We have put some of the quiz stats here.

The mean was about 75%. And I must tell you that that is very impressive. I guess MIT undergrads never cease to amaze me. And this was not an easy quiz.

This was a relatively hard quiz.

And that average implies that you guys did well on a relatively hard quiz. Good.

Let's get back to our final lecture on amplifiers and small signal circuits. And as always let me start with a review. Very quickly -- -- we came up with a notation to represent small signals.

And our notation looked like this.

Our total variable was small and capital, and this was a DC bias and this was a small signal.

This is also called the operating point.

And the small signal is also called the incremental signal.

In general, if you have some function, some variable of interest in the circuit, say a total variable V out, let's say it relates to some input variable as F of VI.

So mathematically we can find out V out by simply finding the slope of this function at the operating point and then multiplying it by the incremental change in the input.

Gold standard math. So we do the slope of this function and evaluate it at the operating point.

So this would give us the slope of the function.

And multiply that by small VI, which is incremental change.

This is standard math. What this will tell you is given a small change in VI this function gives you, this expression gives you the small change in V out.

And in lecture we have pretty much used this method so far, used the math to get to where we wanted it to be.

And then the way we provided biasing and so on was for our amplifier in particular we had a bias voltage, some small signal value, VS.

And this was output which was also given to be some output operating point plus a small change, which was a change in the output voltage. So what we have done here is mathematically computed small V out.

And what I am showing you here is to get the same effect in a circuit is you build your circuit and replace what used to be a total variable with a DC bias plus a small change.

And then you will get your output here.

And this output will relate to this input using this expression.

So this is more review. To continue on with the math review, for our amplifier VO was given to be VS-K/2(vI-VT)^2 RL.

So this was the output versus input relationship for the amplifier. And mathematically I could get the small change in the output VO by simply differentiating this function with respect to VI, evaluating that function, at capital VI and multiplying by the small change in the input. And the resulting expression that we got for small VO -- -- was simply minus K, this was our DC value, and RL times small VI. So we derived all of this the last time. So nothing new so far.

So my small signal output was some function given by K(VI-VT)RL times small vi. And notice that this is how VI relates to VO. And this is a constant with respect to VI. V capital I is a DC bias, so this is a constant. So therefore this is the linear relationship that we had set out to get.

This term here, for reasons we will see today, this term here K(VI-VT) is called gm.

Transconductance. We will look at it in more detail a little later.

Even more review.

So I can draw the transfer function and plot VO versus VI.

Another way to graphically view what is going on is by plotting the load line curve for this circuit, so this is VI.

And I said we draw that by first plotting the -- These were our MOSFET curves. And we know that at some point the MOSFET gets into saturation, so this curve was iDS=K/2 VO^2.

And to the right side of the curve the MOSFET is in saturation. And we said we will adhere to the saturation discipline and operate in this regime.

When the MOSFET gets into this region it is in its triode region. And then we could draw the load line here. The load line codified the following relationship, iDS=VS/RL-VO/RL.

This was a load line. So I have superimposed a load line on the device characteristics, and I am going to show you a little demonstration based on that at this point. So these curves were drawn for increasing values of VI. And if I choose some operating point here then this point would correspond to some bias, this bias point would correspond to some input voltage VI, a corresponding output bias VO and a corresponding current iDS. So iDS capitals, VO capitals, VI capitals represent the operating point values for our little circuit.

So far there is nothing new. One thing we stopped the last time by pointing out that the gain of our amplifier, this is the gain, -K(VI-VT)RL.

That is the gain A of the amplifier.

That gain related to VI. A gain was proportional to VI-VT. So therefore if I increased VI, I would get more gain. So the question is how do we choose a bias point? And in our particular example, let's say we are free to play around with VI.

So we play around with VI and I can choose various bias points.

So where do you set the bias point?

What are the various characteristics of the circuit that relate to my bias point? Well, first, of course, is gain. The gain depends on how I choose VI. I will show you that in a moment. The second important thing, in other words, if I choose a bias point that is a small VI then my gain is going to be smaller.

If I choose a bias point that's at a much higher value of VI, I get a bigger gain. The second important consideration is operating range.

Notice that if I choose a bias point here then as the input changes -- Notice VI in this graph goes up or down, and I would be traversing and following different lines here in my MOSFET characteristic.

And as VI increases the operating point would come up here and so on. So if about this operating point I varied my input voltage VI then, so let's say about this operating point, if my input VI, my small signal VI varied about a small range then correspondingly the output value would vary about this part of my load line. So notice now that the operating range, how far can VI vary before the MOSFET goes out of its saturation discipline?

Well, on the low side my VI can come down to here.

And we looked at the operating ranges for an amplifier.

And I can come all the way down to VT.

At that point the output will come here.

Similarly at the high end VI could get up to a high value.

And we computed that value in the last lecture.

And the corresponding value of the input would be here.

So in some sense I can traverse all the way from here to here and have the MOSFET remain in saturation.

Remember we are not talking about linearity right now, just about the valid operating range based on my definition which is that the MOSFET should stay in saturation.

So if I chose my operating point here then I get this range here. And, on the other hand, if I chose my operating point to be here, for negative excursions of the input signal I have a very small amount before I hit cutoff. So if I chose my operating point here then for negative traversals of VI about the operating point I very quickly hit cutoff.

So if I want symmetric swings then this is the best that I can do in terms of the valid input operating range if I want symmetric swings given that this is my bias point.

On the other hand, if I chose my bias point somewhere here, or very carefully chose my bias point then my input can vary on a much wider region and still get symmetric swings. And so therefore the choice of bias point also influences the maximum swing range of my input signal. I shouldn't call this operating range. I should call it input swing range. We defined the valid input operating range as the range for which the amplifier satisfied the saturation discipline. So the two key issues, gain and the input swing. Let me show you a quick demo and try to point out on a graph some of the characteristics that relate to the matter we have been talking about so far.

So what I show here are these curves for the MOSFET.

This is VO and this iDS. This is the zero point.

Ignore this line down here. This line up here corresponds to the output voltage VO. What I am going to do now is, through some careful circuit hacking, I'm going to show show you a load line and show you the bias point, and show you how the bias point can be moved up and down by changing the input voltage which changes the corresponding output voltage.

It is hardly visible out there.

Is it there? OK.

It is not really clear, but notice that as I increase my input, I am increasing my input.

My output keeps coming down. And I hope your eyesight is better than mine because I don't see a dot up there.

I am amazed. This is the first time this has happened to me. That's OK.

All right. As you can see, as I change the input value the output operating point changes, and the dot out there traverses, articulates a load line. I guess I have to believe that there is a dot out there. Next what I will do is show you some more fun stuff. What I will do is instead of having just a dot by having a DC voltage, let me apply an input sinusoid. So if I apply an input sinusoid at some bias then I should see an articulation of the corresponding region of the load line corresponding to the input.

So, as you can see here, now the bottom line, here is my input and this is my output.

And notice that this the region of the load line articulated when the input is of this magnitude.

Now let's have some fun. As I increase my input, you can see that a larger portion of the load line is articulated, right? There you go.

And as I decrease my input a smaller region of the load line is articulated. Let's leave it here for a moment. And what I will do next, this is the region here that we are looking at, let me increase the bias. If I increase the bias, if I increase VI, what do you think should happen to this line here? Well, if I increase the bias, the line should go up, right?

Because remember the dot? The dot is in the middle of this thing here. If I increase the bias this should move up here. So that line moves up.

Do you expect anything else to happen to that line?

Pardon? It increases, exactly. If I increase the bias point to here then this must also increase because my gain has increased. Let me do that.

So let me increase the input bias.

Indeed notice that the region of the load line articulated is larger now. Let me decrease the bias.

And notice that because the gain is smaller the little segment shown is also smaller. I have shown you two things so far. One is that I as I increase my bias the line indeed rises up corresponding to a higher value for the input operating point. And the second is that I get a larger swing in the output as I increase the bias.

Just to show that for those like me who were visually challenged in terms of viewing that little dot up there, let me get some audio so you can actually hear the sinusoidal tone. It is a big annoying.

As I reduce the bias the gain is decreased.

As I increase the bias you can see that the gain is increased and the tone is louder. Let's have some more fun and let's play some music now. And what I am going to show you with the music -- The reason I play the music is not just for fun. Well, it's 85% fun and 15% learning. Can we turn it on for a second?

What I would like to do is, as we play the music, the reason I am playing the music for that 15% is so you can listen to distortion. I want you to listen to the distortion. That is when the articulation is here you are not going to get much distortion.

But as I get into cutoff you should be getting a bunch of distortion. Similarly, as you get into the triode region you should also be getting distortion because the amplification from being somewhat nonlinear here becomes highly nonlinear at those two points.

So let's just play the signal. So volume increases, or rather the amplitude increases by increasing the bias. Now you should hear the volume go down and distortion.

So notice now that the bias point is way down here.

So the gain is very low, and plus there is a distortion because of cutoff. Now what I will do is blast it up here, and you will see that the volume has gone up but then you see distortion again. Let's see if you can stand the volume here.

Even the CD doesn't like that.

Notice that as I went up here the volume kept increasing because the gain kept increasing, but as I got into the triode region I began to lose my gain because, remember, the amplifier doesn't have gain in the triode region, MOSFET in its triode region, and we also get a bunch of distortion out there. Finally, it turns out that as people are building amplifiers -- I think this was in the mid to late `50s and `60s and so on.

They said man, electrical engineers are not going to get their thing right. So they invented a new kind of music which was much more tolerant to distortion.

And I will play that music for you.

It is called hard rock. I challenge you to tell me it is distorting.

Sounds good to me.

OK. All right.

That'll do it. Thank you.

I hope there are no hard rock musicians in here who will come and beat me up after lecture or something.

All right. Believe it or not most of that was review. There is nothing new today besides some fun and games and so on.

I will give you a breather for five seconds before jumping into something even more fun.

I want you to look at the middle board here.

And, as I told you in the beginning of 6.002, engineering is about building useful systems.

Engineering is not about showing off at math or saying man, I am really cool in math and stuff.

Engineering is about building useful systems, and you want to find the simplest, easiest, cheapest way to get there. Unlike deep areas of math and theory and so on, the beauty is in the simplicity. So the aesthetics are in how simply can we make things and still get to where we want to be? All through the course what you will be seeing happening again and again and again is when things begin to get too grovelly in terms of math, we will step back and say oops, we are engineers, remember? Let's find a much simpler way to do it and use intuition. So time and time and time again, I am going to take you on a simpler path where you can solve things by inspection by pure intuition.

Most circuit designers do that. So take a look at this.

I don't like this nasty differentiation here.

That's getting into late high school calculus and so on.

Let's avoid the math and let's see if you find some way of doing it that is even much more simpler.

And that is what I will do next and show you what is called the small signal circuit view. A purely circuit way of developing the small signal model.

So let me just start by drawing the large signal equivalent circuit for you. I will draw it here for reasons that will be obvious at the end of the class.

All right. This is the large signal equivalent circuit model for our MOSFET amplifier.

VS and here is my current source.

iDS relates to the square of VI minus VT.

So stare at that for a second. And that is a nonlinear circuit. iDS relates to the square of VI minus VT. Let me start by making the following claim. Let me shoot from the hip here and make the following grand claim, and then I will show you how I can prove that claim. The grand claim I am about to make says the following. A bunch of little devices here.

It is a nonlinear circuit. Just suppose for a moment we do a Gedanken experiment. Suppose I replace each of my circuit elements here with its linearized element equivalent.

In other words, here is a VS source, here is a dependent current source, let me replace them with their linear equivalent circuit models.

In other words, with their corresponding small signal element models. And I will show you what those are in a second. The resistor has a corresponding small signal element.

The dependent current source has a corresponding small signal behavioral element model. And what I am going to do is keep the same circuit connections and simply pull out the large signal model for the element and replace it with a small signal element model. And by the nature of the small signal model they are all going to be linear.

So what I am going to be left with is a linear circuit with simple linear circuit elements in there.

And then once I have a linear circuit, I should be able to analyze that linear circuit using methods 1, 2 and 3, superposition, Thevenin, node method and so on. And certainly the intuitive methods like superposition and Thevenin, which make life a lot easier for me with linear models, and thereby get the function that I am looking for very quickly.

Again, my claim is that I can replace each of these large signal models by just small signal equivalents and then just analyze the resultant circuit. And I claim that I should be able to get the same answer. That's a claim.

All right? So what I will do is give you an informal proof for why I can do that.

And I also ask you to refer to Section 8.2.1 of the course notes to go through the foundations of the small circuit model in more detail. The intuition is that, remember KVL and KCL? I can write down KVL and KCL for every loop in that circuit and every node in that circuit.

If I do KVL and KCL, I will end up with something like this. For the input loop I get VI something or the other applying KVL.

For the output loop I get V out something or the other.

And then applying KCL I get some other equation in iDS.

So here are my KVL and KCL equations for that circuit.

Now, KVL and KCL are simply a different representation of the circuit because within those KVL and KCL is encoded the topology of the circuit. Remember each KVL equation represents a loop and each KCL equation represents how nodes are connected together. So KVL and KCL equations encode within them the topology of my circuit.

What I do next is, say, I replace each of these with the bias plus the small signal, so I get the bias plus the small signal and keep the equations the same.

All I have done in my big set of KVL, KCL equations, I have simply replaced the total variable with the large signal variable and the small signal quantity.

Then comes a key trick. The key trick is that because the bias point variables, they are a valid solution to the circuit. The circuit is in this quiescent state, and those are valid solutions to circuit. So therefore I can cancel them out. So the VI, the large signal values can be cancelled out leaving just small signal variables in there. So from the KVL, KCL equations I can cancel out the large signal values, the DC bias points because they satisfy the KVL and KCL themselves. In other words, I could have written VI plus V out and so on.

Since they are satisfied I just strike out the large signal variable from both sides of each of these equations, so what is left is the same KVL, KCL equations but with small variables in place of the big variables.

What that should tell you, this informal proof should tell you is that the small signal variables should then satisfy the same form of the KVL, KCL equations that the total variables satisfy. And because the KVL, KCL equations are a reflection of the topology of the circuit, what that says is that the small signal variables must also satisfy KVL and KCL. And since these arrive from the small signal elements that says that I can replace the big elements with the small elements and KVL and KCL will hold for the resulting circuit. This is a very quick breeze through, an informal proof to show that I can replace the big elements with the corresponding little element models and then simply apply linear techniques. Refer to Section 8.2.1 for more foundations and more discussion about the foundations for why we can do this. That brings up the small signal circuit method. The circuit method for small signal analysis has three steps. The first step is find operating point by using LS. First you analyze your large signal circuit and find the operating point.

You have to do this, because remember, the small signal models depend on the operating point values.

Remember the gain of our amplifier depended on the bias point. Second step is develop small signal models of elements. Second step is take each of the elements in your circuit and find their equivalent small circuit model for each of the elements.

Third step is replace original elements with their small signal model elements. Third step is simply take the large elements and replace them with their small signal equivalent models. Then analyze resulting circuit, and that circuit will be a linear circuit.

So let's do an example. I will just use the amplifier as an example of this method. And convince you that you are going to get the same expression at the end, but just so, so simply without even the smallest amount of grubby math.

Three steps. The first step is to find the operating point using the large signal model.

And let me just do that here. I get my V out = VS-K/2(VI-VT)^2 RL. Let me just write down that out here. Don't worry about copying that down. It is on the last page of your notes. The first step of the method simply applies the large signal model and finds out the behavior of that circuit to find out what the bias point values are.

The second step is to develop the small signal model of my elements. How do I go about developing the small signal models of elements?

Let's start with the MOSFET. The large signal model for the MOSFET looks like this.

Here is my Vgs. This is my gate.

This is my drain. This is my source.

And I know my iDS to be K/2(Vgs-VT)^2.

So this is the large signal model for the MOSFET, again in saturation. I am talking about all of these models are under the saturation discipline.

So Vgs relates to iDS in the following way for the MOSFET.

That is iDS, is K/2 and that is my square law relationship. So what is a corresponding small signal model? I go ahead and start with this.

The corresponding small signal model simply says that iDS relates to Vgs in the following way.

All I have to do is find a small signal equivalent where I need to find out, given a small change in the input Vgs, what is the small change in the iDS?

So I can apply my standard trick to a much simpler expression here, which is iDS simply, I differentiate this function with respect to Vgs.

So I don't completely eliminate the math here, but it is a much simpler problem here.

At Vgs equals the bias point times small vgs.

I can find the small change in iDS corresponding to a small change in the input using this expression.

That gives me iDS as simply K(Vgs-VT) vgs.

I call this gm, and I will tell you why in a second. So what does this expression say? This expression says that if I have a small change in Vgs then this will be my small change in iDS. Notice that the resulting small signal model is also a dependent current source.

It is a voltage controlled dependent current source.

So the output is the current, and it is a dependent current source and it depends on the input voltage.

The good news is that notice that this one, this expression here gm is a constant related to the bias point values. Therefore, notice that the small signal model for the MOSFET in saturation, not surprisingly, is a linear voltage controlled current source according to the following expression.

So iDS=gm Vgs. Gm is a representation for K(Vgs-VT) and is called a transconductance.

It is called a transconductance because it, in some sense, deflects the conductance properties of this based on the input. So it is a transconductance.

So this value is called Vgs. Therefore, I can build the small signal model as follows. Vgs is a voltage controlled current source and iDS is simply gm Vgs.

So this is my gate, drain, source.

So that is the small signal model for my MOSFET.

As a next step what are the other elements in my circuit?

Let's see. I have a voltage source and I have a resistor, so let me find out the corresponding small signal model for a DC supply VS.

This is Page 7. I will do it mathematically for you, but often times it is always good to do a sanity check using intuition. Let me ask you, the large signal for a DC supply looks like this.

The element law for a voltage source is VS equals some capital VS. It is a constant voltage.

So what do you expect to be the small signal model for a voltage source? In other words, for a small change, suppose I have a small change in the current, by how much should the output VS change? It shouldn't change.

It is a voltage source. So what does intuition tell you is a small signal model for the voltage source?

A short. So the key here is that a voltage source behaves like a short circuit for small perturbations. In other words, if I change the current flowing through it by a small amount somehow, the output is still going to held at VS.

In other words, small signals are simply going to scoot through this voltage source without having any impact whatsoever on the voltage. Or mathematically I could also do small vs is del by del IS of VS evaluated at IS equals some capital IS times small IS. And therefore VS equals zero.

What that means is that the small signal model for my voltage source is simply a short circuit.

So in a small circuit voltage sources appear like a short circuit. Finally, I have a resistor, my resistor R. Let me find out its corresponding small signal model.

The large signal model looks like this R, VR, IR. And I know that VR is simply RIR. And to find the small signal equivalent I do del of IRR divided by del IR for IR calculated at some constant value times small IR.

What I am looking to do is to find out what is the change in the voltage across R for a small perturbation in the current?

Again, let me exhort you to rely on intuition to at least sanity check your answers. So what do you think this should look like? It's a resistor and I have a small change in the current, by what do you expect the voltage to change? Think about, for the next five seconds, what the small signal model for this should look like and then I will go ahead and write down the answer.

So differentiating I simply get RIR.

In other words, for a resistor the small signal model is the resistor itself.

So what I have done so far, let me just take you through where we are right now, give you the big picture there.

I began by suggesting that looking to find an even simpler way to do small signal analysis. I gave you an informal proof to show that if I had small signal element models for all of my elements, I could simply replace them in the circuit and then do a corresponding linear circuit analysis phase to get the result I am looking for. There are three steps to the method. As a first step we began by finding small signal models for each of our elements.

For the nonlinear MOSFET the small signal model was a linear dependent current source. For a voltage source the corresponding small signal model was a short circuit.

Again, that makes sense intuitively if I change the current through a voltage source by a small amount.

By how much does the voltage change?

It is a voltage source, silly.

The voltage doesn't change. So the small signal V, the small change in the voltage is zero, and that is the same thing as a short circuit. For a resistor by how much does the voltage change if I change the current by a small amount?

Well, it will change by R times the current change, and that is the property of a resistor, R.

As a final step what I would like to do, on Page 8, I'd like to very quickly draw for you the small signal circuit and then analyze it. This is the large signal circuit. That is a large signal circuit.

And let me draw the small signal circuit.

And the method says simply pluck out, gouge out each of these elements. And simply replace each of these nasty nonlinear elements with the corresponding small signal linear equivalents. So let's do that.

Remember, for the input you replace input with its small signal voltage because I am telling you that it's sourcing a small change in VI. So that is VI.

And then I replace a short for VS.

I replace an R for RL because it is an RL itself for the small signal model. And then for the dependent source, we discovered that the dependent source was a linear dependent source given where ids=gmvi.

Remember, this was my small VO. Here you go.

I have a small signal circuit here where I have simply created that by replacing each of the big elements by little rinky-dink elements. Now these are all linear elements so I can do a really simple linear analysis.

What method shall we use? Well, this is so simple.

I will just go ahead and use the node method.

So applying the node method at the node with voltage VO, what I will do is the current going up, VO divided by RL equals the current going down iDS.

And so the current going up is VO divided by RL and the current going down is -- Oops, I should have done this.

The total current going out is zero, so the sum of these two is zero. That is my good old node method here. And I know that iDS is simply gmvi equals zero. So right there I have the relationship between VO and VI. So VO is simply minus gmviRL.

And remember gm was simply K VI minus VT.

We are done, OK?

What have we here? I created a linear circuit which simply comprised small signal models for each of my big elements. And then I simply did a straightforward linear analysis using any one of the linear techniques I knew about. This is simple enough so I apply the node method. And I've got the equation at this node, simplified it and I directly got the answer.

In one or two steps I directly gave you the output as a function of the input. It can't get any simpler.

Thank you.