8.03 Lecture 17

*Review: Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Polarization: Another way to "add more dimensions"! If wee choose our coordinate system such that the wave is going in the z-direction then:

$$\vec{E}(z,t) = \text{Re}\left[\vec{\psi_0} \cdot e^{i(kz-\omega t)}\right]$$

where $\vec{\psi} = \psi_1\hat{x} + \psi_2\hat{y}$. Can be understand as superposition of two EM waves!:

$$\psi_1 = A_1 e^{i\psi_1} \qquad \psi_2 = A_2 e^{i\psi_2}$$

Or sometimes we write it as

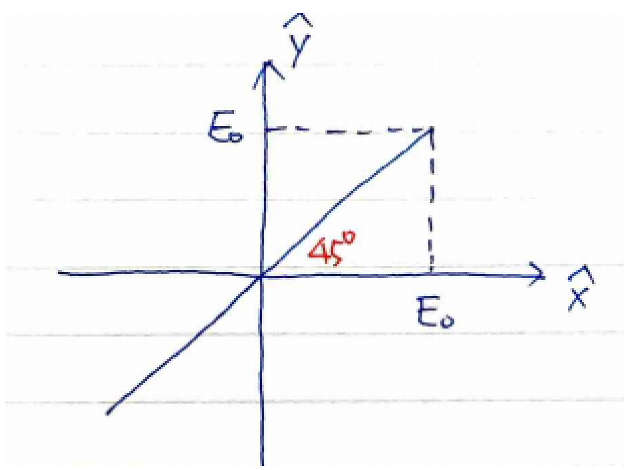
$$\mathbb{E} = \text{Re}\left[\mathbb{Z} \cdot e^{i(kz-\omega t)}\right]$$

Where

$$\mathbb{Z} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

(1.) If we add two waves with no phase difference:

$$\vec{E}_1 = E_0 \cos(kz - \omega t)\hat{x}$$
$$\vec{E}_2 = E_0 \cos(kz - \omega t)\hat{y}$$



$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

Write it in matrix notation:

$$\vec{E} = \text{Re}\left[(E_0\hat{x} + E_0\hat{y})e^{i(kz-\omega t)}\right]$$
$$\mathbb{E} = \text{Re}\left[\begin{pmatrix} E_0 \\ E_0 \end{pmatrix} e^{i(kz-\omega t)}\right]$$

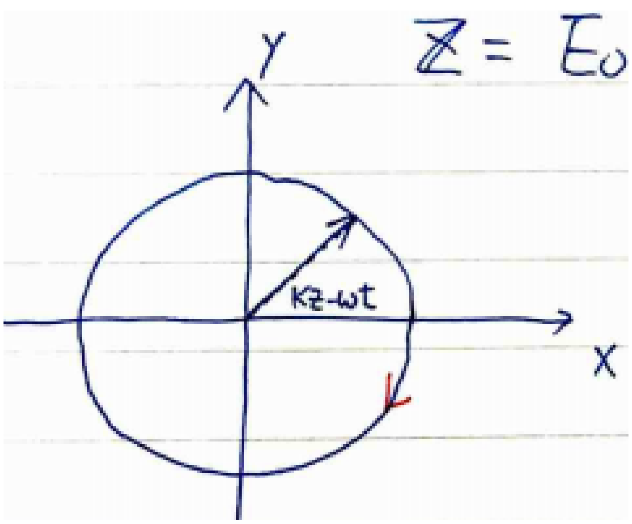
$$Z = E_0 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Linearly polarized! Other examples:

$$\mathbb{Z} = E_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \mathbb{Z} = E_0 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \mathbb{Z} = E_0 \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

(2.) If we add two waves together with the same amplitude but a phase difference of $\pi/2$

$$\begin{aligned}
\vec{E}_1 &= E_0 \cos(kz - \omega t)\hat{x} \\
\vec{E}_2 &= E_0 \sin(kz - \omega t)\hat{y} \\
&= E_0 \cos(kz - \omega t - \pi/2)\hat{y} \\
\vec{E} &= \mathrm{Re}\left[(E_0\hat{x} - iE_0\hat{y})e^{i(kz-\omega t)}\right] \\
\mathbb{E} &= \mathrm{Re}\left[E_0 \begin{pmatrix} 1 \\ -i \end{pmatrix} e^{i(kz-\omega t)}\right] \\
Z &= E_0 \begin{pmatrix} 1 \\ -i \end{pmatrix}
\end{aligned}$$



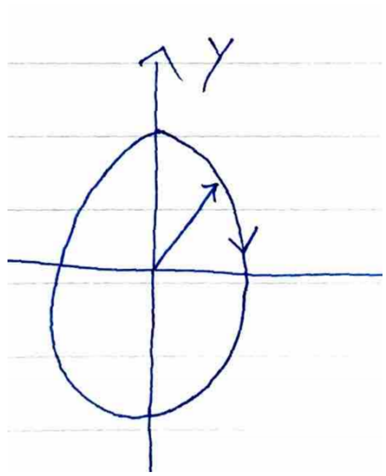
Clockwise "right-handed." Circularly polarized!

Counter-clockwise:

$$\mathbb{Z} = E_0 \begin{pmatrix} 1 \\ i \end{pmatrix}$$

(3.) We can also add two waves with different amplitude

$$\vec{E}_1 = \frac{E_0}{2}\cos(kz - \omega t)\hat{x}$$
$$\vec{E}_2 = E_0 \sin(kz - \omega t)\hat{y}$$
$$\vec{E} = \vec{E}_1 + \vec{E}_2$$
$$\mathbb{E} = \text{Re}\left[E_0 \begin{pmatrix} 1/2 \\ -i \end{pmatrix} e^{i(kz - \omega t)}\right]$$



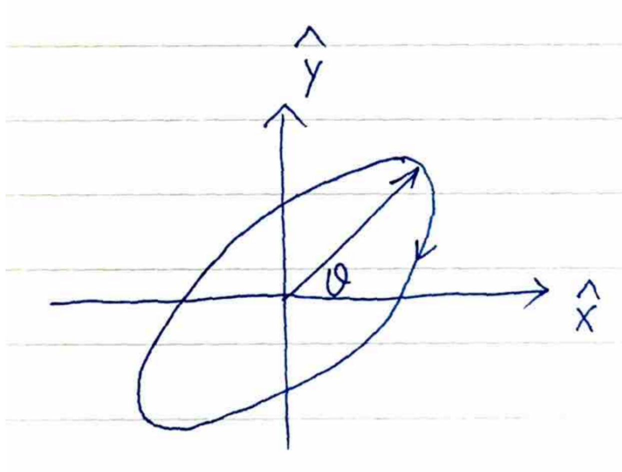
"Elliptically polarized":

$$\mathbb{Z} = E_0 \begin{pmatrix} 1/2 \\ i \end{pmatrix} \qquad \mathbb{Z} = E_0 \begin{pmatrix} A \\ iB \end{pmatrix} \qquad \mathbb{Z} = E_0 \begin{pmatrix} C \\ -iD \end{pmatrix}$$

(4.) There is another way to produce elliptically polarized EM waves: phase difference: $\Delta\phi \neq \frac{\pi}{2}, \frac{3\pi}{2} \cdots$ otherwise, circularly polarized

Example:

$$\vec{E}_1 = E_0 \cos(kz - \omega t)\hat{x}$$
$$\vec{E}_2 = E_0 \cos(kz - \omega t + \Delta\phi)\hat{y}$$

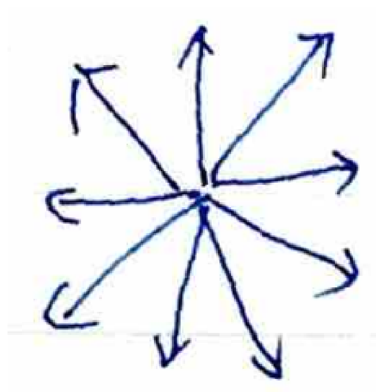


Elliptically polarized

In general: $A \geq |B|$

$$\mathbb{Z} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = e^{i\phi} \begin{pmatrix} A\cos\theta - iB\sin\theta \\ A\sin\theta + iB\cos\theta \end{pmatrix}$$

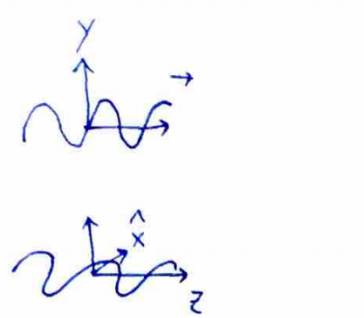
(5.) "Unpolarized" light: EM waves produced independently by a large number of <u>uncorrelated</u> emitters. Not:

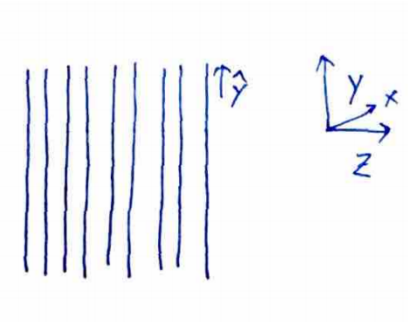


Because that gives zero!

*Emitted at different time with slightly different frequency!

Polarizer Example: grid of metal wires:



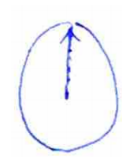


1. If the EM wave is in the $\hat{y}$ direction then it will induce movement of the electron in the $\hat{y}$ direction (in the metal wires). EM wave is reflected like what we worked on before with metal plates.

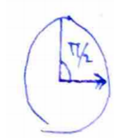
2. EM wave in the $\hat{x}$ direction cannot induce movement of electrons in the $\hat{x}$ direction

In this case, the "Easy Axis" is $\hat{x}$

$P_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ for polarizer with $\hat{x}$ easy axis:

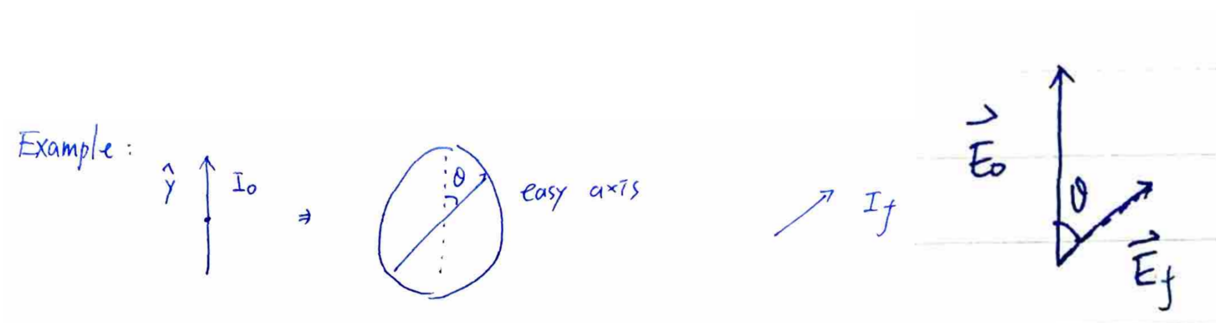


$P_{\pi/2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ for polarizer with $\hat{y}$ easy axis:



In general:

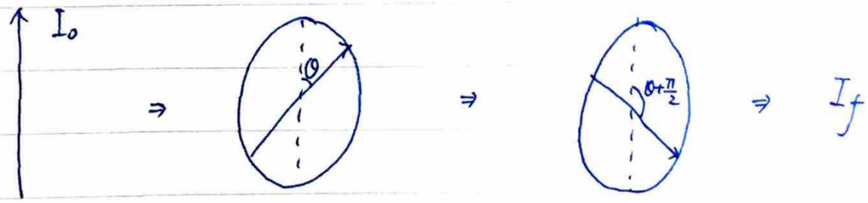
$$P_\theta = \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix} \qquad P_{\pi/4} = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$



Intensity $\propto \langle \vec{E}^2 \rangle$. After passing through the polarizer the perpendicular component is eliminated

$$\vec{E}_0 \Rightarrow |\vec{E}_f| = |\vec{E}_0|\cos\theta \Rightarrow I_f \propto \langle \vec{E}_f \rangle \Rightarrow I_f = I_0 \cos^2\theta$$

Example:



I_0
$\Rightarrow$
θ
$\Rightarrow$
$\theta+\frac{\pi}{2}$
$\Rightarrow$
I_f

$I_f = 0$