MITOCW | MIT8_01F16_L30v03_360p

So when we have a vector product of two vectors, A cross B equals C, let's compute that vector product in different coordinate systems.

So let's begin by choosing two vectors.

I hat, and J hat.

And notice they're at a right angle, and because there is a unit vector, the area here is equal to and 1.

And I want to define K hat to be equal to I hat cross J hat.

Now, our angle theta here was 90 degrees.

And so, if I use the right hand rule, then I hat, cross J hat is-- the right handed unit normal is out of the plane of the figure.

And so I would write K hat like that, and notice K hat is out of plane of figure.

And because I used my right hand rule, this is what we call a right handed coordinate system.

Now, there's something very nice about this cross product definition.

Notice that there's a cyclic order.

IJK is a cyclic order.

And if you interchange any two.

For example, JIK, that's anti-cyclic.

And the cross products satisfy this cyclic rule, in that, J cross K hat is I hat.

And notice JKI, maintains that cyclic order, and K hat cross I hat is J hat.

KIJ maintains that cyclic order, but because of the way we defined a cross-product in general A cross B is minus B cross A, because now you're using the opposite direction, so there's a minus sign.

And therefore, any anti-cyclic permutation of these unit vectors, as an example, K hat cross J hat, has to be-- notice I've-- is minus I hat-- that's anti-communitive property of the cross-product.

Similarly, I hat cross K hat is minus J hat.

And lastly, J hat cross I hat is minus K hat.

And so, in fact, you only need to know one.

And this idea of cyclic, and anti-cyclic, to be able to write down all of the other 6.

So we have one, two, three, four, five, six.

Now, when you want to compute the cross products in Cartesian coordinates, for instance, it can be a little bit messy.

There's going to be a lot of terms.

If I write a vector A as A X I hat plus A Y J hat plus A Z K hat.

And I write a vector B as BX I hat plus BY J hat plus BZ K hat.

And now I want to compute the cross-product of these vectors to get the new one.

Notice that there's going to be six terms, because I have I hat-- well, we actually should say one more thing.

that I hat cross I hat-- the angle between these two vectors is zero.

There's no perpendicular projection.

So that's zero, as is J hat cross J hat, K hat cross K hat.

So of these nine parts, when we take the cross-product, three of them will be 0, by this rule, and we'll we apply our cyclic or anti-cyclic rules for the others.

And so what we have here-- let's just do it in-- so C equals A cross B.

And now let's just go one by one.

I hat cross I hat.

That's 0.

There's no perpendicular part.

The area formed by these two vectors is 0.

I hat cross J hat.

That's cyclic.

IJ is plus K hat.

So our first non-zero term is AXBY K hat I hat cross J hat.

And now let's do I hat cross K hat.

Notice that's anti-cyclic.

I K minus J. So our next term is minus AX BZ J hat.

So there's the first two.

And now let's just continue this process.

J hat cross I hat.

That's anti-cyclic.

So we have minus AY BX K hat.

AY J hat across BX I hat.

J hat cross J hat.

That's 0.

So we have no contribution there.

And J hat crossed K hat, that is cyclic.

So that's plus AY BZ K hat.

And now we have our last two terms K hat cross I hat.

That cyclic.

So that's plus AZ BX J hat.

K hat cross J hat.

That's anti-cyclic.

So there's a minus I hat.

So it's minus AZ BY I hat.

And finally K hat cross K hat, well that's also 0.

So we have six terms.

And we can collect them equal to AX BY minus-- well let's check this one.

Here we used the wrong symbol here.

We have to be a little bit careful here.

AY cross BZ is J hat cross K hat.

That's plus I hat.

So we have AXBY minus AY BX K hat.

And now let's look at the I hat terms.

I'll just check those off.

We have AY BZ minus AZ BY I hat.

And check those two terms off.

And lastly, we have AZ BX minus AXPZ.

So we have AZBX minus AX BZ and that's J hat.

And that's how we calculate the cross-product in Cartesian coordinates.