Let's compare kinetic energies in a two particle one-dimensional collision in different reference frames.

So we could have one reference frame in which particle 1 is coming in and particle 2 is moving like that.

And in particular-- so we can call this the ground frame.

And now let's consider the center of mass frame.

And in the center of mass frame, let's remind you that when we have two different reference frames, the velocity-- we'll call this the ground frame g-- in the ground frame is equal to the velocity of the object-- we'll actually write it this way, just unprime, V1-- the velocity in the center of mass frame-- so this is the velocity in the cm frame-- plus the relative velocities between the frames.

And that's why that's the velocity of the center of mass.

So this was our rule for describing how velocities change in different reference frames.

And so we can draw the picture in the center of mass frame, V1 initial prime and V2 initial prime.

Now let's compare kinetic energies in these different frames.

So we know that the kinetic energy in the center of mass frame is just 1/2 m1 V1 initial squared prime-- put the prime there-- plus 1/2 V2 initial prime squared, kinetic energy in the center of mass frame.

How do we calculate the kinetic energy in the lab frame?

Well, that's a little bit more complicated.

And we'll need a little algebra to start that.

So let's put kinetic energy in the ground frame.

We know is 1/2 m1 V1 initial squared plus 1/2 m2 V2 initial squared.

Now what I have to do is use the law, the velocity relationship.

And this is going to take a little bit algebra.

We have m1-- I'll write V1 prime plus Vcm.

And remember that any quantity squared is the dot product, V dot V. So I'm going to dot this with itself.

V1 initial prime plus the center of mass.

And I have the second term which looks identical to this first term.

I'll write it all the way down here.

1/2 m2 V2 initial prime plus Vcm.

Vector dot scalar dot product of V2 initial prime plus Vcm.

Now when you take a dot product, remember there's four terms here.

There's V1 prime dot V1 prime, which is just V1 prime squared.

There's V center of mass dot V center of mass.

So that's V center of mass squared.

And then there's the cross term.

And because they're identical, there's a factor of 2.

And it will be repeated below.

So the kinetic energy in the ground frame is 1/2 m1.

So we'll take v1 i prime dotted with itself.

That's V1 i prime squared.

We have the cross term, which is a factor of 2, which will cancel this.

So the cross term is . m1.

That canceled the factor of 2.

V1 i prime dot Vcm plus the Vcm with itself.

So that's 1/2 m1 Vcm squared.

Now I have exactly the same thing on the next one.

So we'll write that down.

1/2 V2 i prime squared plus m2 V2i prime dot Vcm.

That's the same in both.

Plus 1/2 m2 Vcm squared.

Now let's look carefully at what we.

Have 1/2 m1 V1 prime squared.

This is an m2.

1/2 m2 V2 i prime squared.

So we have 1/2 m1 V1i prime squared plus 1/2 m2 V2i prime squared.

And you're already noticing that's the kinetic energy in the center of mass frame.

We have the total mass, 1/2 m1 plus m2 Vcm squared.

I can just put a little check to show which terms I've done so far.

And now here's the interesting one.

We have m1 V1i prime plus m2 V2i prime dot Vcm.

That represents this term and this term added together.

But recall that the center of mass reference frame is defined by the condition that the total momentum in that frame is zero.

So this term is zero.

And thus, we get that the kinetic energy in the ground frame is equal to the kinetic energy in the center of mass frame plus 1/2 m1 plus m2 times Vcm squared.

And that's how kinetic energy is in different reference frames.

And the next thing we'll look at is how that changes when we have a collision.