

## LECTURE 16

### Markov Processes – I

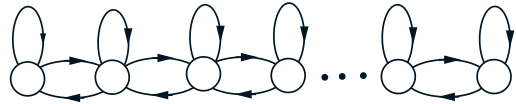
- **Readings:** Sections 7.1–7.2

#### Lecture outline

- Checkout counter example
- Markov process definition
- $n$ -step transition probabilities
- Classification of states

### Checkout counter model

- Discrete time  $n = 0, 1, \dots$
- Customer arrivals: Bernoulli( $p$ )
  - geometric interarrival times
- Customer service times: geometric( $q$ )
- “State”  $X_n$ : number of customers at time  $n$



### Finite state Markov chains

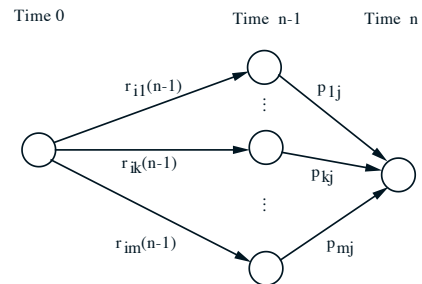
- $X_n$ : state after  $n$  transitions
  - belongs to a finite set, e.g.,  $\{1, \dots, m\}$
  - $X_0$  is either given or random
- **Markov property/assumption:** (given current state, the past does not matter)
 
$$p_{ij} = \mathbf{P}(X_{n+1} = j \mid X_n = i)$$

$$= \mathbf{P}(X_{n+1} = j \mid X_n = i, X_{n-1}, \dots, X_0)$$
- Model specification:
  - identify the possible states
  - identify the possible transitions
  - identify the transition probabilities

### $n$ -step transition probabilities

- State occupancy probabilities, given initial state  $i$ :

$$r_{ij}(n) = \mathbf{P}(X_n = j \mid X_0 = i)$$



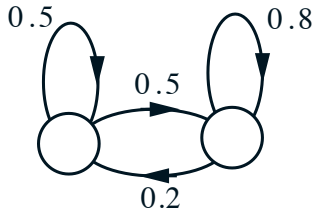
- Key recursion:

$$r_{ij}(n) = \sum_{k=1}^m r_{ik}(n-1)p_{kj}$$

- With random initial state:

$$\mathbf{P}(X_n = j) = \sum_{i=1}^m \mathbf{P}(X_0 = i)r_{ij}(n)$$

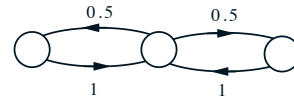
**Example**



	$n = 0$	$n = 1$	$n = 2$	$n = 100$	$n = 101$
$r_{11}(n)$					
$r_{12}(n)$					
$r_{21}(n)$					
$r_{22}(n)$					

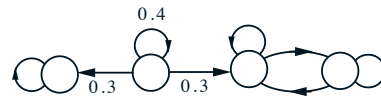
**Generic convergence questions:**

- Does  $r_{ij}(n)$  converge to something?



n odd:  $r_{22}(n) =$       n even:  $r_{22}(n) =$

- Does the limit depend on initial state?



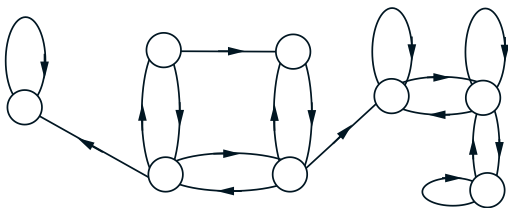
$r_{11}(n) =$

$r_{31}(n) =$

$r_{21}(n) =$

**Recurrent and transient states**

- State  $i$  is **recurrent** if:  
starting from  $i$ ,  
and from wherever you can go,  
there is a way of returning to  $i$
- If not recurrent, called **transient**



–  $i$  transient:  
 $\mathbf{P}(X_n = i) \rightarrow 0$ ,  
 $i$  visited finite number of times

- **Recurrent class:**  
collection of recurrent states that  
“communicate” with each other  
and with no other state

MIT OpenCourseWare  
<http://ocw.mit.edu>

6.041SC Probabilistic Systems Analysis and Applied Probability  
Fall 2013

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.