22.67 2023 Principles of Plasma Diagnostics

Prof. Jack D. Hare

Pset 1: B-dot probes and plasma position

Prof. Puffin at the Texas Institute of Magic (TIM) wishes to use the pulsed-power generator HARE (High Ampere Research Experiment) to study MHD instabilities in a classic Z-pinch. She places a set of b-dot probes at different azimuthal locations a distance R from the plasma to measure the magnetic field around the unstable pinch, which can be decomposed into $m = 0$, $m = 1$, etc. as follows:

$$B(\theta) = C_0/2 + \sum_{m=1} C_m \cos(m\theta) + S_m \sin(m\theta)$$

Consider sketching the first few modes. Model the pinch as a plasma column with radius $a < R$, and initially **assume that the electric current is uniformly distributed within this radius**.

1. What arrangement of b-dot probes, if any, could be used to determine the amplitude of the $m = 0$ mode?
2. What is the minimal arrangement of b-dot probes necessary to sense the $m = 1$ (displacement) mode?
3. What is the minimal arrangement of b-dot probes necessary to sense the $m = 2$ (elongation) mode?
4. What about higher m modes?
5. If the assumption of uniform current inside the plasma is removed, do your results still hold? What can you infer from an array of b-dots?

You may carry out this work analytically or numerically (for example, by generating a magnetic field with a given mode, showing the measurements expected for the b-dot probes, and demonstrating you can reconstruct). You do not have to show your code - I recommend submitting a PDF with the appropriate discussion, equations and figures.

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