

For a rocket and an external force, we had the rocket equation, which we wrote as mass of the rocket-- now remember, this is a function of time-- times the derivative of the rocket dr/dt .

And we also had this second term that came from the ejecting fuel.

Now from our mass conservation equation, we can rewrite this equation as a differential relationship that the change of mass of the fuel in time is equal to minus the change of the rocket.

It doesn't matter which side we put the minus sign on.

So now I want to interpret this equation in a different way, and it will come back to what we mean by our system.

Let's first off bring this other term over to this side.

So we have plus $dM_r/dt u$.

And that's equal to $M_r dv_r/dt$.

Now separately, let's make this substitution again and go back to our fuel term.

So that's minus $dM_f/dt u$ equals $M_r dv_r/dt$.

Now notice that over here we have mass times the acceleration of the rocket.

So if we just rethought our system as simply the rocket-- M_r -- then we have two forces acting on the rocket.

The external force might be the gravitational field, but we have a new term here which we're going to refer to as thrust.

And so our thrust can be thought of as an external force simply on the rocket as a system.

And that's equal to minus the $f/dt u$.

Now again, if I chose \hat{j} up-- let's just look at this in components-- then our thrust as a vector, we'll write it as a y -component, is equal to minus dM_{fuel}/dt .

Now what is that relative speed?

Well, the fuel is being ejected backwards, so that's minus $u \hat{j}$.

The external force, by the way, would be minus $mg \hat{j}$ if it's near the surface of the Earth.

So we have another minus \hat{j} .

And we see that we have a positive thrust force in the vertical direction that is giving us an additional force other than the gravitational field.

And in Cartesian-- in our unit vectors here, we have minus mg plus $\frac{dM_{\text{fuel}}}{dt}$ times u is equal to $M \frac{dV}{dt}$ of the rocket.

And so this is the rocket equation in components.