MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.013 – Electromagnetics and Applications

Problem Set 5

Suggested Reading: Course notes, Sections 9.5.1 - 9.5.3, 9.2.1 - 9.2.4. 9.1, 9.2.6.

Problem 5.1

A certain pair of Polaroid sheets transmits light with minimum transmission (10^{-6}) when their attenuating axes are at right angles, and with an ideal 0.5 transmission when these axes are aligned. We wish to design a light intensity modulator by electrically controlling the permittivity ε_{fast} along the fast axis of a perfectly transparent plate of thickness d inserted between these two polaroids, where $\varepsilon_{fast} = \varepsilon_{slow} + 10^{-5}V$, $\varepsilon_{slow} = 2\varepsilon_o$ is the permittivity of the orthogonal slow axis, and V is the control voltage applied along the fast axis.

(a) In terms of d and ε_o, what control voltage V yields a quarter-wave plate (QWP) between the polaroids, and what is the maximum fraction F of the incident light that can then pass through the modulator when the two Polaroid sheets are at right angles (i.e. blocking ~all light in the absence of the QWP)? By what angle θ is this QWP rotated relative to the polaroids when transmission through the three sheets is maximum? Would a half-wave plate perform better and, if so, how? Explain briefly.

(b) This optimum modulator can then vary the fraction, F, of transmitted power between 10^{-6} and the maximum value found in part (a), depending on V. Find the expression for F(V,d).

(c) If the transparent dielectric plate has $\varepsilon = 4\varepsilon_o$, what fraction F_r of the incident wave power is reflected by the plate's front surface at normal incidence? A nearly comparable amount is reflected by its back surface as well.

(d) What is Brewster's angle θ_B for the plate of part (c)?

Problem 5.2

(a) Consider a metal with roughly one free electron per atom and assume a frequency such that it behaves like a plasma with $\mu = \mu_o$, $\varepsilon = \varepsilon_o$, and $n_e = 10^{30}$ [m^{-3}]. What is the skin or penetration depth δ [m] of radio waves into this plasma in the low-frequency limit ($\omega \to 0$)? Appendix A.4 of the Course Notes contains useful physical constants.

DHS 3/18/09

(b) For a metal with conductivity $\sigma = 10^7$ [S/m], at what frequency f [Hz] is the penetration "skin depth" δ the same as for (a)?

Problem 5.3

John knows that the electron density in the ionosphere generally increases with altitude sufficiently slowly that upward bound radio waves suffer very little reflection except below the plasma frequency and at incidence angles beyond the critical angle θ_c. John can hear his local 1-MHz radio station WMIT out to the suburbs but then loses it as he drives further, until suddenly about 200 miles away he can again hear WMIT; from this he concludes $\theta_c = 45^o$.

(a) What is the ionospheric plasma frequency f_p [Hz] on this occasion?

(b) For those waves impacting the ionosphere at 45° the incident time-averaged intensity, $\langle \overline{S}(t) \rangle$, is 1 mW/m^2. What is the complex Pointing vector, $\underline{\overline{S}}$, just inside the ionosphere at this position when the wave is incident exactly at the critical angle, $\theta_i = \theta_c = 45°$?

Problem 5.4

(a) An x-polarized 1-GHz uniform plane wave of one watt per square meter is propagating in the +z direction in a very slightly lossy medium characterized by ε_o, μ_o, and $\sigma = 10^{-4}$ [S/m]. What are the complex electric and magnetic fields $\underline{\overline{E}}(z)$ and $\underline{\overline{H}}(z)$ in this case? Are they in phase?

(b) The same wave is now propagating in the +z direction in vacuum and is reflected from a perfect conductor at $z = 0$. What now are the complex electric and magnetic fields $\underline{\overline{E}}(z)$ and $\underline{\overline{H}}(z)$ for z < 0? Are $\overline{E}$ and $\overline{H}$ in phase in time and space? Explain briefly.

(c) What are the total $\underline{\overline{E}}(x,z)$ and $\underline{\overline{H}}(x,z)$ for the case where a TM uniform plane wave of wavelength λ is propagating in the x-z plane for z < 0 and is reflected at 45° incidence angle from a perfect conductor at z = 0? Assume $\underline{\overline{H}}(0,0) = H_o$, the maximum value of $\underline{\overline{H}}(x,z)$.

(d) Sketch the total electric field $\overline{E}(x,y,t)$ for z < 0 and t = 0 for the waves of part (c).

6.013 Electromagnetics and Applications
Spring 2009