

We would now like to consider a system of particles.

And let's draw them as 1, 2, 3.

And let's refer to this one as the j particle of mass m_j .

And these particles, system of particles is in a gravitational field.

And I would like to consider the torque about some point s for this system of particles.

And in particular, I'd like to know how do I treat the gravitational force that's acting on all of these individual particles.

Well, the way we'll analyze this is recall that the definition of torque about some point.

Now we have n particles labeled by index j .

It's a vector from the place we're calculating the torque to where the j -th particle is located.

So we draw that vector r_{sj} .

And we cross that with the force that's acting on the j -th particle, which in this case is the gravitational force $m_j g$.

And this is the torque about s , and it's this complicated sum.

But the sum will simplify in the following way.

Let's just write out a couple of terms to see what it actually looks like.

So we have $r_{s1} \times m_1 g$ plus $r_{s2} \times m_2 g$.

And we just keep on going for the n terms.

Now quantity m_1 is the scalar units of kilograms in SI units.

But the cross product is between this vector and that vector.

So I can actually rewrite this as $m_1 r_{s1} \times g$ plus $m_2 r_{s2} \times g$, et cetera.

And you see that the g term is the same for every single object.

So I can write this sum-- j goes from 1 to n as $m_j r_{sj}$ and pull the g out of the sum, because every term has the

same \mathbf{g} in the cross product.

Now let's focus on the meaning of this sum, where we're weighting position by mass.

Recall that if you have the center of mass of an object, and we wanted to find out where is the location of the center of mass with respect to point \mathbf{s} , then we can calculate this vector $\mathbf{r}_{s,cm}$ by our definition that the location of the center of mass with respect to \mathbf{s} is given by the sum $\sum_{j=1}^n m_j \mathbf{r}_{sj}$.

And we divide that by the total mass m_j .

We'll denote this term by m_{total} for the total mass.

So what we see here is the total mass times $\mathbf{R}_{s,cm}$ is equal to exactly the sum that we have in that expression $\sum m_j \mathbf{r}_{sj}$.

That's precisely that term.

And so we can conclude that the torque about the center of mass is $m_{total} \mathbf{R}_{s,cm} \times \mathbf{g}$.

Now remember again m_{total} is a scalar.

The cross product is between these two vectors.

And so finally, we write this as $\mathbf{R}_{s,cm} \times m_{total} \mathbf{g}$.

And this is how we apply the gravitational force to a system of particles.

Now what does that mean?

Well, that's just denote our system like this, and let's say here is the center of mass, and here's the point \mathbf{s} .

So what we need to do is we need to draw the vector $\mathbf{R}_{s,cm}$, and apply all the gravitational force at the center of mass.

And then our torque about \mathbf{s} is the vector \mathbf{R}_s to where the force is acting cross $m_{total} \mathbf{g}$.

So in conclusion, when you have a rigid body or a system of particles, and you want to calculate the torque due to a uniform gravitational field, then we place-- the total gravitational force is acting at the center of mass.

And that gives us the torque about \mathbf{s} to the individual torques of the system of the gravitational force about \mathbf{s} .

All the force is placed at the center of mass.

