We'd like to consider velocity-dependent resistive forces.

So let's imagine a situation where we have an object, and it's falling in a gravitational field, so we have a gravitational force.

And this object is like a marble falling in a vat of oil.

And there is a velocity-dependent resistive force that is acting on this object.

And let's again choose the positive direction down.

And now for objects that are moving very slowly in a fluid, let's make a model for our velocity-dependent resistive force.

We will assume that the resistive force is proportional to the velocity, it's opposing the direction of motion, and we'll put a coefficient alpha in front.

And this is our model for velocity-dependent resistive forces for objects that are moving very slowly.

For our case, let's consider the units of the coefficient alpha.

So the units of alpha have the units of force divided by velocity.

So the units of force are-- we have the units of force divided by the units of velocity, and that gives us kilogram meter per second squared, divided by the units of velocity, which are velocity per second.

And so we see that the coefficient of alpha has the units of kilogram per inverse second.

Now what we'd like to do is apply Newton's Second Law.

So let's analyze the forces and get the equation of motion for this object falling in a viscous medium with a resistive force.

In the j hat direction, we have the gravitational force.

And we have our resistive friction force, which we're writing Vy as the y component of the velocity.

And that's equal to m times dVy dt.

Now notice that the acceleration-- we're not treating as a constant.

It's a derivative of the velocity.

This leads us to our differential equation, which is written as dy dt-- I'll divide through by m-- g minus alpha over m, Vy.

Now this is a linear and velocity first order differential equation with a constant term in here, which is called inhomogeneous linear first order differential equation.

We shall solve this like we did with our other equations by separation of variables.

So what we do is we bring the velocity terms to one side and the time terms to the other side.

And now we can integrate both sides directly.

But before I integrate this side, I'd like to rewrite it by writing it as dVy-- and I'd like to pull out a minus alpha over m.

And this term here becomes minus mg over alpha.

Why did I do this?

For the integration, it just makes my natural logarithm interval look a little bit easier.

And over here, I had dt.

Now what I want to do is integrate.

And we're going to integrate from some initial velocity, which we're going to take to be 0, we're going to let this object be released from rest, and we're going to integrate to some final speed Vy of t.

Over here, let's just call our integration variable t prime.

And we integrate t prime from 0 to some time t.

So this is our variable and the velocity as a function of time.

Now doing this interval-- first off, this constant term comes out.

So we have 1 over minus alpha over m.

And the interval is just a natural log integral of Vy minus mg over alpha.

And we have to divide that in the lower limit, Vy is 0, and we have just minus mg over alpha.

And this side is very simply t.

Now we can rewrite this equation by bringing the m over alpha to the other side.

And we have log of Vy minus mg alpha, over minus mg alpha, equals minus alpha m t.

Now recall that e to the log of any quantity x is equal to x.

So if I exponentiate both sides, I get Vy minus mg over alpha, divided by minus mg alpha, is equal to the exponential e to the minus alpha m t.

Now the last step is simply to bring this over to there, multiply, and do a little bit of algebra.

And so I get that the velocity as a function of y is equal to mg alpha times 1 minus e to the minus alpha m t.

Now there's a few interesting things here that we want to look at.

Suppose we define this exponential e to the minus alpha m t as equal to e to the minus t over tau, where tau is m over alpha.

The units of tau, when we look at the units, it has the units of mass, which is kilograms, over the units of alpha, which we worked out before.

So we have kilograms over kilogram inverse second.

That gives us units of seconds.

And that's what we would expect because t seconds over seconds gives us dimensionless quantity for the exponential.

Now the last thing we'd like to look at is let's just see what happens when we plot this.

Well, if we're plotting Vy as a function of t, we start off at 0.

And we're going to reach some terminal quantity Vy terminal, which is when we have a graph like that.

And we can see what that terminal speed is when we set t equal to infinity, e to the minus infinity is 0.

And so this terminal speed is equal to just this coefficient, mg over alpha.

And that's the terminal speed.

Now notice, that as a check for what we did, we can come back to our differential equation, and what does terminal speed mean?

Notice that when you get to this limit, as t approaches infinity, the slope of the velocity curve is zero.

And that's really the statement over here that Vy terminal is when the velocity is no longer changing in time.

You can see it graphically.

And this is the statement, when we go back to Newton's Second Law, that acceleration is 0.

So the sum of the forces at terminal speed is 0, the resistive force is equal to the gravitational force, and so from this equation here, as a check, we can see that Vy terminal equals mg over alpha, which checks with our calculation.

And so we have confidence that we analyzed this correctly.

And that gives us our motion.

One last thing that's quite interesting.

Notice that the terminal speed depends on mass.

This coefficient alpha is only a function of the properties of the object-- the size, the shape, the geometric properties of the object, not how dense it is.

So if you have two objects, like two balls, same radius, but one was denser than the other, then the other mass would be heavier and the terminal speed would be greater.

So what we see is that two identical objects, one heavier than the other, the heavier object reaches a faster terminal speed than the lighter object, which explains a well observed phenomenon.