just saying, you can put it anywhere. It would definitely make more sense to put it in the direction or against the direction of the magnetic field gradients because you have this advantage that all the different frequencies then very clearly correspond to different places in the plasma.

STUDENT: [INAUDIBLE] plane?

JACK HARE: Yes. I think so. OK. Do you have a question?

STUDENT: Yeah. I was wondering for the higher harmonics, you had the intensity increasing. Would the intensity of the higher harmonics be larger?

JACK HARE: So I haven't really talked too much about what the intensity of the harmonics is. There are formulas to work it out. I think in general they drop off, but we're about to talk about opacity effects on these lines. And so what actually happens is the lower harmonics are absorbed more and the higher harmonics aren't.

And so at least, I think, Hutchinson has a figure in his textbook where the second harmonics are the strongest and the third are actually stronger than the first and things like that. So it's complicated because there's a lot of factors at play in terms of what the final intensity is. Yeah. So here I'm just drawing them as different heights to show that they don't have to all be the same. But I don't mean anything by the specific heights that I chose for anything. Yeah. Shawn?

STUDENT: On tokamaks people tend to neglect the poloidal field effect on the total magnetic field.

JACK HARE: Yeah. So the question is, do you neglect the poloidal field? You do neglect it. Of course, you don't really neglect it. You would take it into account. But in this simple sketch here, I don't need to put the poloidal magnetic field in. But you could imagine that if you did know what the poloidal field was because you've got all your wonderful magnetic diagnostics and you can do a magnetic field reconstruction, then you know what the magnetic field is at every point in your tokamak and then you know what the frequency is and so then you can back this back out again. So it is possible to have poloidal field and still do this technique. It's just the sketch I'm giving you here doesn't include it.

STUDENT: Then is it more complicated to get the theta, because it's now the magnetic field is not just straight around the torus? It has some--

JACK HARE: Well, you're still going to be-- even if you've got a twisted magnetic field line, if you're looking from the outside in it's still going to be at 90 degrees. It's just that it's going to be 90 degrees like that, not 90 degrees like this, because your magnetic field might be tilted up or down. It's still 90 degrees. If you start looking at another angle, not at 90 degrees, then, of course, you have to think about the geometry a lot more carefully.

STUDENT: OK.

JACK HARE: So if you're looking from-- yeah. If you're looking tangentially, right? If you're doing tangential viewing of ECE, electron cyclotron, like trying to look along the field lines, then the poloidal field will make a bigger difference. Here it doesn't make a huge difference.

STUDENT: Thank you.

JACK HARE: Cool. Lots of questions. OK, we'll go with Grant.

STUDENT: I'm just a little confused on the spatial resolution. So is the idea we know the magnetic field so we can back out where the emission came from? Is that?

JACK HARE: Yes.

STUDENT: But that also relies on us knowing the velocity of the electrons to know their rest mass-- or not the rest mass. Their relativistic mass, right?

JACK HARE: What you're saying is these peaks could be sufficiently broadened by these mechanisms that they overlap. Right? And of course, effectively, they will always overlap because I've just gone three spatial locations, but there's an infinite number of spatial locations. And so you're absolutely right. That broadening sets a limit on your frequency resolution. You're effectively convolved your signal with that asymmetric peak. And because you've set a limit on the frequency resolution, it sets a limit on how well you can resolve different magnetic field regions. And because of that, it sets a limit on how well you can resolve different spatial regions.

STUDENT: Sure. But even just for a single particle, so there wouldn't be the broadening effect. If I gave you a set of peaks and a magnetic field profile, would you be able to figure out where those peaks came from? Would you also need to know what energy electron was-- like the mass was different to figure out?

JACK HARE: I think this broadening is very small And so this is not a huge change to your signal. And then to me it seems like a deconvolution problem, which, they're hard, but not impossible to deconvolve signals.

STUDENT: OK. Maybe I'm overcomplicating then.

JACK HARE: I think you're thinking about reasonable complications, but not ones that are the biggest problem with this diagnostic. We're about to go on to the fact that most of these lines are optically thick and so you don't see them anyway. So yeah. But OK. Other questions. Yeah?

STUDENT: What's the wavelength and frequency of these?

JACK HARE: What it would be a typical frame-- yeah, like is there any--

STUDENT: Picking up other--

JACK HARE: Like 100 gigahertz or so.

STUDENT: OK.

JACK HARE: It depends strongly on the magnetic field. So in something like [INAUDIBLE] c model spark, it will be much higher than it is on other devices.

STUDENT: If you get some other kind of radiation that kind of falls in the same spectrum, then--

JACK HARE: We will actually talk about that, yes, when we talk about the accessibility condition. This frequency may be close to other natural frequencies in the plasma, and that may modify what we can observe. So good question. Yeah. OK. Any other questions online or in the room? OK, let's keep going.

OK. So because you know the emissivity or you can know the emissivity-- hopefully you believe me that it is possible to calculate with lots of algebra, the emissivity. We can also calculate alpha. That means that you can ask yourself what is the optical depth of one of these waves propagating through this plasma. You can solve tau is equal to alpha dx. And what you will find if you do that is that for the lowest harmonics m equals 1, m equals 2, often but not always, that tau is much greater than 1. So for these lower frequency harmonics, you are optically thin.

Now that's bad because you can no longer use detailed structure inside these in order to work out clever things about your plasma. But it's actually incredibly useful because what we find in practice is that the intensity at these different frequencies is now just going to be the black body intensity. Right?

So the intensity i of mu is just equal to nu squared upon c squared times t. You'll notice I continuously switch between omega and nu, and hope you realize that omega is just 2 pi nu and are not too confused by this. Hutchinson uses nu because most diagnostics work in Hertz, or at least we think about things in Hertz when we're digitizing them rather than omega. But to be honest, I think you should be comfortable enough to switch between these unit systems.

OK. So the neat thing about this is that you will then have, for this region of the spectra, a signal that maybe looks like this. And again, this is frequency here and intensity here. And that means that at each frequency--what colors did I use? Actually, I will go like that. This is just the black body curve. I've actually gone and done the full thing, not just the nu squared. Do we want to say that? No, don't say that. Ignore that. That's not the scope.

Right. You'll get a spectrum that looks like this. And that means for every frequency, you can measure the intensity. And so therefore for every frequency, you know the black body temperature that corresponds to it. So we have omega. This frequency codes for a very specific magnetic field, which codes for a very specific. Also the plasma. And this intensity divided by nu squared times by c squared codes for a very specific temperature. And so from this, we get out temperature as a function of radius.

So we can convert this spectrum into a plot of temperature inside our tokamak. Again, we are measuring in this toy example that these three frequencies are this. But we can, for example, get out of the peak temperature profile.

So this is extremely powerful. If we look at just the optically thick lines down at low frequencies, the intensity of those optically thick lines depends only on the plasma temperature and not on any other complicated physics. Because of that, because we have this very strong mapping in frequency of magnetic fields position, we can also take the intensity map at the temperature and we can get out the temperature as a function of position. So this is why ECE is so incredibly important. Because of course, as I said before, it's also time resolved.

You get temperature as a function of r and time. And then tokamaks, because of the existence of flux surfaces, once you measure the temperature along the line, you also know the temperature along all the flux surfaces. So from your magnetic reconstruction, you've effectively got the temperature everywhere inside your plasma. OK. This is a little bit hard to get your head around, so I'll pause there, let you think, and ask any questions.

STUDENT: If this is such a great property, why does Thomson scattering exist? What's the issue here?

JACK HARE: Thomson scattering can use as many more plasmas than electron cyclotron. So even in the nonexistence of tokamaks, we would still have Thomson scattering. Why does Thomson scattering exist? Why is Thomson scattering used in tokamaks? Well, this requires plasmas of sufficient temperature to make high enough frequencies to be detected.

So it will not work in the edge, and edge Thomson scattering is a big, important tokamak. This Thomson scattering is defined along a chord by your laser beam. And so you can get truly local measurements of temperature, whereas this diagnostic is kind of a little bit more global. We'll talk in a moment about fluctuations and the fact that this default diagnostic is not good at seeing small temperature fluctuations.

Feel like that's enough. There are probably other good reasons. Oh, we'll talk about accessibility in a moment. So the fact that, in fact, sometimes it's very hard to do electron cyclotron emission because your waves as opposed to exiting the plasma and hitting your detector here get cut off and get reflected by density, regions above the critical density inside the plasma. So we'll talk about accessibility. Yeah. I think those are good reasons. Yeah?

STUDENT: Do we do anything with the higher harmonics, or?

JACK HARE: In this course, I'm no longer talking about the higher harmonics because it gets quite complicated. If you go to Hutchinson you can apparently find out things about high energy electrons in the tail of your distribution from the high harmonics. And in general, your emissivity contains lots of information about the distribution function unless you end up in this optically thick regime.

And so if you look at the shape of the high harmonic lines, you might be able to say something clever about the electron distribution function. So if you want to study runaway electrons or something like that, you can do that. But I had to cut something out of course and that's what I cut out here. But yeah, go to Hutchinson's book if you want to know more. Yeah. Other questions? Anything online? Gone so quiet I can hear the cryopump compressor.

OK. Good. Well, actually, yeah, the only thing I will say here is that this signal will not, of course, be nice and smooth. It'll be very noisy. And the point is that in general noise limits your measurements of delta te over te. These are the temperature fluctuations. And the reason why you might be very interested in measuring temperature fluctuations is because, as we discussed I think a little bit ago with reflectometry, if you have fluctuations in density and temperature driven by turbulence, that will cause anomalous transport of particles and heat out of your plasma and limits its efficiency as a fusion reactor.

So a ongoing problem is how to measure these density fluctuations, temperature fluctuations so that we can understand turbulence better. And so one thing we'll now talk about is correlation electron cyclotron emission, and that is a technique which is specifically designed to measure very small temperature fluctuations inside. Is that what I'm going to talk or am I going to talk about accessibility? I'm going to talk about accessibility. And all that run up.

STUDENT: What is the noise in that [INAUDIBLE]?

I'm used to thinking of thermal noise [INAUDIBLE] like you're looking at a big [INAUDIBLE].

JACK HARE: Obviously there's fluctuations in the temperature, which will produce a different intensity, but there are also thermal fluctuations in the number of photons being produced, this sort of shot noise type thing. There's also noise from the fact that your detectors are in a noisy environment and they're not perfect and things like that.

So yeah, we'll talk a little bit more about the sources of noise when we talk about correlation electron cyclotron emission, which, again, I thought I was going to do now, but it turns out I'm doing accessibility. So we'll get back to that, but yeah, think about just in general, whenever we make a measurement, we're going to have some noise of it. In this case, it just is very difficult to make-- these detectors are very difficult to work with, and at 100 gigahertz or so you tend to have a lot of noise. Yeah. Any other questions? OK.

OK. So the question of accessibility can be phrased as a question of can the electron cyclotron emission reach the detector? If it can't, we can't detect it, so all of this beautiful technique is useless. Now in general, when we're doing something with an MCS device, we're going to be measuring perpendicular to the magnetic field. That's just a simple geometric constraint that our device, whether it's a stellarator or a tokamak, looks like this, and therefore, you'll tend to have magnets that look like this.

And so the only gaps are to look in this direction here. You, of course, can design other geometries. I'm just using this as an example here. Because the magnetic field is predominantly the toroidal magnetic field, our angle theta is equal to pi upon 2. So what modes are propagating-- what modes is the electromagnetic radiation, the ECE radiation propagating as inside our plasma? We discussed four modes. X mode, and we'll start with the O mode. It's much easier. The other mode is easy.

So once again, we think about our plasma as a circular cross-section and a magnetic field, which folds up as roughly 1 upon r. And so what I'm going to plot is a drawing with the r-axis and the frequency of different modes within the plasma here. And I'll plot this r axis. I'll have zero. This is going to be r over r0. r0 is the coordinate to the middle of our circular cross-section.

And so this is the central plasma one here. That's zero. Call that 2. The plasma is going to have some boundaries here and here. Very tight aspect ratio you're talking about. OK. And so for the O mode, we know that the spatial relationship is just n squared equals 1 over omega p squared upon omega squared, like that. If we think about a typical tokamak, we know that the density is going to be peaked.

So we're going to assume that we have a tokamak where we've achieved good particle confinement in the core. We know the density has to go to zero at the edge anyway, otherwise there wouldn't be the end of the plasma. And so we're going to assume that we've got some density profile that's a little bit like this. And that means that at the edges of the plasma, the plasma frequency, omega p, is zero. Remember the plasma frequency goes as any squared plus a few other constants.

And that means in the middle it's going to peak. It's going to be the highest possible frequency. That plasma frequency is important because waves below the plasma frequency are evanescent. So if you have a wave that's propagating in this region of frequency position space, it's going to be evanescent. It won't propagate. It won't reach us. OK. What about the cyclotron harmonics? Well, they go as magnetic fields. They also go as 1 upon r. So this is maybe the first harmonic, the second harmonic, the third harmonic. Let me just label these a bit better. Omega, 2 omega, 3 omega, like that.

So in the picture I've drawn right now, if I have a cyclotron frequency, they're here born at this point near the far edge of the plasma, and I follow the trajectory of that wave outwards, I don't pass through the evanescent region. This wave is going to make it to my detector. Of course, it may also be absorbed as we talked about and become black body, but the general idea is that this wave could propagate out without absorption. And certainly the second and third harmonics could all propagate out.

But what if my plasma was a little bit denser than this? What if I increase the density in the core a little bit more? Then I could end up in a scenario where there is now an evanescent region. And in fact, almost all of this wave will get reflected back to the high field side to the r towards 0 side, and only this region will propagate. So this stuff will be reflected and only this. And the stuff that's born inside here can't even be born. There's no mode in which for that electromagnetic radiation to be emitted. OK. So this can still propagate.

So that means that for a sufficiently high density, if I draw my tokamak cross-section again, there is a region where omega capital omega is less than omega p-- that's this region here-- where I will not get electron cyclotron emission from. So this effectively says at high density what's ECE.

And you could imagine the density could even get so high it blocks the second harmonic or something like that. So this is a challenge if you're designing a tokamak and you want to have some magnetic field and some density. And obviously, those are not free parameters because we have data limits and things like that. If you're designing some tokamak, you might end up in a situation where you don't have accessibility to this first harmonic and so you can't use it as a diagnostic like you wanted to.

STUDENT: Just to ask again, what happens when it's emitted under the plasma frequency?

JACK HARE: I don't think it can be emitted. There's no mode for it to go into.

STUDENT: [INAUDIBLE] so you have you still have orbiting electrons. For example, the far field would get destroyed by the plasma effects or [INAUDIBLE] get reflected back [INAUDIBLE].

JACK HARE: It's a good question. For those online, the question is, what actually happens to the electric fields generated by gyrating particles in a single particle picture in this region? What happens if you try and launch a wave evanescently?

Well, you were telling me the other day that if you're launching your waves from too far away from the plasma, they evanescently decay until they couple into the plasma, where there is a mode that they can fulfill. So what happens to the energy that is lost? Some of the energy is coupled into the plasma. What happens to the rest of the energy from when you do low field side launch? Is it reflected back into the antenna?

STUDENT: Well, I think it's lost in the scrape off layer to some extent, but reflections also.

JACK HARE: Well, so you're saying it's collisionless, but maybe this is one of those places where a small collisionality comes to the rescue and saves us. Right? So there's lots of cases where you're like, if it's collisionless, physics breaks. That probably means that something happens such that the collisionality becomes high enough to save the day again.

What am I thinking of specifically? Back to the sheaths, actually, where you have-- you say my sheath is collisionless and then you violate all sorts of fundamental laws of physics. And then you're like, well, perhaps it's slightly collisional, but not enough that I care about it, but just enough to save the day. So I don't know. That's a great question. I'll have a think about it. I'm probably not going to find the solution to it, but if anyone else does, feel free to put along the answer or something for that.

STUDENT: But radiation does need to be admitted or emitted at least, right? Because it's still a moving charge?

JACK HARE: Right. So the question is like, does the particle lose energy? Does it just decide not to emit because it's moving? Yeah. I don't know. I suspect there are also some limitations to the WKB picture that we're using here and things like that. So I don't know the answer yet.

STUDENT: I have an even more evil question.

JACK HARE: OK, go for it. I see.

STUDENT: [INAUDIBLE] towards the process that generates the harmonic components?

JACK HARE: Oh, yeah. That would be good, wouldn't it? Yeah. No, so I guess don't know the answer to that one either, but that's a great question. Yeah. Cool. Nikola?

STUDENT: So we can look at it from the way we looked last class and when you entered the evanescent [INAUDIBLE]?

Anyway, the point is you never get oscillations. You still get an energy release via the wave, but it's not a wave. It's just an exponential decay.

JACK HARE: I don't think you do get energy released by the wave. The point of the evanescent wave is that that energy that you've lost gets reflected back. It's not absorbed inside the plasma. That's for sure. And that's true whether it's a plasma or a block of wax with some microwaves. Right? So that's just the property of whether the waves can propagate or not. But I don't think the energy gets dissipated in the medium.

STUDENT: And that's what's spooky about when you have--

JACK HARE: Yes. That's what's spooky about having a gap that the wave just sort of goes through and then appears out the other side. Yeah. I mean, so in this case here, you could imagine if you had a small enough evanescent region-- oh, that's interesting. OK. If you have a lot of evanescent region, you could have some radiation that couples to the other side where there's a mode you can propagate into.

Similarly, if you have an evanescent length scale which is long enough, then the plasma inside this region could couple to some modes outside, right? And evanescentally produce radiation. I will say this is much more complicated than what Hutchinson has in his notes. I am not a ECE person. These are great questions.

I'm going to keep going before someone asks another question. I've almost run out of tea as well. I'm so stressed. Good. Right. Let's do the X mode. So the X mode has n squared equals 1 minus-- you really shouldn't write this down. We've talked about modes before. I just want to write on the board so that we are staring at it a little bit. OK.

And just to remind you with the symbol mode example, this surface is when n equals 0. And so this is the cut off. So the question-- and it's pretty obvious that the cutoff, in this case, is when omega equals omega p squared. What's much less obvious for the X mode is where the cutoffs are. Right? So this is a complicated formula. We can go and rearrange it and find out what we get out here.

And what we find out is that we have cut offs for a wave frequency omega less than omega l. And I'll explain that to the left resonance. And also for a wave frequency that is between what's called the upper hybrid and the right resonance as well. And I haven't defined any of those. I will do that in a moment. But just to say there are now actually two regions in which we have cutoffs. Previously we just had this region with a cutoff for omega less than omega p. OK.

So you can go to Chen and you can go look up all of these and you find that the upper hybrid is the combination of, well, upper hybrid squared is equal to the electron gyrotron frequency squared plus the plasma frequency squared. That's why it's called a hybrid frequency. It's like a sum of two other important plasma frequencies. And then these left and right cutoffs, which I'll write in one go, with left over right meaning there's going to be plus and minus signs.

And for the left way, if you take the top sign, and for the right way, if you take the bottom sign, this is equal to a 1/2 minus a plus omega plus omega squared plus for omega p squared to 1/2. So looking at that you think, oh, someone had to solve a quadratic. Indeed, that's what happened. OK. Doesn't necessarily mean very much to you at the moment apart from these-- I'm just going to move this 2 down a little bit so it's closer to the symbol it's actually meant to be referring to. It's clear that we have a complicated set of different cut offs here.

So what I'm going to do is draw again this diagram upon on r0 frequency of the mode inside plasma 1, 0, 2. And we'll do the edges of the plasma here and here. Now it turns out that the lower cutoff here is going to look very similar to this O mode cutoff.

And the reason is that if we go to a region where ne at the edge is equal to zero, then omega p is equal to zero and we have minus gyrotron frequency plus gyrotron frequency. Oh, that's zero. OK. And then we can also start this a little bit and go, OK, it's going to peak in the center. So this is the lower evanescent region down here. So far so good. And again, we can draw on the same 1 over r falling modes for their cyclotron emission.

Now, what's very interesting is when we go and ask ourselves, well, where is this second region here? This is the region bounded by the upper hybrid and omega r. Well, let's do the same thing. Let's ask ourselves at the edge of the plasma where the density is zero and the plasma frequency is zero, the frequency of the upper hybrid is just going to be the frequency of the first harmonic here.

And actually, conveniently, the frequency of the lower hybrid is also going to be the frequency of the lower harmonic. We can play the same game at the other side here and here. And if you stare at these for a little while, you'll realize that the upper harmonic is always slightly higher than-- sorry. The upper hybrid is always slightly higher than the first harmonic.

And the right hand resonance is even slightly higher than that. So this is the right hand frequency and this is the upper hybrid. Of course, let me write it like that. It's all based on some excellent notes that Hutchinson made for 22-611, which is still floating around amongst the grad students if you can get a hold of them. These are very nice diagrams. They are in some form in the textbook, but the ones in his notes are better. What does this mean for accessibility?

STUDENT: You need a minimum of the second harmonic.

JACK HARE: Yeah. If we're going to observe from the high field side-- sorry, the low field side out at large R, which is where we tend to put our detectors, there's no way this wave is going to get through this evanescent region, evanescent coupling notwithstanding. Right? Because if we emit here and it's got to travel in this direction up, it's hit the region. If we emit here, it's going to travel up. It's hit the region. So we have got a complete cut off in assembly here. And if I'd left myself slightly more space, I would draw it.

I'll draw it directly underneath this one. So now, I, again, have my tokamak cross-section like this, and I effectively have an evanescent region that looks a little bit like a banana right in the middle here. This is the upper hybrid. This is the right hand resonance. So if I have any emission anywhere at the fundamental capital omega, that's going to reflect back. Say it.

STUDENT: Say what?

JACK HARE: I don't know. I thought you're going to tell me I should put my detector on the high field side.

STUDENT: I was going to, but now I'm not. Is this helpful in the sense of because you now have no first harmonics electron emission you can very confidently do your black body thing?

JACK HARE: Well, no, you can't because that's a mission, which becomes black body.

STUDENT: OK.

JACK HARE: If that's blocked out you can't see it at all.

STUDENT: OK.

JACK HARE: Basically what it's saying is, so there is a black body spectrum at some frequency that the plasma should be emitting here at some frequency with that black body spectrum, but that frequency gets blocked. It cannot get through the evanescent region. So we can't see any of the radiation from this side on the far side. We can only see radiation from here.

STUDENT: So yeah. High field side sounds like a great idea.

JACK HARE: High field site is a good idea for this. Yeah. Exactly. Now didn't go into this at all, and it seems slightly obscure the exact place where it crops up in Hutchinson. But it turns out we had this conversation before about whether you have O mode or X mode. And of course, it's not at all clear to us which of these two modes the gyrating particle couples into, right? There's no reason-- we haven't done nearly enough mathematics to know that. It turns out that predominantly ECE is in the X mode by, like, a factor of 10.

STUDENT: That's extremely unfortunate.

JACK HARE: Extremely unfortunate for us. And yet, we soldier on. So yeah. If you have high enough density, this region, of course, could go like that. Block out the second mode as well. Completely possible, so. It depends an awful lot on the density of your tokamak or whatever system you do. All right. I want to do the correlation ECE, but I don't think we're going to-- I really want to start and not finish. Questions? Yeah?

STUDENT: Do we have a general rule or intuition for the second harmonic intensity in terms of if we can completely block the fundamental frequency, if we go up to try to detect the second harmonic frequency, would that be possible practically, or is it just too low?

JACK HARE: No, no, no. It's definitely possible. If you look in Hutchinson's book with his example spectra, you will see a spectrum with second harmonics in, and they're very clear. The trouble is they may be optically thick, which may be good. So that may be fine. You'd be happy if they're optically thick because you will see some emission from them.

So it just is complicated is the short answer. So they should be detectable, but they may have been attenuated by the optical thickness. They may have been blocked off because our density is too high. And so there's an evanescent region. So yeah. There's lots of things. You need to do the calculations for your plasma to work out whether you're going to see them or not. Yeah. Any questions online? OK. I think we will leave it-- oh, yeah. Go on.

STUDENT: Why is it that the upper hybrid [INAUDIBLE] and--

JACK HARE: The left and right resonances. Yeah.

STUDENT: Yeah. Why do they have to meet the cyclotron frequency?

JACK HARE: Yeah, absolutely. No, it's a great question. It's not completely obvious. The upper hybrid one, imagine we're at the edge of the plasma here. So this is the plasma edge, and at this point, ne at the edge is equal to zero, which means that our plasma frequency at the edge is equal to zero.

STUDENT: OK. Yeah.

JACK HARE: And so then if you stare at this for long enough, you get-- you get the idea? Cool. But yeah. When I first saw this picture, I'm like, wait, that's too much of a coincidence. Is it always like that? And the answer is, yes, it is always like that. There's no combination of density and magnetic field you can come up with where that's not true, which is kind of cool. OK. I think we're good for today then. If anyone does have any questions, they can come and ask them. See you on Tuesday.