In kinematics, if we know acceleration, which we generally can get from Newton's second law, then we'd like to integrate the acceleration to get velocity and position.

How does that work for circular motion?

Well, when you think about circular motion and you think about the tangential direction, then that, in some sense, is a one-dimensional motion.

And so what we'll see is that the tangential description of this motion can be integrated to get the tangential velocity and position, just in a similar way that we did with linear motion, where if we were given the acceleration, we integrated velocity and position.

So let's look at a particular example.

Suppose we're given that the tangential component of the acceleration is given by some simple polynomial, A minus Bt.

Now, in this case, the acceleration is certainly non-constant.

What we'd like to do-- remember, this is equal to rd squared theta dt squared.

So if we integrate the tangential component of the acceleration with some variable from t prime going from some initial value, which we can call 0, to some final value, then that will give us the change in the tangential velocity.

So for this case, V theta of t is equal to V theta t0 plus the integral of A minus B t prime d t prime.

Now, we needed an integration variable because recall that our functional dependence is the upper limit of the integral.

We saw that in one-dimensional kinematics.

So we're going from some t0 0 to some prime t prime.

Well, this integral is now-- we'll write this V theta 0, where, again, t0 we're calling 0.

And when we do this integral, A minus B t prime and d t prime-- that's an integral from 0 to t prime equals t.

t prime equals 0.

This integral is not hard to do.

We get A of t minus 1/2 B of t squared.

And there's that constant initial term.

And so the velocity at time t of the tangential component of the velocity is given by that expression.

In exactly the same fashion, the angle-- how much angle does this go through?

Well, we have to be careful there.

So if we want to do the arc length, s of t, we have to multiply the angle by the radius.

And that is just the integral of the tangential component of the velocity from t prime equal 0 to time t prime t.

And so in this example, we have a slightly complicated integral, V0 d t prime, 0 to t.

I'm going to drop-- well, I'll write it in-- t prime plus the integrals A t prime minus 1/2 B t prime squared.

I didn't need to split this up.

But I'm splitting it up, anyway.

And I have really one, two, three integrals to do.

And what I get is V theta 0 t-- the result of the first integral.

The result of the second interval is 1/2 At squared.

And the result of the third integral is minus 1/6 B t cubed.

And that gives me this.

I have to be a little careful on this side.

It's how much has the arc length changed as we go from time 0 to time t.

So that's the change in arc length.

And that's a typical example of where we're given tangential acceleration.

We integrate it to get the tangential velocity.

And then when we integrate the tangential velocity, we're getting the change in arc length around the circle.