Let's review what we've done this week.

We've seen that the work done by a force along a trajectory from point A to point B can be written as the integral of the dot product of that force, with the displacement going from point A to point B, where we note that this integral, in general, must be evaluated over the particular path taken from point A to point B. This is called a line integral or a path integral.

And again, I want to emphasize the value of this integral depends, in general, on the path taken from point A to point B. And you could understand that if you considered the fact that, in general, the force is a function of position.

And so if you go along different paths, the dot product of the force with a particular path may be different, even though you're going between the same end points.

Now, the reason that the work is a useful concept is that it allows us to understand how the kinetic energy of an object-- so the kinetic energy is 1/2mv squared.

The work allows us to understand how the kinetic energy evolves under the action of a force.

And we see that through what's called the work-kinetic energy theorem, which states that the work done-- so again, the integral of F dot ds from point A to point B, evaluated along the specific path-- is equal to the change in the kinetic energy.

So that's 1/2mvB squared, since B is the final point, minus 1/2mvA squared, where A is the initial point.

And this is equal to the change in the kinetic energy.

And we derived this theorem from Newton's second law, F equals ma.

So this is essentially just a restatement of Newton's second law.

Now, two things to note about the work and this work integral-- the first is that the work integral, when evaluated, it just gives a number, because a dot product is a scalar.

And that number can be either positive, or negative, or even zero, depending upon whether the velocity increases, decreases, or stays the same under the action of the force, as you go from point A to point B.

The second point to emphasize is that, again, this integral, in general, must be evaluated along the specific path taken from point A to point B, which means that in order to evaluate the integral, we need to know what that path is.

Now, if we have a situation where we don't know in advance what the path is, then we can't evaluate the integral.

So in a way, it might seem like a not very useful concept, that we haven't really made progress, because we need to know the answer about what the trajectory is before we can calculate the work done.

And it's not clear why that's useful.

However, what I want to point out is that there are two special cases that are particularly useful.

The first special case is that of constrained motion.

Sometimes we know in advance what the path is going to be, because the motion is constrained.

We know that it's going to be moving along-- the object is going to be moving along a circular path or along some particular track of some particular shape.

And as long as we can describe that trajectory, we can evaluate our work integral along that trajectory.

So that's a case where even though the work is a path-dependent integral, we know the path, and so we can evaluate the integral.

The second special case we've seen is that there is a special class of forces called conservative forces.

And conservative forces have the property that the work integral is independent of the path.

So for a conservative force, to work integral only depends upon the endpoints A and B, and not upon the path you take to get from A to B. So for any path, you'll always get the same value of the work integral if the force is conservative.

OK, so those are the two cases in which we can usefully apply work and the work-energy theorem.

Now in general, one might have a combination of conservative and non-conservative forces acting on a system.

And in that case, we can actually write the work as consisting of two separate terms.

The work done by conservative forces, what I'll write as Wc, plus the work done by the non-conservative forces, Wnc.

And so the conservative work is just the work integral evaluated for the conservative forces, so Fc, for conservative force, dot ds, going from A to B.

And because this force is conservative, this integral is path independent.

Let me emphasize that here.

This is a path-independent integral.

It's not a line integral.

It only depends upon the endpoints, because the force is conservative.

The second term is the non-conservative work.

And that is the work done by the non-conservative force, F sub mc, dotted with ds, between the same end points, A and B.

But this integral does depend upon the specific path.

The path matters for this term, because there the force is non-conservatives.

So in general, you can compute the work by separating out the conservative force and the non-conservative force acting on the system, if both are acting.

If the force is only conservative, then you only have to worry about the path-independent integral, which in general is much simpler to evaluate.

If the force is non-conservative, then you need to specify the path in order to evaluate the integral.

Again, a path-dependent integral is often called a path integral or a line integral.