

[SQUEAKING]

[RUSTLING]

[CLICKING]

JACK HARE: Right. So we talked already about free-free radiation, bremsstrahlung, electron, cyclotron radiation. We talked a little bit about free bounds or recombination radiation. And these are all relatively simple compared to the complexity of bound-bound radiation, which is what we're going to be dealing with today and maybe for the next few classes.

The bound-bound radiation is the radiation that we get when electrons move between discrete energy levels within what we're going to call it Eta So we have maybe four energy levels like this. We might have an electron in some excited state up here that drops down to this energy level, and it will emit a photon. Or there could be an electron in the same energy level here that drops down all the way to the ground state, and emits a photon of a different energy. And what we would see on our spectrometer would be some distinct lines corresponding to these different photons.

And so one of the things we might want to know is how strong should these lines be? Where should they be? If I see these lines, what does it tell me about the temperature of my plasma, about the density of the plasma, things like this. And so, again, bound-bound radiation here, we might also call this line radiation. The reason being that when people first did spectroscopy on films, you would get these peaks showing up as very discrete lines on their spectra. So they're called spectroscopic lines.

And just a note on notation here, or at least on the [INAUDIBLE], we're going to be using the word atom a lot here. And for the purposes of this section, an atom is a nucleus with one or more electrons. So this could be 73rd states ionized tungsten. As long as it's still got an electron, we're still going to have spectroscopic lines coming out of here. So this is not an atom as in an unionized ion. This is a slightly different definition here. So, yeah, this could be something like argon 5 plus. That still counts as an atom.

So in order to get a feel for some of the physics that's involved in these transitions, we're going to go to a very simple system. We're going to go to a two level system and consider the processes which exist in that two level system. So in this two level system, we just have a level L lower, and another level U for upper. The upper level is an energy E_U , and the lower level is energy at E_L . So this is a two level system, and we're going to consider three processes that go on inside this two level system.

OK. So two levels, three processes. And some of you may have seen this before already, we're going to be looking at things called the Einstein coefficients. Can't be bothered to write out coefficients. OK. So the three processes that we're going to be looking at, the first process I'll just draw on here, is what is called spontaneous emission. This is probably the one that we're most often thinking about when we're thinking about spectroscopy here.

We have an electron in the upper level, and it drops spontaneously the lower level and emits a photon. There's a frequency $h\nu$, which is equal to the difference in energy between the upper level and the lower level here. So this is spontaneous emission.

Just want to note as an aside here that Hutchinson calls these levels I and J. This is his notation, and I've swapped it around because then you start writing like N subscript I for the number of states. But then we also use the ion density, and I found it confusing. And I just found it easier to think of them as the lower and upper states instead.

OK. And this process here has a rate, which it occurs at an Einstein coefficient that we call A subscript U subscript L. So this is the rate of spontaneous emission going from level U to level L. Of course, it's a two level system so that's the only spontaneous emission process we can have. But when we expand this to a multi-level system, we have A subscript, sub number subscript, some other number, and that will be the spontaneous emission from one level to another level.

OK. If we have spontaneous emission, we should have a process which looks the exact opposite, and this is spontaneous absorption. We tend to drop the spontaneous bit and just call it absorption. And indeed, it looks just like the mirror of this process. We have some electron in the ground state, some photon that happens to have exactly the right energy comes in. The electron is excited up, and the photon is absorbed here.

And this has a rate, which is the Einstein E coefficient between the lower and the upper level. But this is now multiplied, as well, by the density of photons with this specific energy. So we'll use ρ to be the density, the number of photons per meter cubed, of photons with exactly the right energy ρ_{UL} , which can do that excitation. There's an important difference here. This happens spontaneously. It doesn't matter how many photons or electrons are around in this system.

This one has to happen only when this photon comes in. And so the rate at which this process occurs is going to depend on the number of photons. If there aren't any photons around to do this, this process just won't happen. OK. And then the final process that we're going to consider, instead of spontaneous emission, is stimulated emission. Stimulated emission is a very interesting process, because here, we have electron in the excited state up here in the upper state, and some photon comes through with energy that is the same as this band gap here.

Now, of course, this photon can't be absorbed. There's only this upper level. There's nowhere for that energy to be absorbed. What it does, instead, is it encourages this electron to drop down, and it stimulates the emission of a second photon. So the first photon comes through still at this energy, and we have a second photon at exactly the same energy. And this is the process by which lasers work.

So this has Einstein coefficient B_{UL} . So we have a transition from the upper state to the lower state. And again, the rate at which this process occurs depends on the density of photons. We have to have some photons to stimulate the emission so it has the same ρ density of photons with that specific energy. So note these are actually different coefficients. The rate of absorption, the L_U , is not the same, in general, as the rate of emission B_{UL} .

OK. So those are the three processes we're going to consider, and we'll then start deriving what these coefficients actually are. So any questions before we keep going? Yes.

AUDIENCE: [INAUDIBLE]

JACK HARE: Basically, every subscript is going to be U and L. So UL here. So this new UL is just to say it's a wave with energy which is equal, the difference, between the energy of the two levels here. So the total energy in the system is conserved. Yeah, I guess I could write these as uppercase L and uppercase V like that. Yes.

Yeah. So the important thing about stimulated emission is that you stimulate a photon with exactly the same energy. And, in fact, the photon in the same mode of your cavity. So polarization and things like that are also the same. Yes.

AUDIENCE: [INAUDIBLE]

JACK HARE: Excellent question. We will get on to that when we do line broadening in a few classes. The rough answer is, roughly, there's a sort of uncertainty principle type thing involved if you do-- if the process happens quickly, you can be less sure about the energy. So, yeah. But these lines can be extraordinarily narrow, right? And so if you look at a spectra, then these lines could be extremely narrow. That would correspond to needing very precisely the same energy. Yeah. OK. Any other questions? Yeah.

Is what we're going to get onto, you're quite right. But, no, no, you're right. They can't just all be random because in steady state, they're going to have to match up. Otherwise, our system will be driven in one direction or the other. And so we are, indeed, thank you for the transition, going to use thermodynamic arguments, which we did once before already to match the emissivity and the absorption. So now, we're going to use thermodynamics, and we're going to do that to link these coefficients A_{UL} , B_{LU} , and B_{UL} .

OK. I think it's best if I just cover this off and start. OK. So one thing we want to be able to write down is the number of electrons in the upper state versus the number of electrons in the lower state here. And to do this, we're going to use a Boltzmann distribution for that. So this is a Boltzmann type argument where we say that the number of electrons in the upper state-- imagine we have an ensemble of these two level systems non-interacting a large number of them, on average, the number of these systems, which are in the upper state, is going to be proportional to the exponential of minus the energy of that upper state divided by temperature.

And again, we're writing temperature in energy units, like, joules, or eV . I folded the Boltzmann constant inside the T here so I don't have to keep writing. This doesn't have any normalization on it. But for a two level system, the normalization is very simple. We'll just divide an upper by and lower, and we will get exponential of minus $E_{upper} - E_{lower}$ upon temperature like that. And you'll notice, of course, straight away that this quantity is just Planck's constant $h \nu$.

There's a slight subtlety to this. If you have systems with degeneracy so that there are multiple different ways the energy can be in these states. So it could be spin, or something like that. Then you have an extra factor due to degeneracy at the front here. This degeneracy factor is like G_U over G_L like that. So these could be numbers like 1 or 2, things like that. So this is just making this slightly more generic for a broader range of two level systems. For now, you can just consider this factor to be one for what we're doing. But this factor is due to degeneracy.

Then we also need to have an expression for the density of photons with a given frequency here. And here, we're going to go back to our blackbody spectrum. And that density of photons is going to be the standard result $8\pi h \nu^3$ over c^3 , 1 over the exponential of $h \nu$ on T minus 1 . So this is for a system at temperature T . What is the density of photons with frequency ν , or what is the density of photons of energy $h \nu$.

And finally, we're going to use the thermodynamic argument that these systems should be in equilibrium. And so that says that the rate at which atoms fall from their top level to the bottom level, which is A_{UL} plus $B_{UL} \rho$ of ν_{UL} times by the number of atoms in the top level must be equal to the inverse process, which is the rate of absorption B_{LU} of photons with energy ν_{UL} times the number of atoms in the lower level.

So this just says the rate of de-excitation is equal to the rate of excitation our system is in steady state. And if you put all of these together, these effectively form a set of simultaneous equations, you find that, indeed, our Einstein coefficient for spontaneous emission A_{UL} is equal to $8\pi h \nu_{UL}^3 / (15 V B_{UL})$, like this. This means that we have something linking the spontaneous emission to the stimulated emission.

And more simply, we also find that A_{UL} / B_{UL} is equal to A_{LU} / B_{LU} . So again, this is the two different types of-- sorry. This is linking the processes going up and down and this is the absorption. And this is stimulated emission. So similar to how we did last time, we have used a system in complete thermodynamic equilibrium with Boltzmann occupation, steady state, and blackbody photons. And we've used that to pin these different coefficients to each other.

So if we calculate even one of them, like, B_{UL} , we now immediately know B_{LU} and A_{UL} . And so that's good because we get all of them for free. But it also means that all the processes are linked together in this system. OK. Any questions on that? I then tell you in a moment how we actually calculate even one of these so that we can get the [INAUDIBLE]. But now, it's important just to realize that those three are [INAUDIBLE].

Also, the cool thing is, this has no material physics in the relationship between these, has no materials physics in it. This is related to lasers. For laser, you need a three or a four level system because you have to pump the excited state. And then you have to have a metal stable state halfway down. You can't actually get lasing in a two level system. But in a three level, four level system, you have. So the systems which are favorable to lasing have specific energy levels that are spaced in a way that you can get stimulated emission.

I just mentioned stimulated emission as that sort of-- I think when you look at this diagram here, you initially think to yourself, do I need stimulated emission? It seems a bit niche, like excessive bit of physics that I could throw in there. Can't I just get away with balancing spontaneous emission with absorption? They look quite similar. But it turns out that for the thermodynamic balance, and this is the thing that Einstein realized, which is why the Einstein coefficients, you have to include the stimulated emission in there. Otherwise, the theory doesn't work. You can't get them balanced properly.

So I mentioned this as a thing which is important that lasers in the two level system. It doesn't play an important role and, also, this is material independent, right? Once we've calculated one of these, the other one depends purely on the energy level gaps. It has nothing to do with the crystal structure or something fun like that. Yes.

Any reason it isn't stimulated absorption? What would that look like? So at the moment for spontaneous absorption, we have one photon come in, and the electron goes up. So with stimulated absorption with two photons coming in, one getting absorbed and one going forwards, like, the opposite of this process. I don't have a good answer for why that doesn't occur. But it's an interesting one.

It would go as the square of the photon density field because you'd have to have two photons in the same volume to do it. So possibly, for most reasonable systems where we don't have that much radiation density, that wouldn't be a big-- that wouldn't be a very important process. But I see what you're saying. I hadn't thought about that. Yeah.

AUDIENCE: [INAUDIBLE].

JACK HARE: Not in this, because in order for that to happen, you need to have some ill-defined, metastable level halfway up like this. But there has to be some level. It has to-- it doesn't have to be exactly there. Because of the uncertainty, it could be like this, or something like that. But it has vaguely possibly exist. But in this model, I've said very clearly I only have two levels. And so I don't have to think about that.

But you're right, if you-- in a real system, you can have two photon absorption where one photon takes up to there. And before the atom has time to realize that this energy level doesn't really exist, the second photon comes in and takes it up even further and then everything is OK. So there's like-- yeah. But in this, I've been very clear I only have two levels so I don't have to worry about that. But, in general, [INAUDIBLE]. Any other questions?

AUDIENCE: [INAUDIBLE].

JACK HARE: Yeah. So actually, none of this is about ionization at this point. So these are just two levels within an atom. The number of electrons is conserved. And you're right, they're just energy levels. So this could be-- actually, the simplest way to think about this, maybe the simplest quantum system, would be a spin which is flipping up and down. So these could be the hyperfine splitting of the ground state, or the fine splitting of the ground state and the magnetic field, or something like that.

Yeah. So that's why this is quite generic. The way I'm thinking about it is about energy levels within an atom that isn't undergoing ionization. And we'll talk about all the ionization processes in a moment. But you're right, this applies more generally to other systems that have discrete energy levels. They've got two discrete energy levels. All of this math still works. Cool.

OK. So, unfortunately, if you want to work out any of these, you're going to have to do some quantum, right? And the reason for that is, if you go into Hutchinson's book and you look at an expression, for example, BUL here, you find out that it has $8\pi^3$ cubed over $3H^2$ squared $4\pi\epsilon_0$. Don't ask me where any of these factors come from. And this is times a quantity called SUL . SUL is like the dimensionless rate, and then this is all the constants that you need in order to get all of your equations to match up properly.

SUL is the thing that depends on quantum. And that is to do with the transition between your two states. So we're calculate an overlap integral, the integral between your upper state and your lower state. But we're not just doing the overlap between those two things because there is an operator, which acts on the upper state to transform it into the lower state. And that, for most electromagnetic radiation, is the dipole operator, which is the charge times the position vector here.

We integrate that, as we always do, over all space, and we square it because we're interested in the power here, rather than the electric field. So this-- you can think of this as representing a transition probability. How likely is it that the emission of a photon can take you from the upper level to the lower level? And so levels which are more alike, you're more likely to have transitions between them. Of course, there are some levels which are very alike, which we can't have a transition to, where we have S_{UL} equals zero. And these are called forbidden transitions.

And these forbidden transitions don't do things like conserve angular momentum, other important things like that. And the way that that manifests itself in the mathematics is that under the operation R on the lower energy level, the Ψ of U is orthogonal to R of Ψ of L . And so when you do the integral, you get zero out. So effectively, this keeps track of angular momentum conservation, or momentum conservation for you. So you have S_{UL} there equals zero.

So you might have, for example, a system with three energy levels where you can have transitions like this that both produce photons with different energies. But for some reason, this transition here, as S_{UL} , equals zero. And you can't have a transition like that at all. So these are forbidden transitions, and sometimes people call these selection rules.

So if you've done any amount of quantum physics or atomic physics, I think you'll probably have come across this before. If you haven't done a class on that before, I'm just trying to give you a very brief flavor of it. In general, this is a difficult problem. For a two level system, you only have to calculate S_{UL} once. But if you imagine for a three level system, now, we have to calculate the overlap between this level and this level, this level and this level, and this level and this level.

And for a four level system-- and then you get the idea. So this quickly becomes very, very challenging. If you want to actually do this, you have to calculate this overlap integral for all the different wave functions. To make it worse, you also need to know what the wave functions are. And so you have to solve the Schrodinger equation, first of all, to know what the states are allowed in your system. But, of course, if you have more than one electron in your system, not just one more than one level, more than one electron, this is now a many body problem and so you can't even calculate the wave functions properly.

So very quickly you can see that spectroscopy is going to be a horrible mess compared to all of the other types of radiation we've dealt with before. Well, line emission is going to be a horrible mess compared to all these other types of radiation. But say you have managed to overcome all of these problems, the main takeaway from this is, if we know B_{UL} by calculating this overlap integral, then we also get the other two coefficients A_{UL} and B_{LU} for free.

And that means that even in a system that is not in thermodynamic equilibrium where we don't-- where we can't assume Boltzmann occupation, we can still calculate now the occupation of the upper and the lower energy levels through a rate equation where these are the rates. This is effectively a steady state rate equation here. And that means we now know how many electrons are in the upper level, how many are in the lower level, and we can make a prediction of how many photons we're going to get per second. And so that means that we can predict the intensity of one of these spectral lines here.

Again, this is an extremely simple system. There's already quite a lot of work to get there, but we can do that. So that's a very thorough explanation of just the two level atom before we go on to more general systems with multiple levels and multiple electrons. So any questions on that before we move on? Yes, go ahead. Pointed to the wrong person. There you go. You get slightly different frames, I think.

Yeah. So the question was, do you have to do this every time, or is it tabulated somewhere? So the answer is, someone may have already done the tabulation for you and made that available in what's called an atomic code.

AUDIENCE: [INAUDIBLE].

JACK HARE: Oh, generally, these are experimentally-- these are theoretically predicted, and then experimentally confirmed. For some of the transitions, they're so weak it's really hard to do it experimentally. If you go on the NIST website, then NIST has a huge table of all the different transitions here and they are like oscillator strength. And sometimes, they have a star next to them, being like this was theoretically predicted, or they have something else being like, no, this has actually been experimentally checked, or like someone tried to check it but we don't think it's a very good check.

Because these are just-- there are so many of these lines, it's impossible to check all of them. And, of course, you can theoretically predict these four states of matter, which are very hard to reach in a real plasma. So for very, very hot very, very dense plasmas made out of plutonium, or something like that. We probably do actually have good measurements of that, but not unclassified ones. So there are things that maybe it's hard to experimentally verify. But actually, you can still theoretically.

So, yeah, but if you are doing something where that hasn't already been tabulated, you may have to do it yourself. So you may actually have to do these calculations yourself. And there are people who build careers out of doing these calculations more and more precisely. Because as I mentioned, this is a very hard calculation to do. So there's always going to be some approximation. And so then, it's like you could make your career out of doing a slightly better approximation than previously, and you'll get slightly better results. And that will be worthwhile to some people.

Yeah. You have a question?

AUDIENCE: [INAUDIBLE].

JACK HARE: Yes, this is-- you should think of it as a number density of photons. We're using ρ here, which often is used for mass density, but I'm using it here for number density because that's what [INAUDIBLE]. So, yeah.

AUDIENCE: [INAUDIBLE].

JACK HARE: We will actually talk about that later, you're right. And unless your spectrometer is absolutely calibrated, it's very, very hard to get-- it's very, very hard to get it just from a single measurement of a line. What's interesting is, if you have two lines here, this isn't the best drawing, actually, to show this. A better drawing would be if I had one of these transitions coming from this state here.

So then you'll have two lines which have a downward transition, which are both emitting photons at different energies. There are two things you'll need to know then. You'll need to know what the strength of this transition is, which is the strength of the spontaneous emission AUL, which will be different for these two different lines. But then you'll also want to know what the occupation probability is of these two upper states.

And that will, in general, be a function of temperature. So it will be-- this state will more likely to be occupied than this state just from a Boltzmann argument. If you happen to be able to use a Boltzmann type argument, that's what you predict. So that means that the ratio between the amount of light in this blue line and the amount of light in the orange line here will tell you, once you've taken out the factor of AUL that you've precalculated, it will tell you the occupation of this upper state versus this state.

And that will be related to temperature. And then you have your plasma temperature. And it's a relative ratio. You don't need to know the absolute intensity. So in reality in spectroscopy, we normally look at line ratios. They're much better to work with. There'll be many, many lines, not like in this two level system. There'll be many, many lines out there. We look at line ratios like a differential measurement again. The thing that we love to do with experimental.

Does that make sense? Good. We will now go through lots of other processes which can excite and de-excite your electrons. Not just radiative processes like this. And then we will use a subset of those to define different sorts of equilibrium, which help you to calculate what this occupation probability is for the different energy levels. So it's quite an involved series of steps, and I'm trying to go through it quite slowly. Any questions before we move on to all the other processes out there that aren't just these simple radiative [INAUDIBLE]? OK.

OK. So the first set of processes we're going to look at are processes which excite or de-excite electrons. So in this case, we're looking at a change in the electron energy, but, or and, the number of electrons is constant. So we are not dealing with ionization here. We're going to get on to ionization and recombination in a moment. So the first set of processes we've actually already covered. But I'm going to draw them on the board here so that we have a nice complete set. These are the radiative processes.

So these are processes where we have, for example, some energy levels, again, lower and upper. We have different processes going between them. We have spontaneous emission, absorption, and stimulated emission. And again, we just covered this, but I'm just writing it down here so that we've got a full set of [INAUDIBLE]. Continuous emission, absorption, and stimulated emission. And I'll put the density of photon states on here, as well, well, ρ of new UL.

Competing with these processes, or ongoing with these processes, a set of processes which are collisional. And in some cases, the collisional processes may dominate, and in other cases, the radiative processes will dominate. And we'll talk about those in a moment once we've got through this whole zoo. So the collisional processes, again, between the lower and the upper levels like this, are called things like-- this one here has a coefficient C_{LU} , which is proportional to the density of electrons.

Because this happens when an electron comes nearby the atom. Maybe it's an electron which is a free electron so it's up here at some high energy. It interacts with the electric field of the ion, and then the atom and flies off. But that interaction with the electric field promotes an electron from a lower energy level to a higher energy level, taking some energy from the free electron here. And this process is called electron impact ionization.

So I'm just going to write that as P minus impact-- not ionization, excitation-- excitation. And actually, exactly the same process can happen in reverse. This electron can come in, its electric field can interact with the electric field and the wave functions of the atom. And this can drive a downwards transition, as well. And this has a coefficient C_{UL} , which is proportional to the density of electrons, as well. And so this would be the same name as above, but the excitation.

So you can imagine in a very dense plasma where there's a lot of electrons flying around, you may well end up with these processes being very dominant. And they will dominate the population of these different excited states. And so when you're trying to calculate the excited state population, these are the ones you're going to have to focus on. We'll talk a little bit about how we actually calculate these in a moment.

So those two-- well, we've seen these already. Hopefully, they make sense. Does this make sense? OK. These were the excitation, and the de-excitation. Now, we're going to have processes which actually change the number of electrons. So these are ionization. And we don't tend to call it de-ionization. We call it recombination, instead. So in general, you can think of these as processes that give an electron enough energy to be free, so ΔE , and that makes it free, or they absorb free electrons energy so that our electron becomes bound.

So that first process is an ionization process where we lose an electron from our atom. And the second one is a recombination process where we gain an electron onto our atom. So I'm going to draw this as one long sort of [INAUDIBLE] tapestry of various different processes here. This is the lower energy level and this is the-- now the ionization energy level I . And I'm going to shade the region above here, because if an electron is in this region, it's now a free electron.

But you can think of this as the ionization energy you get to this point. And I've called this L . This may not necessarily be the lowest energy state in the system. It's just the lower state. There could be other states down below. There could be states up above it. We're just considering one level L at the moment here. OK. So the first process, we're going to consider is where we have an electron in the ground state-- oh, sorry, in the lower state down here, and we have a free electron that swings by. And it gives up enough of its energy to excite this electron, not to the upper energy level, but actually, up so that it's ionized, and then two electrons come out.

And this we give a coefficient C_{LI} . And we give it a superscript CI here. And that is for collisional ionization. And we call this electron impact ionization. You can see this is analogous to electron impact excitation here. I'm just going to make it clear that this is a lowercase I . But there's another coefficient here. What's this coefficient proportional to? Remember previously, we've had density, we've had photon density, all sorts of things like that. What's this coefficient proportional to?

Yeah, exactly the same as previously. There's nothing special. Yeah. OK. Now, we have a slightly different one. We actually have two electrons coming in. As they get close to the atom, one of the electrons runs off with a load of energy, and the other electron has lost so much energy to the first one that it drops down into this energy state here. So this is a recombination process, and this is called three body recombination.

And we give it a rate coefficient C_{IL} , and a superscript three body recombination. What is this proportional to? Someone else. But I will come back to you. Yeah. What are the neutrals in this?

AUDIENCE: [INAUDIBLE].

JACK HARE: Why didn't the atoms come in here, in that case? It's a good point. In fact, all of these are proportional to the atom density, as well. I've just left it off because every single one of these is proportional to the atom density. So we don't need [INAUDIBLE]. Yeah.

AUDIENCE: [INAUDIBLE].

JACK HARE: Why is that? Yeah, exactly. So we've actually got to get two electrons in the same place close to the atom. And so you're going to square the density in order to work out the probability of that happening. So this three body recombination can be a really important process in dense plasma. It's actually particularly important in dense cold plasma. So plasmas with very low ionization tend to have dominant three-body recombination.

OK. The next one is also quite odd. We imagine that there's some intermediate energy level like this in which there are two electrons. And there is some spontaneous process by which this electron drops down to this energy level, which provides exactly enough energy for this electron to bump up and be free. In reality, this will happen as long as this intermediate level is somewhere above halfway, because then one electron dropping down, will have enough energy to boost this off and it will be not just a free electron but a free electron with some bonus kinetic energy, as well.

This is still written as a coefficient from lower to upper, even though we should probably think of a name for this intermediate energy level. So we call it CLI. And its superscript is AI because this is a process called autoionization. Does this have any dependence on the electron density? I saw a shaking head. It doesn't have any dependence on the electron density. We don't require any free electrons to do this. This is something the atom has just decided to do by itself.

Of course, for this to happen, there has to be an unoccupied lower energy state. So this has to be some excited atom, right? So some process like electron impact excitation has moved an electron from the lower level to this intermediate level here, leaving a state for this electron to fall back into. So this can't just happen in us. We're not going to suddenly start emitting electrons by autoionization. Because in general, most of our electrons, if we're well behaved, stay in their ground state, OK.

But if in a plasma where these processes are happening, we very well may have a hole in a lower energy level, which allows this process to happen. Pretty cool. OK. I saw Nicola.

AUDIENCE: [INAUDIBLE].

JACK HARE: Yeah. So basically, it needs to be more than halfway. It needs to be greater than energy lower ionization divided by 2, right, as you say. Otherwise, there won't be enough energy for-- I don't know if there's auto excitation that can happen. That wasn't in the book. But it seems like it might be possible, right, where you could have excitation of this electron up to some higher energy level. Maybe that doesn't happen for some reason, I don't know. Did you have a question, as well? OK.

Yeah, we're coming to that. That's a question I'm about to ask in a moment. You're absolutely right, that can still happen. But it's subtly different from these four. I'm up to the fourth process in a way which I think is informative. And then we will go on and cover what you're talking about, yes. Any other questions on this before I do the full process?

OK. You may have noticed there's a sort of symmetry to these processes so far. And so this is the symmetric partner of this process. This is where we have some intermediate energy level. We have an electron drop down out of the unbound states, and this spontaneously promotes an electron for this intermediate state here. OK. And this is called-- well, this is, again, from-- got LI written here. I'm going to go with it. But maybe it should be IL. And this has a superscript DR, and this is called dielectronic recombination.

What is it proportional to? Yeah. Then see electrons, again, it relies on there being electrons here, free electrons. When we're writing any, obviously, we're talking about free electrons here, ones which are not bound by atoms lies on that. So a couple of interesting things here. Notice that these reverse processes do not have the same scaling with density, and so we don't necessarily expect these processes to balance.

In fact, in an arbitrary plasma, they will not balance. In steady state overall, you have to have balance between the upward processes and the downward processes. But it doesn't mean that each process has to be balanced by its mirror because they are not symmetric. These recombination methods here require one more electron than the ionization mechanism here.

What else is interesting about all of these ionization and recombination processes from the point of view of a spectroscopist? How would-- yes, go on. They are non-radiative. Does that mean, given this is a diagnostics course, that I just wasted your time by telling you about them because we cannot diagnose them? I've wasted your time, OK. Any other votes for wasting your time? I've been known to do it before. Yeah.

They are absolutely key in some plasmas. But understanding the ionization state of the plasma, and also for understanding, in the case of these ones, the excitation state of each ion. And if you don't know that, you can't predict how many photons you're going to get out. So even if your signature is a radiated signature, it will be strongly modified by this. So these are still important for the population. And in this case, this is the population of excited states and ionization states.

Eh, I definitely ran out of room. That says states. OK. Cool. Any questions on these before we go on to some [? radiative ?] ones that we can directly [INAUDIBLE] and also [INAUDIBLE]. Yes. Yeah. I mean, you-- what I did-- what I did here without making it very explicit, and we will go on to this in a moment, is write a rate equation. And this is just a rate equation with two levels. But instead, you would write a set of coupled rate equations which has the rate equation for each pair of energy levels. And you'd have to write that for each pair of energy levels within the hydrogen or the deuterium, and each pair within the tritium.

Now, deuterium and tritium are close enough that we don't have to worry about. But you will have tungsten in your reactor, or something like that. And then you will want to solve the tungsten rate equation so you know how much ionizing radiation is coming out of the tungsten that can then reionize hydrogen and you'll scrape off layer. I don't know, something like that. This stuff may be important.

And so, yes, of course, as soon as you do anything more interesting than these very simple systems, you're going to have even more complicated equations. And I will write down the full-- think, the full-rate equation towards the end of this class if we get to it. But it looks like this, but with lots of some symbols to indicate the fact that you have to do it lots of times. Yeah. Any other questions? OK. Right. Now, we're getting on to processes that, I think, Grant was asking about that.

So these are, again, ionization and recombination processes. So again, I have this shaded region with free electrons in it. I'm going to call the highest energy level the ionization level. And the lower level L. And now, I'm going to look at processes, for example, which involve protons. So here is a photon coming in and it's got energy $h\nu_{IL}$. So it's got an energy which is matched to this gap between the lower state and the excited state. And as you might expect, this takes an electron and ionizes it.

OK. And so this process here is called photoionization. It has a rate coefficient $C_{L \rightarrow I}$ lower to ionized state. It has a superscript ν_i . And what is it proportional to? Yeah, thank you. With that specific energy. So R_{HO} of ν [$? LIL. ?$]

OK. Now, we have the opposite of that process. We have some electron, which is in some excited state. And then it drops down to here. We've already covered this, right? This is spontaneous radiative recombination. We talked about this briefly, but now we're putting it into our more quantum picture of what's going on. And we will give it a rate, as well, so that we can include it in our rate equation here.

So this is C_{IL} radiative recombination. I'll write that up here, C_{IL} radiative recombination. So technically, this is spontaneous radiative recombination. What's this proportional to? Thank you. Note again, big asymmetry between these two processes which look like mirrors of each other. One of them depends on the photon density, one of them depends on the electron density. There's no good reason to believe that these two processes balance each other.

OK. The final process is, again, you can see these if you want to as the ionization recombination analogs of the excitation and de-excitation processes. And indeed, we do have a process where we come in with a photon with energy $h\nu_{IL}$. We have an electron drop down, and then we get two photons out here, $h\nu_{IL}$. This is stimulated radiative recombination. You see, I'm getting lazier and lazier as the class goes on. I can't be bothered to write out any full words anymore.

And this has a coefficient C_{IL} stimulated radiative recombination. That is proportional not only to the electron density, but also, the density of photon states with frequency ν_{IL} , like that. And these processes, as we discussed, are radiative. So we can look out for them, but there are nice diagnostic signature. But they are also important for setting the occupation of the different energy and ionization states. Question.

AUDIENCE: You mentioned that the process of [INAUDIBLE] there is some [INAUDIBLE] by [INAUDIBLE] but like [INAUDIBLE] far above [INAUDIBLE] having [INAUDIBLE] was this really like a quality [$? type? ?$]

JACK HARE: Yeah, absolutely. The process up to some energy state I, which could be a free state up here. So this could be some other state like that. We can think of the free states as a continuum of states. So once you get above that ionization energy, you don't have discrete states, you have a continuum of states. And that would depend on this energy density here, which of course, for very high energy photons will be very low. We don't have very many gamma rays from a black body unless it's very, very hot.

The only thing I would say, and I'm not sure exactly how to make this not hand-wavy, is that your gamma rays, although, they are good at ionizing, they tend to just go through stuff, right? I mean, the point is if you put a bit of paper in the way, you're like great. The gamma ray hits a bit of paper. It's got all the energy it needs. It should just be absorbed and ionize something. But in fact, it goes through that, and it goes through lots of other things.

So I think there is a quantum mechanical effect about the interaction of this very high energy radiation with these energy levels, which makes that a less likely process. So the gamma ray's, in general, not going to have very high cross-section for that interaction. But I don't know how to take that hand-wavy argument and turn it into something mathematically. Yeah. Yeah. Yeah.

And so I'd have to average over-- integrate over all the energies, which is what I'm about to do. So, no, it's a great question. It's a great question. So for the collisional processes, so these non-radiative ones, these ones. And the ones on this side, as well, at least this set here. We actually-- we have a different way. We don't tend to write these coefficients in terms like this. We tend to think about them in terms of reactivities, right? So we have some cross section for the interaction which we could write as σ_{IJ} . So that is the cross section for an interaction going from state I to state J.

We multiply that by the velocity of the free electron that we're working with multiplied by our distribution function of electrons. And then we integrate that up over all the electrons in our distribution function. And that will give us something that looks like, for example, any $\sigma_{ij} v$ some reactivity here, times the density.

If we're dealing with a process that involves two electrons, like three-body recombination, we'd actually have to put the distribution function in twice and integrate over the interactions between particles going from distribution 1 V_1 , and particles going from distribution 2 at V_2 . So you don't just square this, you actually have to integrate and then do a double integral over these two distributions.

AUDIENCE: [INAUDIBLE].

JACK HARE: Per nucleus, yeah. So the total rate would have an additional times N. I'm going to go with I here, as in whatever. This is an electron causing a change in the atom state from I to J, and so we need to know how many atoms there were initially in state I. And in this case, when we're writing this in terms of I and J, so this is transition from I to J, and that could be excitation or de-excitation, or it could be ionization, as well.

The reaction rates, the reactivities, have the same form as that. So you'll still end up with something like that. OK. And so then, we're now in a position to write down our full balance. So we can say the change in particles, atoms in state I-- for this example is an excited state, or it could be an ionization state with respect to time is going to be equal to the sum over states not equal to I states. $J \neq I$, because if we have a transition from state I to state I, we don't change the number of particles in state I, so we don't want to double count.

This is going to be the number of particles in I, the spontaneous emission process from I to J minus the number of particles in J, the spontaneous emission from some other J to I. These coefficients may be zero if spontaneous emission is forbidden. For example, if spontaneous emission would involve going up an energy level. So in this case, we're looking at spontaneous emission from I downwards and from a different J back upwards here.

So this is I. This is I like that. Then we would also have a term that's to do with our absorption and stimulated emission. So that is $n_i B_{ij}$ minus $n_j B_{ji}$ density of photons with energy, or with frequency IJ like that. So that covers these processes, as well. Then we have the two-body processes, which are these collisions, impact ionization, autoionization, this sort of thing. Actually, autoionization is not a two-body one. OK. We'll get that back flat.

Well, we're going to have the density of electrons now, the density of ions in state I, and then this σ_{IJV} that we just derived, or at least handwaved, into existence up here. And that's also going to be balanced by the opposite process driving the system in the opposite direction. So σ_{JI} instead of σ_{IJ} . That should be times V. These processes are two-body. These processes on the top line here are one-body, maybe plus 1 photon, but that doesn't count.

And then we have the three-body processes at the end here. And E^2 and $I \sigma_{IJV}$. And the meaning of the angle brackets here is actually an average over both the distribution functions, or at least over the distribution function with itself. And then, of course, there'll be the opposite process here. And E^2 and J is an opposite process. I'm not convinced there is. I'm going to put it in anyway because it's my notes. I can think about that in a moment. And these are the three-body processes.

Now, in general, we allow ourselves a small simplification, and we tend to look for plasmas in steady state. And that steady state doesn't have to be an absolute steady state. It can be on some time scale of interest. The plasma does not change its occupation state very much. And so this is a reasonable approximation, quasi steady state. Because even with this approximation, I think it's pretty clear that this is a very long equation that you would have to solve in order to work out the-- and this, I believe, is just what we need to get the excitation state of a single atom.

So this is the excitation, for example, of an atom at state Z, like that. We'd have to have another coupled equation, which would tell us whether we're going from NI of Z to NIZ, plus 1, or NIZ minus 1. So that'll be our ionization balance equation, as well. And you have to couple all of those equations together. And this is where I start thinking that I've mixed up the excited states in the ionization states, because if I'm talking about excited states, I don't have these three-body processes.

So it's a bit of a mess. OK. But my general point is a very, very large number of equations. What we will do in the next lecture is, we will work out some types of equilibrium where some of these processes are unimportant and, therefore, we drop them. And that simplification allows us to make a calculation of the occupation states in a much simpler way that is actually tractable. But, in general, for some arbitrary plasma where we can't make some of the assumptions we'll make in the next class, you will need to solve this full-rate equation in order to work out the occupation of excited states, the ionization states in your plasma and, therefore, what radiative signatures you're going to get out of it so that you can do spectroscopy. So all gets a bit complicated.

We will leave it there. Does anyone have any questions? Yeah.

AUDIENCE: [INAUDIBLE].

JACK HARE: Yeah. Oh. Yeah. Thank you, that's great, good point. You were paying attention. Very good. Fixed it. Yeah. NI in this case, yes. I would prefer to think of it as atoms in a certain electronic configuration. OK. So imagine you've got two electrons here. They're indistinguishable, so it doesn't really matter what state the electrons are in, as long as the configurations are indistinguishable. Yeah.

So this is an atom in some state which has some energy. And so, for example, in a three-level system with two electrons, this would be your ground state, and this would be another state. But because we're not tracking which electron is which, this would be an identical state. So this would actually be like a degeneracy. When we're talking about that G factor before, these two states would be degenerate here. So this is-- so these are all different-- or this is, for example, N_0 , and this could be N_3 , or something like that.

And maybe I would label my states in terms of their increasing overall energy. And so you can imagine there's some intermediate states with different energies like this. OK. Pick three out of a hat. I think it's probably two. Anyway, yeah, Nicola.

AUDIENCE: [INAUDIBLE].

JACK HARE: Which one?

AUDIENCE: [INAUDIBLE].

JACK HARE: Stimulated radiative recombination. This process is possible for any of those, but it doesn't-- there's a proportionality here, but there'll be a constant that depends on quantum in front of it. And I would have thought for very high energy electrons, this is not going to be a very significant process. So-- yeah, that could work. But the trouble is, of course, if you make your photon energy density field very large in order for this process to be big, then your photoionization also goes back up. And so if you have a radiation dominated plasma like that, it will still come to some equilibrium, and that equilibrium will not be everything in the ground state. It will be something different. And we'll talk about that.

But, yes, you're-- I don't know anyone who's done this in the lab, just pumping in photons and somehow reducing the total energy of the system and getting light out. I think it's just one process that takes place with all the others. So you're unlikely to be able to engineer a plasma that is dominated by this. Do any of the other ones have a proportionality to both electron density and photon density? No.

So there is scope here for making a very dense, very photon dominated plasma where you could imagine that this coefficient could be larger than any of the others. But I don't know how dense and how energetic, or how dense your photon field and your electron density needs to be for that to happen. Cool question.