

**PROFESSOR:** We all regularly drive with a car. We want to get to from point A to point B. Usually, we like to drive with a constant speed. But what happens if there's a traffic light? Well, we have to stop eventually, which means we have to brake. And that makes the car decelerate. So there are different parts of motion when we ride a car. It can be constant. It can be accelerating. It can be decelerating. So let's consider this in a little graphic.

We have a little car here. And it drives with constant speed until someone slams into the brakes, and the car will eventually stop. Let's add a timeline to this. Here we're going to put  $t$  equals 0. The point in time where the car stops driving with a constant speed is  $t_1$ , and the point where the car has come to a complete stop is  $t_2$ .

And the problem now asks for what is the position function? What is the distance at any kind of time here as the car goes through these two different phases of motion? So ultimately we want to get to  $x$  of  $t$ . But we can't actually just write this up. We need to first look at other information that we have about this kind of set up. And what we have is information on the acceleration.

And we have to consider these two phases separately. So we have here phase one and phase two. And phase one is from 0 to  $t_1$ . And phase two is from  $t_1$  to  $t_2$ . And we know when a car is going with a constant velocity, its acceleration is going to be 0. So that keeps life easy for phase one.

And then, when it breaks, well, we know it decelerates. So we're definitely going to have a minus sign. And it breaks in between  $t$  and  $t_1$ . And we need to multiply this factor here with a constant. So this is our acceleration information that the problem has given us. And we know, of course, that if we integrate the acceleration, we're going to get to the velocity function. And if we integrate that one again, we're going to get to our position function.

So let's do that first for phase one. And what is also important here is that we need to carefully consider the initial conditions. Again, what additional information is this problem giving us? So initial conditions-- they always make life a lot easier. So it's important to carefully look for them. And what that means is we want to know what the distance is at  $t$  equal 0, and what the velocity at time 0 is.

And we know the distance here is 0. And we know that the problem starts when the car is already driving with a constant speed. So that's going to be  $v$  not. So let's then make use of

this, and start integrating our acceleration. We want to get to the velocities. So  $v(t) - v(0)$  is going to be that integral. And here already we know that we have  $v(0) = v_0$ .

So this already looks pretty nice. And now we have our integral here of  $a$ . And of course, now  $t$  is an integration variable. So we have to give this a prime. And we're integrating from  $t = 0$  to  $t = t$ . And when we plug this in, well,  $a$  is 0 in phase one, which means actually, our integral is going to be 0. That's handy. And from that, we will find that  $v(t) = v_0$ . All good.

Now we need to integrate this to get to our position function. Same thing again--  $x(t) - x(0) = \int_0^t v dt$ . Let's quickly check here what we know about  $x(0)$ . Well, it's 0 from the initial conditions. And now we have to integrate our velocity function here,  $v dt$ , again, from  $t = 0$  to  $t = t$ . These are our boundaries.

And this is simply going to be  $v_0 t$  evaluated from 0 to  $t$ . And we can add that-- we can add that in there. And it's just going to be  $v_0 t$ . So  $x(t) = v_0 t$ . I can just write it here again,  $x(t) = v_0 t$ . OK. Good. So half of the question is-- half of the problem is solved. Let's look at the same things for phase two.

So let's consider the initial conditions for phase two now. And again, now we want to-- in an analogous way-- look at the distance of what we know about the distance at  $t_1$  and the velocity at  $t_1$ . And about the position, we know from over there that this is  $v_0 t_1$ . So our previous result now actually becomes the initial condition for the next step, right? Because whatever happens here sets whatever happens afterwards. So that's nice. And we know that, up to this moment here, this car goes with the velocity of  $v_0$ .