

There's another vector operation, which we call the vector product and sometimes this operation is called the cross product.

And this is taking two vectors A , the operation cross, B . And this will give a new vector C .

And now we want to define that new vector C .

So let's draw a three dimensional picture where we have a plane.

And on this plane, we have two vectors A and B .

And those vectors are forming an angle θ between them.

Now a plane defines two unit normals.

I'll draw one up, which I'm going to call \hat{n} right hand rule.

And the reason for that right-hand rule is if we take $A \times B$, then our right hand thumb is pointing in the perpendicular direction to the plane.

Now I could have chosen a unit vector down, \hat{n} left-hand rule.

And this would correspond to taking my left hand and having $A \times B$ pointing down.

So the way we're going to define our cross-product is with the right-hand rule.

And so we define it like this.

That the vector C has magnitude, the magnitude of A times sine of θ , the magnitude of B , and its direction is given by the right-hand rule.

Now one of the reasons for this definition is let's draw a vector A and a vector B .

When you have two vectors, they define a magnitude in the following way.

That we can think about any two vectors define an area of a parallelogram.

And we can define that area as follows.

Let's drop a perpendicular.

And let's call this $B \perp$.

Then the area, which is a positive quantity, is given by-- and by the way, our angle θ here will be always positive-- so we're going to make it $0 < \theta < \pi$.

And that way $\sin \theta$ is always a positive quantity.

The area is the height, so that $B \sin \theta$ times the base, which is the magnitude of A , and so we can write that as $A \cdot B \sin \theta$, and the magnitude of $B \sin \theta$ is $B \sin \theta$.

So this quantity, $B \sin \theta$, is precisely what's occurring there.

So the magnitude of C is equal to the area formed by the vectors A and B . And we have a choice of which way we want to pick C to point.

And that's where, as a convention, we're choosing the right-hand rule.

Now again, there is a symmetry here in that I can also define the area of that triangle in the following way.

Let's write this, θ .

Instead of taking how much of B is perpendicular to the direction of A , let's drop the perpendicular this way, and write that as $A \sin \theta$.

And then the same area can be expressed as the magnitude of-- well, we'll express it as $A \sin \theta$ times B . And $A \sin \theta$ is the magnitude of $A \sin \theta$ times the magnitude of B .

And that's our same definition as before.

And so you see, a $\sin \theta$ is appearing over here.

So in either choice, our vector operation says take any two vectors.

Any two vectors forms a parallelogram.

The area of that parallelogram is the magnitude of the vector product.

And the direction of this new vector C is given by the right-hand rule with respect to that parallelogram in that sense.