

So far we've defined work in one dimension.

But let's consider the path of a particle in more than one dimension.

And on this screen it looks like a two-dimensional path, and here's our particle.

And what we'd like to do is define the concept of work for this type of path.

Now let's say that at this instant our particle is experiencing a force F .

And the particle is being displaced by a distance, ds , which is always tangent to the path.

And what we'd like to do now is talk about the work done for this particle.

Now what we'd like to first say is what we're going to do is imagine that we're going to break the path down into a bunch of small intervals.

And let's call this the j -th point and the j -th plus 1 point.

Then our particle will start at j and we'll displace it Δs_j to the j -th plus 1 position on the path.

And here, let's denote the force by F_j .

And now what we'd like to do is define the work for this particular section.

Now our concept of work is the force in the direction of the displacement.

So what I'd like to do is I'd like to define the quantity F_j , and I put a little parentheses upstairs-- parallel lines upstairs.

This is the component of the force F_j in the direction of Δs_j .

Now the way we denote that is we can then say that if I take the dot product of $F_j \cdot \Delta s_j$, this is the component of the force in the direction of the motion.

Because remember, our dot product, if we call this angle θ_j , is taking how much of one vector is in the direction of the other times the magnitude of that displacement.

And again, when we write that component in our notation, if we call this angle θ_j , we know that that's the magnitude of F_j times sine of-- cosine of θ_j .

Now because this is the amount of force in the direction of the motion, that's how we've defined work done.

And we'll symbolize the work done in taking the particle from j to $j + 1$ by the symbol W_j .

And now, this is a scalar quantity.

And when we want to find the total work, what we have to add up is from some initial point to some final point, how much work is being done in taking our particle from the initial position to the final position.

And it's just the sum for all of these j 's, and let's say we have n of them of W_j .

And right now we're going to call this W , it's just that scalar sum, And, that's equal to the sum j goes from 1 to n of $F_j \cdot \Delta s_j$.

So now we'll use the vector dot product because we already know that this is how much of the force is in the direction of the motion.

However, this answer depends on how fine we broke up this path.

And what we've seen many times now is that the actual work that we want, let's write this as n because it depends on the number of paths, is the limit as n goes to infinity of this sum of scalar amounts of work.

Now when we take that limit, as n goes to infinity, of this sum, j equals 1 to n of $F_j \cdot \Delta s_j$, this quantity is formerly what we mean by a line integral.

And we denote that with a new notation.

We'll denote a line from our initial to final and it's a dot product of the force dotted into the small displacement.

Now whenever we see the dot product in an integral that distinguishes it from a normal integral, and this is what we call a line integral.

Now again, why did we switch from our Δs 's to ds 's?

Because when we take the limit, as n goes to infinity, our grid becomes finer, and finer, and finer until we're shrinking down to a point where we can just talk about the differential displacement, ds , and the force acting on a point.

And that's what our line integral is.

Now, what we'll learn next is how to calculate this line integral for a number of important physical examples that

occur in our interest in mechanics.