We've seen the rocket equation, and we've studied the example when there's no external forces.

Now, let's consider an example when the external force is not 0.

Well, one of our examples is just a rocket taking off in a gravitational field.

And so when we want to focus on the external forces, let's draw our rocket at time, t.

And now, consider-- what are the force diagrams on this rocket?

Well, we have a gravitational force, mg.

So we're going to choose j hat up.

So now, what does our rocket equation look like?

We wrote the exhaust velocity as minus mu j hat.

And so if we write this now as a vector equation in the j hat component, we get minus mg.

We have the rate that the rocket mass is changing, and this is in our-- we have this as-- remember, dm dr/dt is going to be negative.

So there's a minus.

And over here, we have another-- this is u because u is minus u.

And finally, we have m rocket dvr/dt.

Now, once again, we're going to try to integrate-- this is actually equal.

And we're going to try to integrate this equation to find a solution for the velocity as a function of time when all the fuel is burned.

So what we do is we'll multiply through by dt, and we have minus dmr times u equals mr dvr.

And this is mass of the rocket.

And now, let's divide through by mass of the rocket, so we have minus g minus dmr over mru equals dvr.

And once again, we can integrate this equation from some initial time to some final time.

I'll just denote that by i initial and i final, and what we see here is we have minus g times t final minus t initial because we're integrating with respect to time.

Here, we're integrating mass, so that's minus mu natural log.

And I'm going to write this as mr final over mr initial.

And over here, we have vr final minus vr initial.

And let's just remove that time, t.

And so now, here is our rocket equation.

Now, what we notice here is-- let's look at some rocket taking off.

So suppose that, at 10 initial equals 0, we have the special condition that vr initial equals 0 and mr initial equals the mass of the rocket-- dry mass.

That's all.

That's not counting the fuel.

Plus the total mass of the fuel.

And the final-- at t final, when the engine turns off-- we're going to try to find this velocity at t final.

And our condition for the final mass is all the fuel has been burned, so this is just the dry mass of the rocket.

And so what we get is we get minus g t final.

Now, I'm going to switch the sign here, so we have-- if I invert the log, that will give a minus sign.

And I have plus u log of mr of d plus mass of the fuel.

That's the initial mass of the rocket.

Divided by just the dry mass, and that's equal to vr at t final.

So one of the interesting questions that you might ask is-- how fast should you burn the fuel, and what will that have to do with the final speed?

So we can see from this expression that, if you burn the fuel for a very long period of time-- so t final is big-- then this piece will diminish the final velocity.

So if you want the fastest speed after you've burned the fuel, you want to burn the fuel so that t final is as short as possible.

We have the shortest burn time, will give us the largest velocity when all the fuel has been burned.