You're standing at a traffic intersection.

And you start to accelerate when the light turns green.

Suppose that your acceleration as a function of time is a constant for some time interval t less than t one.

And after that, it's zero for a time after t one less than t less than some time t two.

At the exact same instant the light turns green, a bicyclist is coming through the intersection.

And the bicyclist has some initial speed and is braking with an acceleration of minus b two for the entire time interval t two.

And at time t two, the bicyclist comes to rest exactly where you are located.

And we also know some initial conditions.

So our initial conditions in this problem are that you're accelerating b one at a rate two meters per second squared.

And you do this for time t one equals one second.

And the bicyclist comes into the intersection.

We'll call that b two naught.

That's the initial speed of the bicyclist at three meters per second.

And the question is, what is the rate of deceleration of the bicyclist b two?

Now this can be quite a complicated problem.

So the first thing we want to do is just make a sketch and think about what's involved.

This problem involves two objects.

You and the bicyclist.

The person-- that's you-- has two stages of motion.

And the bicyclist only has one stage of motion.

So to get started, it always helps to choose a coordinate system and to make some sketches of the problem.

So let's say we choose a-- it's all one dimensional motion.

Two objects.

One dimensional motion.

And so we'll pick an origin at the light at the one side of the intersection.

And we have two objects which we'll talk about.

You, x one.

And the bicycle is x two.

Actually we don't know yet who's in front of the other.

The bicyclist will be first in front of you.

So now how do we sketch the motion of these two stages of motion?

So let's make a sketch.

And let's start with the person.

Well, the person-- if we plotted their position as a function of time-- this would be position in general.

I'll just draw the person function.

They're accelerating to time t one.

And then they're moving at a constant speed at time t two.

Now the bicyclist is a little more complicated.

Because initially the bicyclist has a-- this at time t, person one.

Initially the bicyclist has a non-zero slope.

And they're decelerating.

And they reach you with a zero slope.

So this graph, this is the x two.

That is the bicyclist.

And right here we have the person.

x one.

So now to build a strategy, we can even look at our graph and see that from our initial conditions, we have some special conditions that the-- our strategy will be to-- one-- figure out what this time is.

And we know that the bicyclist at time t two has come to a stop.

So that's one condition.

And we also know that the bicyclist comes to stop exactly at the same position as the person.

x one of t two equals x two of t two.

So those are two conditions that we can deduce from all of this given information.

And now we comply our kinematic relationships for both the bicyclist and the person and try to see if these conditions will enable us to deduce what b two is.

So let's begin with the bicyclist.

So the velocity of the bicyclist as a function of time is simply the integration of that bicyclist b t prime from zero to t two.

This is one stage of motion.

The acceleration is minus b two.

So this is a very straightforward interval.

This is just b of two t two.

b two.

This is minus the initial speed equals that.

b of t two minus V of the initial is that.

And because we want this to be zero, we have the condition that t two equals V two naught divided by b two.

So that's our first condition for the bicyclist.

Now we have to separately solve for the bicyclist's position.

That's easy.

x two of t is the integral of V two t prime dt prime from zero to t two.

And that's just minus one half.

We want to make sure that we get the displacement.

But x two naught is zero.

So we have x two at t two equals the integral of the velocity function, which is V two naught minus b two of t prime, d t prime from zero to t two.

And so we get b two naught t two minus one half b two t two squared.

And when we input this condition in for t two, this becomes very simply V two naught squared over two b two.

Substituting t two into each of these expressions gives us that relationship.

So that's the position of the cyclist at time t two.

Now this is a little bit trickier to get the position of the person.

So in order to do that, we first find the velocity of the person function.

It's a two stage motion.

So for the first stage of motion, the velocity two-- the velocity of person one-- minus their initial velocity, which is zero minus one zero.

That's zero.

Equals the integral of b one dt prime from zero to t one, which is just b one t one.

And this velocity remains constant throughout the next interval.

So we can write the velocity function in the following way.

V one t equals b one t for zero less than t less than t one.

And afterwards, a constant velocity.

Now this is the function that we need to integrate to get the displacement.

So let's get ourselves a little room here and integrate that.

And we have x of t is two integrals.

First from zero to t one, the velocity function during that time interval.

And then for the second time interval dt two, the velocity function is constant b-- this is b one.

b one t one, dt prime.

Notice this is not a variable.

But it is the time at the end of the interval.

And when we make these two intervals, we get one half b one t one squared.

Let's make this the velocity at time t.

This first integral goes from zero to t one.

And the second interval, we're going to make this the position at time t two.

And we get plus V one times t one times t two minus t one.

We have a common term, t one squared, b, one half b one t one squared.

b one minus b one, t one squared.

So this reduces to one half b one t one squared plus b one, t one, t two.

And that's how we find the position of the person for our interval.

Let's just review that to make sure.

Because we had to get the velocity function first.

And then we integrated the velocity in each time interval correctly in order to get the position function.

Now we can apply our conditions.

Notice we already know t two here.

And we can now apply the second condition which says that the position of the bicyclist at time t two, which we found to be d naught squared over two b two is equal to the position of the person at that same time.

So that's minus one half b one t one squared plus b one t one.

Now let's make that substitution for time t two.

So that's V two naught over b two.

And now our problem is to solve for this time b two.

And we're given b one.

We're given t one.

We're given V two naught.

And the only variable here is b two.

It's a little bit of algebra to rearrange terms.

What I'll do is I'll bring this term over to here.

So now we'll just do a little bit of algebra.

We have to a b two we can pull out.

I have a minus b one t one b two naught.

And that's equal to minus one half b one t one squared.

And now I can solve for b two.

And so I get b two is equal to Vt naught squared minus b one t one b two naught over minus one half b one t one squared.

Now let's just do a quick dimensional check.

b times t has the dimensions of velocity.

So this is velocity squared, velocity squared.

That's OK upstairs.

b times t squared is dimensions of position.

So what we have is meters squared per second squared divided by meters.

That gives us meters per second squared.

So we're pretty confident that we at least didn't make an algebraic mistake.

And now our last step is to substitute in the numbers.

And what we get is, if we put in the three meters a second squared minus b one times t one times three, we get upstairs is minus 3/2.

And downstairs is two times one second.

Two's cancel.

So we get to 3/2 meters per second squared when we put in the numbers.

If we wanted to check our result, we can then see what time we get.

t two is is three meters per second divided by our b two which is 3/2 meters per second squared.

So that's 2 seconds.

And now the last check would be to see that the position functions correspond to that.

Let's see if we can just do that quickly in our heads.

Our position function for the person is V naught squared over two b two.

So that's 9 meters per second.

9 meters squared seconds squared over two times 3/2 meters second squared.

And that comes out to x two of t.

This is a check, is 3 meters.

And the x one of t.

We left out the one there.

We should have had it.

It's a little more complicated to put in here.

But we'll just run the numbers quickly through.

Two times one seconds minus.

That's a minus one.

Two times one times two.

That's two times two.

So this is also three meters.

And we actually have the right answer here.