

Let's consider a gravitational slingshot.

What is that?

Well, once in a while, people like to send spacecrafts out into the solar system, particularly the outer solar system, to explore what's going on there.

And because we can't on Earth give enough speed to these little spacecrafts, we need the big planets around us to help us a little.

And so we can consider a large planet like Jupiter or Saturn.

Here we have Saturn.

And if we have a little spacecraft and we make it fly by close, then what actually happens is it will, due to the gravitational attraction of Saturn, acquire a kick in velocity.

And that is a gravitational slingshot.

So let's look at that.

So our little spacecraft comes in with an initial velocity.

And Saturn, of course, also has a velocity.

And once it has passed, our little spacecraft will have a final velocity.

And in order to calculate what this final velocity is going to be, what the increase in speed is going to be, we need the concept of relative velocity.

And for that, we need to first consider some initial state.

So we have the relative velocity initially.

And that is the difference between those two velocities, between the spacecraft and Saturn.

And what becomes very important here is that we look at the coordinate system.

And keep that in mind.

Otherwise, we're going to get a few sign errors.

So the initial velocity of the spacecraft goes in the \hat{i} direction.

And the velocity-- the relative velocity is, of course, the difference, so minus the velocity of Saturn.

But that one goes in the minus \hat{i} direction.

And then we have the final state.

So V final relative.

And here we have the final velocity of the spacecraft now going in the minus \hat{i} direction minus the velocity of Saturn that also goes in the minus \hat{i} direction.

Now, there is one thing that we need to consider, which is this velocity of Saturn.

This one actually is, of course, here, the initial velocity of Saturn, and this one is the final one.

But because the mass of Saturn is much larger than the mass of the spacecraft, we can actually set the initial velocity of Saturn to the final velocity of Saturn.

So we can turn this-- we can take this away here again, and just consider one velocity of Saturn.

OK, good.

There's one more thing that we need in order to solve this, because we need to know how the relative velocities are actually related.

And the energy momentum law helps us there, because that gives us that the initial relative velocity equals minus the final relative velocity.

OK, so we can plug that in now.

What do we have here for the initial velocity?

We have V_i minus two \hat{i} 's gives us a plus the Saturn velocity.

And that equals-- minus minus gives us a plus-- the final velocity of the spacecraft.

And then we have three minuses here, so that gives us a minus the Saturn velocity.

And we know from the problem that the initial velocity here of the spacecraft was actually given at three Saturn velocities.

So we can tally this up now.

We have three-- we have three plus one is four.

And we'll put this over on the other side.

That gives us five.

Five Saturn velocities equals our final velocity.

So that's quite a good gain, I would say.

And it nicely illustrates why big planets like Jupiter and Saturn are really, really helpful for the exploration of the solar system.

And that is actually how the New Horizons mission made it out to Pluto, all the way out there.