When we analyzed a one-dimensional elastic collision in any frame and we had a velocity V1 initial and V2 initial, that we saw that we could reduce-- this is a one-dimensional elastic collision-- we always have the energy condition and the momentum condition.

But when we combine the energy movement together, we found the following idea that the relative velocity V1, 2, which we called the relative velocity initial, was just equal to the final relative velocity.

The statement there was that V1 initial minus V2 initial is equal to minus V1 final minus V2 final.

And we call this the energy momentum relation for our classical mechanics.

And then we could combine that with our conservation of energy law, conservation of momentum.

And we've got a linear system of equations that is much easier to solve than the quadratic system.

Now what I'd like to show is that this concept of relative velocity, that V1, 2 relative velocity is independent of the choice of reference frame.

And so if we want to analyze a collision in any other reference frame, then we always can keep this result. So now let's look at that.

So again, let's imagine that we have two particles, where we have particle 1 and particle 2.

We have some origin, r1 and r2.

And now we want to choose another reference frame.

So suppose that we pick a second reference frame, which has maybe some origin over here.

And we have the vectors r1 prime and r2 prime.

And the relative vector from the center of one reference frame to the center of the other reference frame.

So what we have are the two conditions that r1 is equal to capital R plus r1 prime.

r1 is capital R plus r1 prime.

r2 is capital R plus r2 prime.

And now if we subtract these two equations, we have r1 minus r2 equals r1 prime minus r2 prime.

And this shows us that the relative position vectors-- and let's draw that this one from 2 to 1.

And even in our diagram, we can see that the vector from 2 to 1 does not depend on the choice of reference frame.

And even more importantly, when we differentiate, we get that V1 minus V2 is equal to V1 prime minus V2 prime.

And that becomes our statement that the relative velocity vector is independent of the choice of reference frame.