Let's now consider two dimensional motion, and let's try to analyze how to describe the change in velocity.

So again, let's choose a coordinate system.

We have an origin plus y plus x.

And let's draw the trajectory of our object.

And now let's draw the object at two different times.

So for instance, if I call this the location at time t1, and a little bit later here, this is the location of the object at time t2.

We'll call our unit vectors i hat and j hat.

We know that the direction of the velocity is tangent to this curve.

So if we draw v at time t1-- and over here, notice the direction has changed v at time t2.

And what we'd like to do now is describe, just as before, that our acceleration a of t is the derivative of the velocity as a function of time.

What that means is the limit as delta t goes to 0 of delta v over delta t.

Now, it's much harder to visualize the delta v in this drawing.

And partly, the reason for that is these velocity vectors are located at two different points.

And right now, the backs of these vectors have different places in space.

But remember that delta v is just v, in this case, at time t2 minus v at time t1.

And our principle for subtracting two vectors at different locations in space is to draw the vectors where we put the tails at the same location.

So here's a tail at this vector.

We're just going to translate that vector in space.

That is still v at time t1.

These vectors are equal.

They have the same length, and they have the same direction.

And so delta v is just the vector that connects here to there.

That's what we mean by delta v.

And so you can see in this particular case that it's not obvious from looking at the orbit what the delta v is.

So what we need to do is just trust our calculus.

And so when we write the velocity as dx dt i hat plus dy dy j hat, and we're now treating each direction independently.

We call this vx i hat plus vy j hat.

So that's our velocity vector.

Then our acceleration is just the derivative of the velocity.

We take each direction separately, so we have dv x dt i hat plus dv y dt j hat.

Now, again, notice that velocity v of x is already the first derivative of the position of the exponent function.

So what we really have here is the second derivative of the position function in the i hat direction and the second derivative of the component function in the y direction.

And that is what we call the instantaneous acceleration.

Now, again, this is sometimes awkward to draw, but you always must remember that this x component of the acceleration by definition is the second derivative of the component function or the first derivative of the component function for the velocity.

And likewise, the y component of the acceleration ay is the second derivative of the component function for position.

And that's also equal, by definition, to the first derivative of the component of the velocity vector.

And that's how we describe the acceleration.

As before, we can talk about the magnitude of a vector.

And the magnitude of a we'll just write as a.

It's the components squared, added together, taken square root.

And that's our magnitude.

And so now we've described all of our kinematic quantities in two dimensions-- the position, the velocity as the derivative of the position, and the acceleration as the derivative of the velocity where each direction is treated independently.