22.67 2023 Principles of Plasma Diagnostics

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Pset 3: X-ray imaging

This problem set is based on the diagnostic presented in:

Schaeffer, D. B. et al. Measurements of electron temperature in high-energy-density plasmas using gated x-ray pinhole imaging. Review of Scientific Instruments 92, 043524 (2021) https://doi.org/10.1063/5.0043833

...which details the use of a pair of imaging diagnostics with different spectral filters to estimate the temperature of an optically-thin, bremsstrahlung emitting plasma. You do not need to read the entire paper, but the first two pages may be of interest. Our overall goal is to assess how well this diagnostic predicts plasma temperature in the case of inhomogeneous plasmas.

Start by considering a one-dimensional hydrogen (Z = 1) plasma of length L, with a uniform density n and temperature T, emitting bremsstrahlung radiation with emissivity $j(E) = n^2 T^{-1/2} \exp(-E/T)$ (resolved in terms of photon energy in eV, and dropping the constants).

1.1 Convince yourself that the total emission from the plasma for an observer looking along L is $I(E) = \int_0^L j(E) dx$. Neglect any opacity effects.

1.2 Grab filter tables from https://henke.lbl.gov/optical_constants/filter2.html for 6 µm and 12 µm Beryllium (Be). I recommend you generate a table with photon energies E between 10–3000 eV with 1000 linearly spaced data points. You can now calculate j and I for these energies (this saves interpolating between energies later on).

1.3 Calculate $N_i(T) = \int j(E,T) W_i(E) dE$ (eq. 1 of Schaeffer RSI 2021 above) for the two filter transmissions $W_{1,2}$ and for a range of plasma temperatures (I suggest 10–1000 eV). [1] Plot the ratio $R(T) = N_1(T)/N_2(T)$ of the two filters. What range of plasma temperatures are readily distinguishable using this filter pair?

1.4 For the plasma above, calculate the total emission through filters W_1 and W_2 using a range of temperatures (maybe 10 temperatures, including a couple of points above and below the range you think the filters work well in). Use the $R(T)$ calculated in the previous step as a look-up table to infer the plasma temperature T from the synthetically measured ratio R, for the ten or so temperatures you are using. They should match almost perfectly, but this step serves as a check that your code is functioning properly before the next steps. Plot a scatter plot of predicted temperature vs true temperature to be sure they match well.

[1] Note we assume the detector response $K(E)$ is flat for this exercise

In the citation above, the assumption is that T is uniform along the line of sight. Let us now consider what will happen when that assumption is relaxed.

2.1 Generate density and temperature profiles $n = n_0(1 + x^2/a^2)$ and $T = T_0/(1 + x^2/a^2)$ where $n(x > a) = T(x > a) = 0$ which approximates a chord through the center of a spherically symmetric isobaric ($p = nT = p_0$) plasma such as an ICF hotspot at stagnation.

2.2 Evaluate the spectrally resolved emission $I(E) = \int j(x, E)dx$ for a the same range of plasma temperatures you used above.

2.3 Calculate the total filtered emission $N_i = \int I(E)W_i(E)dE$ through the same filters W_1 and W_2, and by comparing their ratio R to the $R(T)$ found in 1.3 above, estimate the temperature in the plasma. Is it an over- or an under-estimate of the peak plasma temperature T_0?

2.4 A better comparison might be the density weighted average temperature, $\bar{T} = \int nTdx / \int ndx$. Is the predicted temperature closer to this quantity?

However, if we are really imaging an approximately spherically symmetric system, then we should be able to use an Abel inversion to back out a better estimate of the plasma temperature profile.

3.1 On an (x, y) grid, generate density and temperature profiles $n = n_0(1 + r^2/a^2)$ and $T = T_0/(1 + r^2/a^2)$ where $r = \sqrt{x^2 + y^2}$ and again we set $n(x > a) = T(x > a) = 0$ so that the Abel inversion can be performed.[2]

3.2 Evaluate the spectrally resolved emissivity on the grid $j(x, y, E) = n(x, y)^2 T(x, y)^{-1/2} \exp(-E/T)$ and then calculate the line integrated emission $I(y, E) = \int j(x, y, E)dx$ for a range of plasma temperatures.[3]

3.3 Calculate $N_i(y) = \int I(y, E)W_i(E)dE$ for the two filters as before. You can try to calculate their ratio R, but this will give a bad estimate for temperatures as we haven't tried to invert the forward Abel transform yet.

3.4 Have a go at Abel inverting $N_1(y)$ and $N_2(y)$.[4] Let's call the Abel inverted profiles $\tilde{N}_1(r)$ and $\tilde{N}_2(r)$.

3.5 By taking the ratio of $\tilde{N}_1(r)$ and $\tilde{N}_2(r)$ and comparing it to the ratio $R(T)$ found in 1.3, estimate $T(r)$.

3.6 Where there any steps in this procedure which you felt were hard to justify? Does the quality of the agreement between your initial and predicted temperature profile help to sooth any qualms you might have about this?

[2]See Hutchinson Fig 4.26 to remind you of the geometry.

[3]You could use an existing software package to carry out this step as a forward Abel transform, but it's much more fun to see how the sausage is actually made.

[4]You can do this numerically with Hutchinson's eqn 4.4.3 (think carefully about the limits!) or using a readily available module. You'll learn more by doing the former procedure, and it may be faster than learning how to wrangle your data into a format the existing software can use.

22.67J Principles of Plasma Diagnostics
Fall 2023