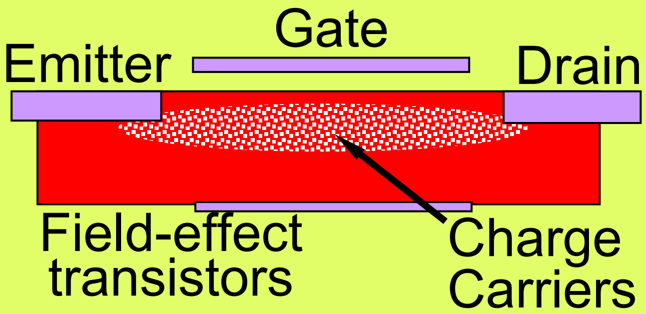
# LIMITS TO COMPUTATION SPEED

## Devices:



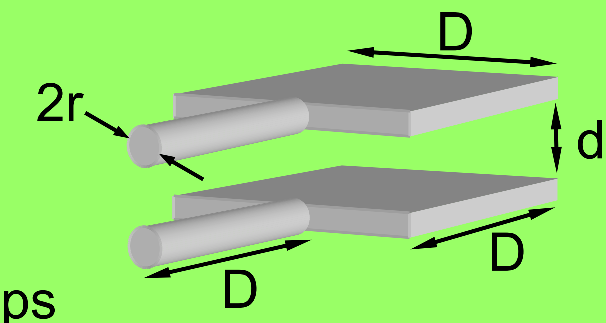

Carrier transit and diffusion times

($f = ma$, $v < c$)

$RC \cong \varepsilon/\sigma$; RL, LC time constants

Beyond scope of 6.013 (read Section 8.2)

## Interconnect, short lines $<<\lambda$:




Wire resistance $R \propto D/r^2$

Capacitance $C = \varepsilon A/d \propto D^2/d$

$\tau = RC \propto D^3/r^2d \cong$ const. if D:r:d = const.

R is high for polysilicon, C is high for thin gaps

L/R and $\tau = \sqrt{LC}$ scale well with size and do not limit speed

## Interconnect, long lines $>\sim\lambda/8$:

Propagation delay: $c = 1/\sqrt{\mu\varepsilon} < 3 \times 10^8$ [m s$^{-1}$] ($\varepsilon$ might be $\sim 2\varepsilon_o$)

Reflections at wire and device junctions, unless carefully designed

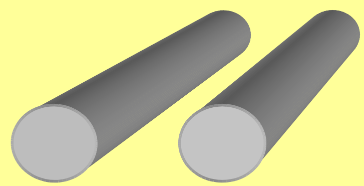
Resistive loss

Radiation and cross-talk (3-GHz clocks imply 30-GHz harmonics)

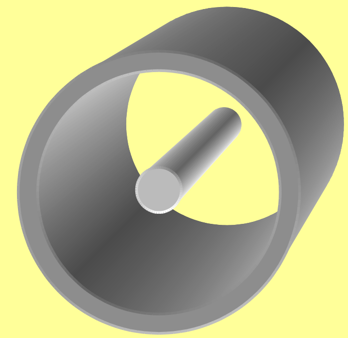
# WIRED INTERCONNECTIONS

## Transverse EM Transmission Lines:

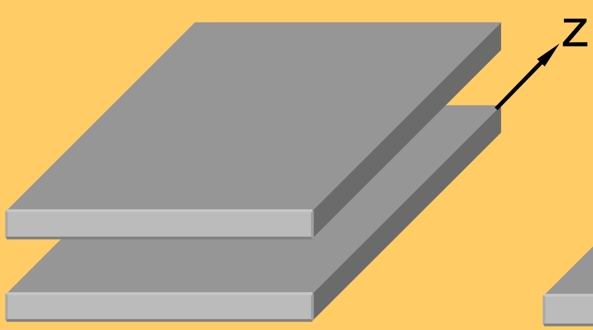
TEM: $\overline{E}_z = \overline{H}_z = 0$



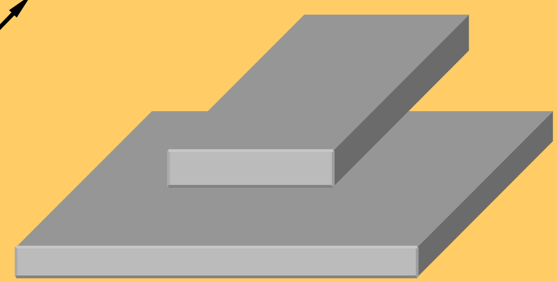
Parallel wires



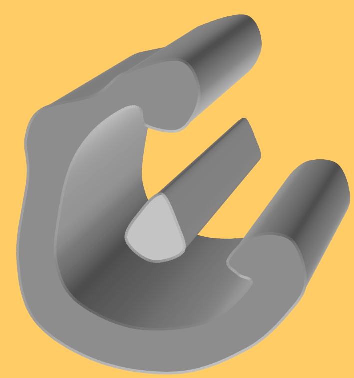
Coaxial cable




Parallel
plates



Stripline



Arbitrary cross-section
$\neq f(z)$

# PARALLEL-PLATE TRANSMISSION LINE

## Boundary Conditions:

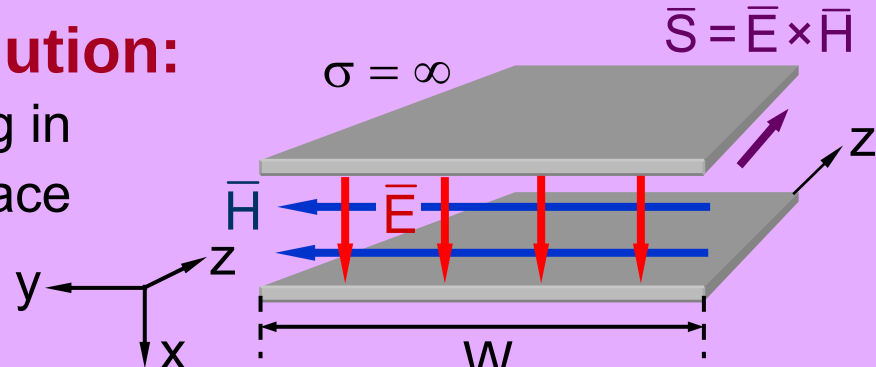
$\overline{E}_{//} = \overline{H}_{\perp} = 0$ at perfect conductors

## Uniform Plane Wave Solution:

x-polarized wave propagating in +z direction in free space

$$\overline{E} = \hat{x}\, E_{+}\left(t - \frac{z}{c}\right)$$

$$\overline{H} = \hat{y}\, \left(\frac{1}{\eta_o}\right) E_{+}\left(t - \frac{z}{c}\right)$$



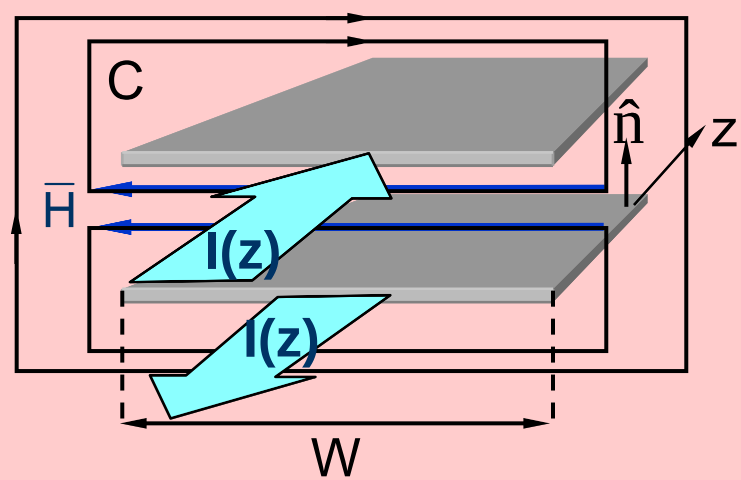

## Currents in Plates:

$$\oint_C \overline{H} \bullet d\overline{s} = \iint_A \overline{J} \bullet d\overline{a} = I(z)$$

I(z) = H(z)W, independent of path C

## Surface Currents $J_s$(A m$^{-1}$):

$$\overline{J}_s(z) = \hat{n} \times \overline{H}(z) \quad [\mathrm{A\ m^{-1}}]$$



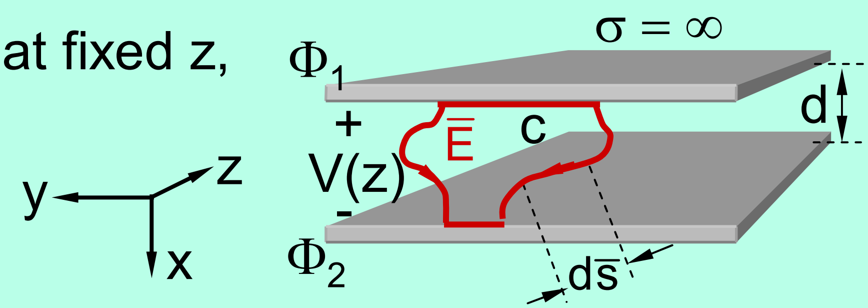
# TRANSMISSION LINE VOLTAGES

## Voltages between plates:

Since $H_z = 0 \Rightarrow \oint_c \overline{E} \bullet d\overline{s} = 0$ at fixed z,

$\int_1^2 \overline{E} \bullet d\overline{s} = \Phi_1 - \Phi_2 = V(z)$

V(z) is uniquely defined




## Surface charge density $\rho_s(z)$ [C m$^{-2}$]:

$\hat{n} \bullet \varepsilon \overline{E}(z) = \rho_s(z)$ (Boundary condition; from $\nabla \bullet \overline{D} = \rho$

## Integrate $\overline{E}, \overline{H}$ to find $v(t,z), i(t,z)$

$v(t,z) = \int_1^2 \overline{E} \bullet d\overline{s} = d \times E_+(t - z/c)$ here, where $\overline{E} = \hat{x}E_+(t - z/c)$

$i(t,z) = \oint_c \overline{H} \bullet d\overline{s} = (W/\eta_o)E_+(t - z/c)$, where $\overline{H} = \hat{y}E_+(t - z/c)$

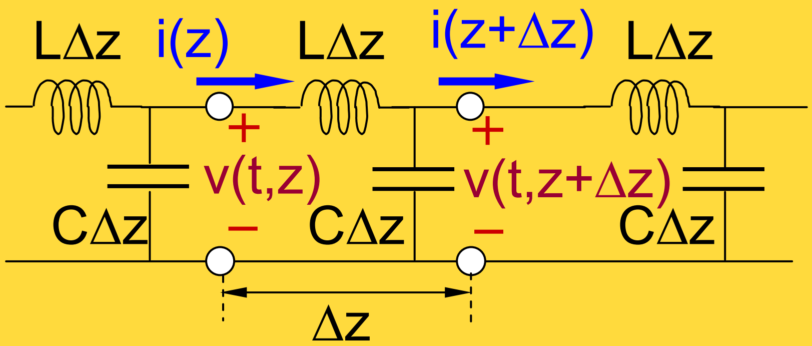
$v(t,z) = Z_o\, i(t,z)$ [if there is no backward propagating wave]

$Z_o = \eta_o d/W$ [ohms] "Characteristic impedance"

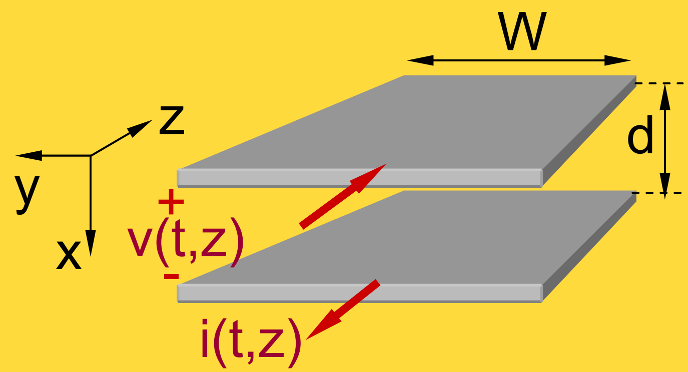
Note: v(z) violates KVL, and i(z) violates KCL

# TELEGRAPHER'S EQUATIONS

## Equivalent Circuit:




L [Henries m$^{-1}$], C [Farads m$^{-1}$]




$$\overline{E}\bullet\hat{z} = \overline{H}\bullet\hat{z} = 0 \quad \text{(TEM)}$$

## Difference Equations:

$$v(z+\Delta z) - v(z) = -L\Delta z\frac{di(z)}{dt}$$

$$i(z+\Delta z) - i(z) = -C\Delta z\frac{dv(z)}{dt}$$

Limit as $\Delta z \to 0$:

$$\underbrace{\frac{dv(z)}{dz} = -L\frac{di(z)}{dt} \qquad \frac{di(z)}{dz} = -C\frac{dv(z)}{dt}}$$

Wave Equation: $$\boxed{\frac{d^2v}{dz^2} = LC\frac{d^2v}{dt^2}}$$

# SOLUTION: TELEGRAPHER'S EQUATIONS

**Wave Equation:** $\frac{d^2v}{dz^2} = LC\frac{d^2v}{dt^2}$

**Solution:**

$v(z,t) = f_+(t - z/c) + f_-(t + z/c)$

$f_+$ and $f_-$ are arbitrary functions

**Substituting into Wave Equation:**

$(1/c^2)\,[f_+''(t - z/c) + f_-''(t + z/c)] = LC\,[f_+''(t - z/c) + f_-''(t + z/c)]$

Therefore: $c = 1/\sqrt{LC} = 1/\sqrt{\mu\varepsilon}$

**Current I(z,t):**

Recall: $\frac{di(z)}{dz} = -C\frac{dv(z)}{dt} = -C[f_+'(t - z/c) + f_-'(t + z/c)]$

Therefore: $i(z,t) = cC\,[f_+(t - z/c) - f_-(t + z/c)]$

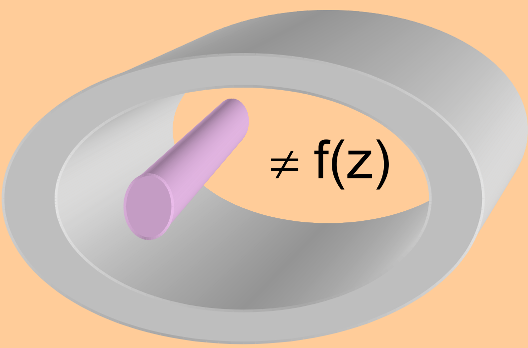
Where: $cC = C/\sqrt{LC} = \sqrt{C/L} = Y_o$ "Characteristic admittance"

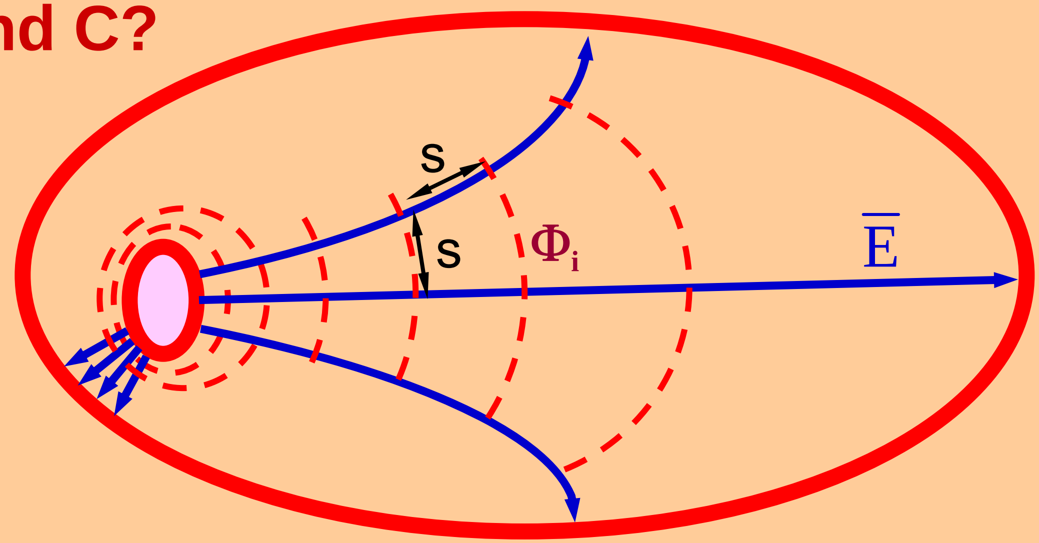
$Z_o = 1/Y_o = \sqrt{L/C}$ ohms "Characteristic impedance"

Therefore: $i(z,t) = Y_o[f_+(t - z/c) - f_-(t + z/c)]$

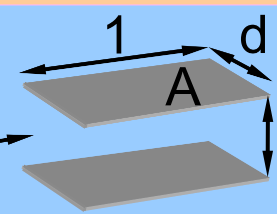
# ARBITRARY TEM LINES

**Can we estimate L and C?**



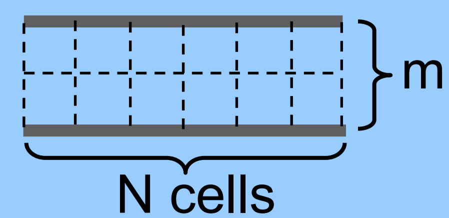

$Z_o = \sqrt{\dfrac{L}{C}} = \dfrac{\sqrt{LC}}{C}$ where $LC = \mu\varepsilon$




$C_{\square}$ = capacitance/m = $\dfrac{\varepsilon A}{d} = \varepsilon$

$C = nC_{\square}$ for n parallel square incremental capacitors ($C_{\square} = \varepsilon$ [Fm$^{-1}$])

$C = C_{\square}/m$ for m capacitors $C_{\square}$ in series

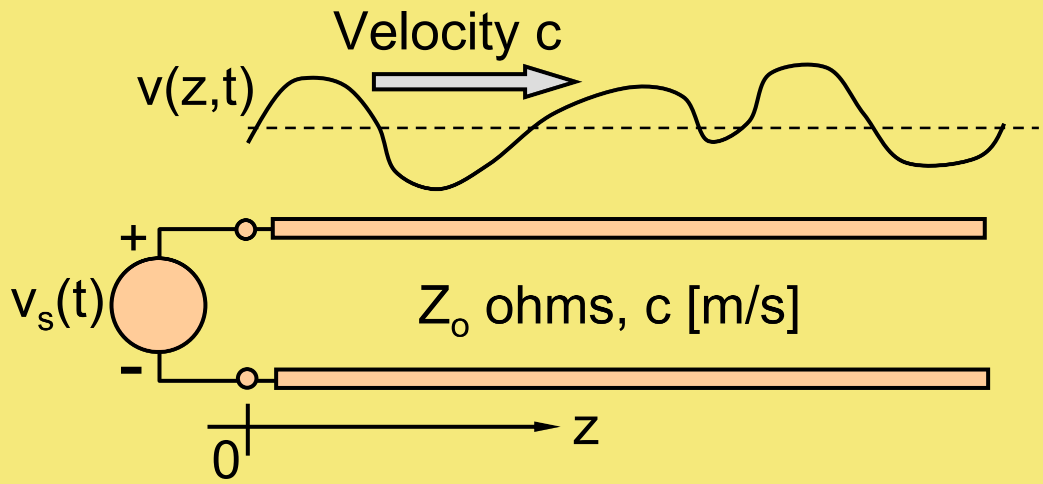



Therefore:

$C = nC_{\square}/m = n\varepsilon/m$ [Farads/m]

$Z_o = \dfrac{\sqrt{LC}}{C} = \dfrac{\sqrt{\mu\varepsilon}}{n\varepsilon/m} = \eta_o \dfrac{m}{n}$ ohms [ $\eta_o = \sqrt{\mu/\varepsilon}$]; $Z_{o\,single\,cell} = 377\Omega$

# TRANSMISSION LINE VOLTAGES




Matching boundary conditions:

v(t) and I(t) are continuous at z = 0

$$\Rightarrow \quad v(z,t) = v_s(t - z/c)$$

$$i(z,t) = \frac{1}{Z_o} v_s(t - z/c)$$