8.02 **Spring 2006**

Experiment 7: Undriven & Driven RLC Circuits

OBJECTIVES

1. To explore the time dependent behavior of RLC Circuits, both driven (with an AC function generator) and undriven
2. To understand the idea of resonance, and to determine the behavior of current and voltage in a driven RLC circuit above, below and at the resonant frequency

PRE-LAB READING

INTRODUCTION

As most children know, if you get a push on a swing and just sit still on it, you will go back and forth, gradually slowing down to a stop. If, on the other hand, you move your body back and forth you can drive the swing, making it swing higher and higher. This only works if you move at the correct rate though – too fast or too slow and the swing will do nothing.

This is an example of resonance in a mechanical system. In this lab we will explore its electrical analog – the RLC (resistor, inductor, capacitor) circuit – and better understand what happens when it is undriven and when it is driven above, below and at the resonant frequency.

### The Details: Oscillations

In this lab you will be investigating current and voltages (EMFs) in RLC circuits. These oscillate as a function of time, either continuously (Fig. 1a, for driven circuits) or in a decaying fashion (Fig. 1b, for undriven circuits).
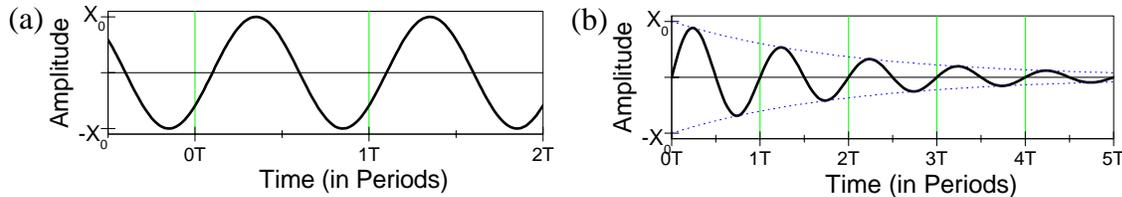
Figure 1 Oscillating Functions. (a) A purely oscillating function $x = x_0 \sin(\omega t + \varphi)$ has fixed amplitude x_0, angular frequency ω (period $T = 2\pi/\omega$ and frequency $f = \omega/2\pi$), and phase ϕ (in this case $\phi = -0.2\pi$). (b) The amplitude of a damped oscillating function decays exponentially (amplitude *envelope* indicated by dotted lines)

Undriven Circuits: Thinking about Oscillations

Consider the RLC circuit of fig. 2 below. The capacitor has an initial charge Q_0 (it was charged by a battery no longer in the circuit), but it can't go anywhere because the switch is open. When the switch is closed, the positive charge will flow off the top plate of the capacitor, through the resistor and inductor, and on to the bottom plate of the capacitor. This is the same behavior that we saw in RC circuits. In those circuits, however, the current flow stops as soon as all the positive charge has flowed to the negatively charged plate, leaving both plates with zero charge. The addition of an inductor, however, introduces inertia into the circuit, keeping the current flowing even when the capacitor is completely discharged, and forcing it to charge in the opposite polarity (Fig 2b).
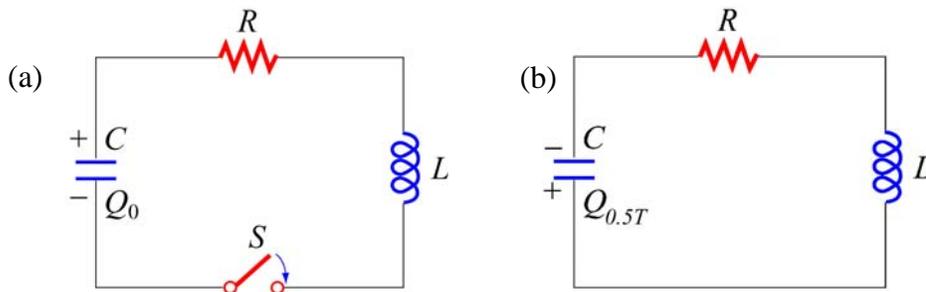

Figure 2 Undriven RLC circuit. (a) For t<0 the switch S is open and although the capacitor is charged ($Q = Q_0$) no current flows in the circuit. (b) A half period after closing the switch the capacitor again comes to a maximum charge, this time with the positive charge on the lower plate.

This oscillation of positive charge from the upper to lower plate of the capacitor is only one of the oscillations occurring in the circuit. For the two times pictured above (*t*=0 and *t*=0.5 *T*) the charge on the capacitor is a maximum and no current flows in the circuit. At intermediate times current is flowing, and, for example, at *t* = 0.25 *T* the current is a maximum and the charge on the capacitor is zero. Thus another oscillation in the circuit is between charge on the capacitor and current in the circuit. This corresponds to yet another oscillation in the circuit, that of energy between the capacitor and the inductor. When the capacitor is fully charged and the current is zero, the capacitor stores energy but the inductor doesn't ($U_C = Q^2/2C$; $U_L = \frac{1}{2}LI^2 = 0$). A quarter period later the current *I* is a maximum, charge *Q* = 0, and all the energy is in the inductor: $U_C = Q^2/2C = 0$; $U_L = \frac{1}{2}LI^2$. If there were no resistance in the circuit this swapping of energy between the capacitor and inductor would be perfect and the current (and voltage across the capacitor and EMF induced by the inductor) would oscillate as in Fig. 1a. A resistor, however, damps the circuit, removing energy by dissipating power through Joule heating ($P=I^2R$), and eventually ringing the current down to zero, as in Fig. 1b. Note that only the resistor dissipates power. The capacitor and inductor both store energy during half the cycle and then completely release it during the other half.

Driven Circuits: Resonance

Instead of simply charging the capacitor and then letting the system go, we could instead add a battery that periodically pushed current through the system. Such a battery is called an *AC* (alternating current) *function generator*, and the voltage it generates can oscillate with a given amplitude, frequency and shape (in this lab we will use a sine wave). When hooked up to an RLC circuit we get a driven RLC circuit (Fig. 3a) where the current oscillates at the same frequency as, but not necessarily in phase with, the driving voltage. The amplitude of the current depends on the driving frequency, reaching a maximum when the function generator drives at the resonant frequency, just like a swing (Fig. 3b)
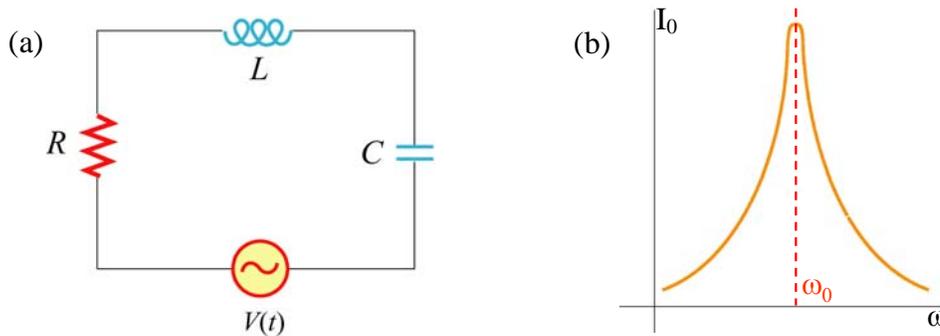
Figure 3 Driven *RLC* Circuit. (a) The circuit (b) The magnitude of the oscillating current I_0 reaches a maximum when the circuit is driven at its resonant frequency

One Element at a Time

In order to understand how this resonance happens in an RLC circuit, its easiest to build up an intuition of how each individual circuit element responds to oscillating currents. A resistor obeys Ohm's law: $V = IR$. It doesn't care whether the current is constant or oscillating – the amplitude of voltage doesn't depend on the frequency and neither does the phase (the response voltage is always in phase with the current).

A capacitor is different. Here if you drive current at a low frequency the capacitor will fill up and have a large voltage across it, whereas if you drive current at a high frequency the capacitor will begin discharging before it has a chance to completely charge, and hence it won't build up as large a voltage. We see that the voltage is frequency dependent and that the current *leads* the voltage (with an uncharged capacitor you see the current flow and then the charge/potential on the capacitor build up).
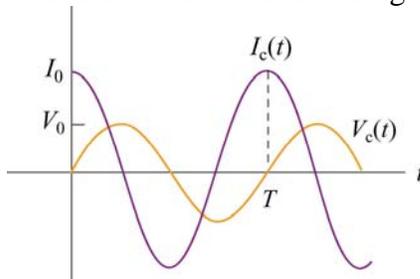
Figure 4 Current and Voltage for a Capacitor
A capacitor driven with a sinusoidal current will develop a voltage that lags the current by 90º (the voltage peak comes ¼ period later than the current peak).

An inductor is similar to a capacitor but the opposite. The voltage is still frequency dependent but the inductor will have a larger voltage when the frequency is high (it doesn't like change and high frequency means lots of change). Now the current *lags* the voltage – if you try to drive a current through an inductor with no current in it, the inductor will immediately put up a fight (create an EMF) and then later allow current to flow.

When we put these elements together we will see that at low frequencies the capacitor will "dominate" (it fills up limiting the current) and current will lead whereas at high frequencies the inductor will dominate (it fights the rapid changes) and current will lag. At resonance the frequency is such that these two effects balance and the current will be largest in the circuit. Also at this frequency the current is in phase with the driving voltage (the AC function generator).

Resistance, Reactance and Impedance

We can make the relationship between the magnitude of the current through a circuit element and magnitude of the voltage drop across it (or EMF generated by it for an inductor) more concrete by introducing the idea of impedance. Impedance (usually denoted by Z) is a generalized resistance, and is composed of two parts – resistance (R) and reactance (X). All of these terms refer to a constant of proportionality between the magnitude of current through and voltage across (EMF generated by) a circuit element: $V_0 = I_0 Z$, $V = IR$, $V_0 = I_0 X$. The difference is in the phase between the current and voltage. In an element with only resistance (a resistor) the current through it is in phase with the voltage across it. In an element with only reactance (capacitor, inductor) the current leads or lags the voltage by 90º. A combination of these elements in series or parallel will lead to a circuit with impedance $Z = \sqrt{R^2 + X^2}$ and a phase that depends on the ratio of the reactance and resistance: $\tan\varphi = X/R$ (note that the phase ϕ has the correct behavior as $X \rightarrow 0$ or $R \rightarrow 0$).

The reactance of an inductor $X_L = \omega L$ and of a capacitor $X_C = -1/\omega C$. First of all, note that these have the correct frequency dependence. An inductor has a high reactance at high frequencies (it takes a lot of effort to change the current through an inductor at high frequencies) whereas a capacitor has a high reactance at low frequencies (it "fills up" to have a large potential across it). The sign on the capacitive reactance is a convention, indicating that it leads to the current leading rather than lagging ($V(t) = V_0 \sin(\omega t + \phi)$ & $I(t) = I_0 \sin \omega t$, so phase ϕ is negative for capacitors). Some people instead write $X = X_L - X_C$ and keep all reactances positive – feel free to use whichever convention you prefer.

APPARATUS

1. Science Workshop 750 Interface

In this lab we will again use the Science Workshop 750 interface as an AC function generator, whose voltage we can set and current we can measure. We will also use it to measure the voltage across the capacitor using a voltage probe.

2. AC/DC Electronics Lab Circuit Board

We will also again use the circuit board, set up with a 100 µF capacitor in series with the coil (which serves both as the resistor and inductor in the circuit), as pictured at left.

Figure 5 Setup of the AC/DC Electronics Lab Circuit Board. In addition, we will connect a voltage probe in parallel with the capacitor (not pictured).

GENERALIZED PROCEDURE

In this lab you will measure the behavior of a series RLC circuit, both when driven sinusoidally by a function generator and when undriven.

Part 1: Free Oscillations in an Undriven RLC Circuit
The capacitor is charged with a DC battery which is then turned off, allowing the circuit to ring down.

Part 2: Energy Ringdown in an Undriven RLC Circuit
Part 1 is repeated, except that the energy is reported instead of current and voltage.

Part 3: Driving the RLC Circuit on Resonance
Now the circuit is driven with a sinusoidal voltage and you will adjust to frequency while monitoring plots of $I(t)$ and $V(t)$ as well as V vs. I.

Part 4: What's The Frequency?
The circuit is driven with an unknown frequency and you must determine if its above or below resonance.

Part 5: What's That Trace?
Current and voltage across the function generator and capacitor are recorded, but you must determine which trace is which.

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END OF PRE-LAB READING

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