YEN-JIE LEE: OK, happy to see you again. Welcome back to 8.03. Today, as you see on the slide, we're going to continue the discussion of dispersive medium-- how the waves and vibration should be sent through this medium. And also, we will learn about uncertainty principle today. Kind of interesting. That is connected back here to what we discuss here. And finally, if we have time, we'll move to two-dimensional system and three-dimensional system to look at two-dimensional waves and three-dimensional waves. OK, that's the plan for today.

Just a quick review about what we have learned so far. Last time, we discussed about shaking one end of this dispersive medium which is actually a string with stiffness. And basically you would see that the strategy that we have been following is to do a Fourier transform to actually decompose the motion of the hand, which is actually holding one end of the string, and then decompose that into wave population in frequency space. OK, so that's what we have been doing. And then, we know based on the property of this medium, the dispersion relation, which is omega as a function of k, we can propagate waves with different frequency at different speeds. Then we can see how this system will evolve as a function of time. That's the whole idea and the strategy we approach this interesting problem.

Last time, we also introduced AM radio. As we discussed before, if we have a very simple-minded strategy to just send the pulse-- which is containing information-- directly through this medium, due to the dispersion relation which we have this medium, different component would be traveling at different speed. Therefore, the information is smeared out after it travels through a long distance. OK? That's the problem.

And then the solution was to use this approach, which is amplitude modulation mixer. That's actually how AM radio works. So basically, we have a slowly oscillating message or signal like music or voice which we want to send, and then as we multiply that by a really fast oscillating cosine tan. If we do this, assuming that omega of 0 is actually much, much higher or larger than the typical scale of your signal-- which is omega s-- then, what is going to happen is the following.

Up to all the calculations we have done last time, we found that the resulting wave which is the amplitude as a function of time and space, you can see that this can factorize into two components. The first component is virtually the original signal you are trying to send. Since you're traveling at the speed of group velocity, and finally, the right hand side-- the second component-- is actually the contribution, the really small structure of these high frequency oscillation. We call it carrier, and the carrier is still traveling at the of face velocity. That's how we actually finally understand what is the meaning of group velocity and the face velocity through this example.

What I am going to do today is to guide you through another example which will ensure we can learn some more insight from this calculation. Today, we are going to have another test of function, which actually I can do Fourier transforms really easily. And this function I'm trying to introduce here, I have this functional form exponential minus gamma times absolute value of t, OK? The reason why I choose absolute value of t is because I would like to make it symmetric around 0.

I can now do the usual Fourier transform and then to extract the wave population. The function of angular frequency, c omega. c as a function of omega. And according to the formula here, which we introduced last time, we can quickly write it down like this. Basically you get 1 over 2 pi integration from minus to infinity to infinity integrating over time. This is the original function, f of t. And multiply that by exponential I omega t. And that's the way we extract c omega. OK? Since we have this absolute value here, basically the trick is to change the interval, split the interval into two pieces. So, y is actually the negative t part, therefore, you get the exponential plus t here, and the other part is from 0 to infinity. Then the absolute value doesn't change side. You have the original exponential minus gamma times t. Then you can go ahead and do with integration, and you get the two turns and you get the functional form, which is c omega equal to gamma over pi times gamma square plus omega square. OK?

From this simple exercise, they are interesting things which we can learn from here. If I go ahead and draw f of t as a function of time, this is what you will get. Suppose I set gamma to be equal to 0.1. And I would like to visualize this function and that's what we did here. You can see that from the left hand side here, is f of t as a function of time. And you can see that this like exponential of t k but symmetric that mirror at the t equal to 0. And with a small gamma value I choose, that means this exponential decay will be really slow, therefore, you have a pretty wide distribution as a function of time. However, if you look at the right hand side, what did I show you in the right hand side? Right hand side is c omega, c as a function of omega, it's the population in frequency space. And you can see that, if I plug in gamma equal to 0.1 into that equation, then you would get a distribution which is actually pretty narrow, around 0. That's actually quite interesting. And now, if I change gamma, I increase the gamma slowly so it changes to 0.2, you see aha, that's what I expect-- the f function graphed in the coordinate space becomes narrower. But, on the other hand, you pay the price that the wave population in the frequency space becomes wider. OK, the distribution become wider. I can increase and increase. Now it's gamma equal to 0.5. Gamma equal to 1. And now I have a rather large gamma. Now it says 2.0, and you can see that as a function of gamma, if I set the gamma to be 5, and you can see that the wave, or say the waves of the wave in the coordinate space, becomes really small. But if you look at the corresponding c function, you can see that waves becomes really large.

This seems to be telling us something interesting. It seems to me that I could not choose a gamma value which simultaneously make waves in a coordinate space narrow and those wave populations in the frequency space narrow at the same time. I cannot actually do that based on this simple-minded exercise. And what you are going to do in your p-set is to go through another parameterization, which is a Gaussian distribution. And you will see very similar, hope for these very similar conclusion from your exercise.

So what is going on? And how do we interpret this result? And why is this result actually related to uncertainty principle? That's the first part of the lecture, which we are going to discuss today.

We can demonstrate this in fact by one example of f of t, which is showing here. And we go through and change the waves of this distribution. Of course, we can also try to show this in a much more precise mathematical definition. That's what we are going to do now. The first thing which we would need to do is to define how to quantify the waves of the distribution in frequency space and in coordinate space.

First, we define that the intensity of the signal is proportional to f of t squared. OK, that is the to estimate the size of the intensity. It kind of makes sense because, for example, the energy of the electromagnetic wave is actually proportional to the wave function squared. That's kind of reasonable to choose this definition. And then, once we have that the definition of intensity, then I can now calculate the average of some operator function. For example, I can calculate g of t is a average of the g function. And in this definition of intensity, how to calculate the average is to do integration over minus infinity to infinity over t. And this g function is put right there and all the components are weighted by this intensity estimator, which f of t squared.

Of course, since we are actually calculating the average, we need to take out the sum of the intensity. So, the sum of all the intensity is an integration from minus infinity to infinity-- dt f of t squared. With this definition, we can calculate the average. OK and don't forget our goal is to have an estimator estimate the waves of sum distribution. Therefore, you are probably very familiar with that. We use standard deviation. So basically that's also usually associated with the exam, but this time it's associated with some physical quantity. what is the estimator of spread of time? Right. We can actually make use of this definition and I can write this notation that t-squared to be a quantity which is associated with the size of spread in time. And now as you define to be the average of t minus average of t squared. Basically you calculate that difference with respect to the mean value, square it, and then do the two averages again. All right, everybody is following? Any questions? OK.

If I have this so-called standard deviation or spread of time definition here, then I can write it down explicitly, and this will become minus infinity to infinity, to disintegration over t, and I have t minus n value of t, half of t squared. And of course I would take out that normalization, which is a minus infinity to infinity dt f of t squared

And I can also do a similar exercise for the frequency space. Basically, I can define the spread of the frequency spectrum. And that I define it to be delta omega squared, and this will be defined as the average of omega minus the mean value of omega squared. And with this definition, we have an estimator of this spread of time, and we have an estimator of spread the frequency. The phenomenon which we see from here, from this exercise, going from low gamma value to a large gamma value is that it seems to us that the spread of the time or in coordinate space and the spread of the distribution in the frequency space cannot simultaneously be small. OK. Therefore, based on this mathematical definition, our goal is now to show that we can prove that delta omega times delta t will be larger or equal to 1/2. That is an interesting consequence based on this definition of spread. We can actually achieve the lecture today. That's our goal. And we are going to try to achieve this goal.

Before we go ahead and prove this relation delta omega times delta t greater or equal to 1/2, we also realize that when we discuss this spread of the frequencies spectrum, if I write it down here, if I try to calculate the average of omega, then what I'm going to do is to do the integration from minus infinity to infinity, d omega. because now I'm trying to calculate the mean value of omega. I have the omega times c omega. And the exponential is i omega t.

If I go ahead and evaluate this integral, I integrate over omega, and I have omega times c omega times exponential i omega t. And you can see that this omega can actually be extracted from this exponential function. If I do differentiation, which is spread through time, then I can actually extract one omega out of the initial function. Therefore, what I'm going to get is this will be equal to i partial t minus infinity to infinity d omega, c omega, exponential minus i omega t. So you can see that this is the design if I do a partiality relative to with respect to t, then I take minus i omega out of this exponential function and this i will make minus i become 1. Therefore, you can see that this integral, which I construct, is equal to i partial, partial t. This function.

OK, and you can quickly realize that we know what this integral is doing right. According to the form which I just did here, f of t is equal to this integral which I actually just highlight there. Therefore, this is just f of t. that's kind of interesting because that would give me i partial, partial t, f of t. Basically you can see that I don't need to deal with omega, I can actually do a partial relative with respect to time, then I can take one omega out of the function which I have constructed. Any questions? All right, now I can calculate what will be the mean omega. What would be the mean omega? The mean omega, according to this definition here, this is how we calculate the mean of some quantity, mean omega will be equal to minus infinity to infinity, tt f star, t i partial, partial t, f of t. OK sorry that this is kind of close to here.

The original definition I should put omega got here, right? But instead of putting omega there, I used the trick that this i times partial partial t can generate an omega for me. Therefore, instead of putting omega explicitly into the integral, I put i partial partial t into the integral, then I get 1 omega out of it, and that's equivalent to the calculation with g of t equal to omega. OK, everybody's following? Therefore, I of course still need to normalize the calculation. This is the denominator, which is minus infinity to infinity, integral over tt f of t squared. OK, you can see that instead of using omega directly, the I used this trick to use i partial partial t to extract 1 omega and I can calculate the mean value of omega.

Therefore I can also calculate explicitly what would be the delta omega square based on the definition which I outlined before. This would be the average value of omega minus mean omega. Mean omega is a number, and if I'd write it down explicitly I get minus infinity to infinity, tt, i partial partial t, minus average value omega, f of t squared divided by minus infinity to infinity disintegration over dt f of t squared. The take home message is that I'm using this trick to replace all the omega by i partial partial t. Therefore, in my formula, you will see that originally, this is supposed to be omega and now we were using that trick. Therefore, it can be written as pi partial partial t. And you'll realize what this is used for afterwards.

All right, so those are just preparation. What we have done is that my goal is to show that delta omega times delta t is greater than or equal to 1/2. OK, that's my goal and I'm preparing for that. And I have that definition of delta t and delta omega. Yes?

AUDIENCE: What do you think [INAUDIBLE]

YEN-JIE LEE: Oh sorry, there should be-- it should be like this. So I am taking partial partial t out of f. OK, sorry. Good question. Any other mistakes? Very good. Not yet? All right.

So now you can see that I have the definition in my hand, and I am almost there to show you that delta omega times delta t is going to be greater or equal to 1/2. And what I'm going to do after this-- maybe you will be even more mad at me-- is to use exactly the same trick which would be used to show Heisenberg's Uncertainty Principle in quantum mechanics. basically what I'm going to do is to consider a function which is r of t. r as a function kappa and t. and the definition of this r function is like this. I define this r function to be t minus average t minus i kappa i partial partial t minus omega. f of t. If you don't know where is this relationship coming from, don't be worried because you don't really need to. This is just to guide us through this mathematical calculation. But if you can see directly how this will help, the maybe you are Heisenberg. Maybe. So that's very nice. It's a test.

What I am going to do is to employ this r as a function of kappa of t. And the 2 for our purpose to show that the delta omega and delta t greater than 1/2. And first, to make my life easier, I would define this to be capital T, and I would define this thing to be capital omega. So that my mathematical expression doesn't explode. Now I can consider this ratio function r of kappa. This is defined as minus infinity to infinity integrating over t r kappa t divided by minus infinity to infinity dt f of t squared. This is r function which is the ratio of the area of r function and the area of the f function. You may say that, professor, this is really crazy. Today is telling about all the crazy things, but that is because I would like to let you know that we are going to see a very interesting result. So that's why I'm doing this.

And if I construct this r function, this r function will have an interesting property. What is the interesting property? I entered an integral over something squared in the numerator and the denominator. Now it means, what would be the value of this r function? The r function would be always positive. Right? Because this is a square, this is a square, therefore, r is going to be positive. That means r is going to be greater than or equal to 0. That's why we have this r function. And the miracle will happen because if I go ahead and calculate this r-- before I calculate this capital R function-- what's the function of kappa, I need to actually deal with this small r as a function of kappa and t squared. If I extract this component and then calculate that, r kappa t squared. What I am going to do is to use this expression r is equal to t, capital T minus i kappa omega times f of t. So that my life would be easier.

Then basically you get t minus i kappa omega f. And then you need tje complex conjugate. Basically, you get T cross i kappa omega star f star. You can have T star, but T is a real number. Therefore, it doesn't do anything. Then, I can now go ahead and collect all the terms. Then the first terms which I can collect is everything related to T times f. Then basically you get the T f squared. That is coming from this T times f times T times f. This term times this term times this term. to give you the first term. And you also you can connect another term which omega f squared. Right. Basically, you can find that contribution. Use should have a kappa square in front of it. Any questions so far? Basically, I collect the terms related to omega times f and put it here.

Finally, you have the third term, which is i kappa T f omega star f star minus omega f T f star. Basically, this small r function squared can be written in this functional form. We are almost there.

What I'm going to discuss first is that now I have these three terms. Number one, number two, and number three. I can now attack number three first. Number three, I'm going to get i kappa Tf minus i partial partial t minus omega f star. Basically what I'm doing is to take this omega here. This is omega star. And then use that definition, write down the expression for omega-- typical omega-- explicitly. Since I am writing omega star, therefore, you get a minus i partial partial t minus average omega out of it. That's why here you have this expression and then multiple it by f, which is the original expression. I also write this omega capital Omega explicitly. I partial partial t minus average Omega. f t f star. And you can immediately realize that-- OK, this whole thing is multiplied by i times kappa.

You can immediately recognize that this term actually canceled because they are-- actually they are literally the same. And then what is actually left over is the two terms, which is in the middle. So basically, you are going to get now I can multiply i and cancel this minus i. Basically what you get is kappa time T equals-- both terms have a T, so I can extract this T out of it. f partial f star partial T cross partial f partial T f star.

After all those works, you can see that this one looks pretty nice. This says what? This is not bad at all after all those calculations basically these will be equal to kappa T partial partial t f f star. Everybody's following or everybody already lost? We are almost there. All right. Now, we have these three. Three originally is a beast. Looks really horrible and after I write it down explicitly, it looks OK, not perfect. Yes?

AUDIENCE: [INAUDIBLE]

YEN-JIE LEE: The complex conjugate of the f function. All right. Now I can put one, two, and three into this integral. Then we are done. Now let's put numbers 3 into the integral first. I do a minus infinity to infinity, number three, dt. What is going to happen? This will give you minus infinity to infinity kappa T partial partial t f f star. And I can use integration by parts so what I'm going to get is kappa T f f star evaluating minus infinity and then plus infinity minus kappa minus infinity to infinity f square partial t partial capital T partial t d t.

Let's look at this. Basically, what I'm doing is to put in the numbers written back into this integral and then use integration by parts. Basically you can see that this is what you would expect. The interesting thing is that this function is evaluated at crossing at infinity and minus infinity. If you assume that your f function is localized-- it's confined in some specific range of time, instead of spreading out over the whole universe. That means this term will be equal to 0 because it's evaluated at plus infinity time and minus infinity time. If the f function is localized, then at the boundary of time, you are going to get 0. This term disappears. Very good. We've solved one problem. And this looks horrible, but partial capital T, partial t, what is capital T? T is small t minus average of t. Average of t is a number and t is just t. Therefore, partial t partial small d is just 1. You can see that there are hopes, things are becoming simpler and simpler.

Therefore, what I'm going to get is this-- minus kappa minus infinity to infinity t t f squared. And then if you divide this by this term, you can see that 3-- number 3 term-- will give you a contribution of minus kappa. That's all. Because once you plug this integral back into this function, the third term contribution gives you minus kappa. That's a very good news because it's actually pretty simple. Any questions?

AUDIENCE: [INAUDIBLE]

YEN-JIE LEE: Oh, you mean this one?

AUDIENCE: No.

YEN-JIE LEE: This one?

AUDIENCE: To the left.

YEN-JIE LEE: Oh, yeah. You are right. I missed a dt. Thank you very much. Very good. Yeah. Basically what I'm trying to do is plug in the expression here into the integral. You can see that the contribution from the third term that number 2 is rather simple. It's just minus kappa.

Let's also take a look at the computation from the first and the second. Wife Number one, will give you minus infinity to infinity t minus average of t squared f of t squared dt. And this is divided by minus infinity to infinity dt, f of t. This is not crazy at all because this just the definition of delta t squared. Just a reminder that the definition of delta t squared is written here. Therefore, this is just delta t squared-- the first term, which looks really strange there, but in reality, it's actually very simple.

Let's look at the second term. This is kappa squared minus infinity to infinity i partial partial t minus average of omega f of t. And then square that. Divide it by minus infinity to infinity dt, f of t squared. And that will give you kappa squared delta omega squared. Basically, our conclusion that this r function is a function of kappa. Essentially equal to the first terms here delta t squared, the second term is plus kappa squared of delta omega squared . And finally, the third term is there. Minus kappa. And this would be greater or equal to 0. Because what I am doing is just summing all those positive functions. Then, take the rest. . Any questions?

AUDIENCE: Why does the integral from negative infinity to infinity dt f squared equal?

YEN-JIE LEE: This one? This one? This is equal to zero, right? Oh, here?

AUDIENCE: Yeah. Why does that--

YEN-JIE LEE: Oh, I see. I see your point. This is an integrated minus infinity to infinity number 3 dt. It's the contribution here. Then, if I take a ratio between this term and that term, then this is canceled by the denominator. Therefore, what is actually left over is minus kappa.

AUDIENCE: OK.

YEN-JIE LEE: This 3, the contribution of 3 in green is already taking the ratio when I evaluate the capital R function. Good question. Now you can see that you can safely ignore what I have said so far. Everything you can ignore. Those are just mathematics tricks. But what is very important is that now I have this relation-- delta t squared plus kappa square plus delta omega squared minus k. This is a function of k. And I can actually minimize it. I can minimize R if I carefully choose a kappa value. This kappa equal to kappa mean value which makes the minimize the R function is equal to 1/2 delta omega squared, which I would not go over this calculation because this is just a minimization problem.

That means if plug that in, what I'm getting is R kappa min will be equal to delta T squared minus 1 over 4 delta omega squared. That is greater or equal to. 0. We arrive there. If I multiple both sides by 4 delta omega squared you get delta t squared delta omega squared greater or equal to 1 over 4. If you take the square root of that, the you get delta t delta omega greater or equal to 1/2. That's actually what we started to try to prove right? You can see that after all those works a lot of complicated mathematic calculations, you can see that we make no assumption, we are just using the definition of the spread of time and the spread of frequency. We follow that definition and the use of mathematical trick which we used to prove Heisenberg's Uncertainty Principle and we arrive there. This means that this is an intrinsic property of wave function. Intrinsic property means that it's a mathematic property of wave function. What do I mean by this equation, which we finally did right?

After all those hard work, we have to enjoy what we have learned right from all of those crazy things. What do we learn? Look at this function. Delta t times delta omega, greater or equal to 1/2. That means if I construct a function, which is how I oscillate the stream as a function of time, if I construct a really narrow one to this very fast and then I stop-- very narrow-- then you will have a very small delta t. Now it sounds really nice. I produce a delta function, delta t, but the delta omega space is going to be a mess. It's going to be a super wide distribution because delta t is really very, very small. That means you have to compensate that by a rather large delta omega because if you multiple delta t times delta omega, that is going to be great or equal to 1/2. And is the consequence of this, for example, for the discussion of AM radio.

If I have an AM radio with bandwidth delta omega. This is 2 pi delta nu and that is something like 3 times 10 to the 4 Hz. If I have some kind of bandwidth which is actually roughly this value. I can now immediately calculate what will be the resulting delta t. The resulting delta t will be a few times 10 to the minus 5 seconds based on this equation. This means that if I'm trying to send two signals in sequence through this AM radio. that mean if the delta t-- the time difference between the first and the second information-- if the time difference is large, if that delta-t between these two much, much larger than 10 to the minus to the minus 5 seconds. Then I can actually easily separate these two signals.

On the other hand, if I send then really, the two signal really close to each other, if it looks like this, then the receiver, the ones who will receive the signal, will not be able to separate, if this is just one signal or two signals, or one pulse or two pulse which you are trying to send.

Any questions so far? So you can see that we can actually quantify what will be the limitation in the resolution, tiny resolution, due to the limitation of bandwidth delta omega.

Before we take a break, I would like to make a connection to quantum physics. So if I look at this delta t times delta omega greater than or equal to 1 over 2, this expression, I can rewrite it. I can multiply t by velocity v. And I get v times velocity and I can have omega divided by v. And this would be better or equal to 1 over 2. So I just multiply v and divide by v, then actually you can solve.

And that means this will become delta x. And that, the second term, will become delta k. And that would be greater or equal to 1 over 2. In the quantum physics, momentum is equal to h bar times k. Momentum will be equal to h bar times k. And h bar is actually the Planck constant.

So that, actually you will see that a few times in L4. OK. So if I have p equal to h bar times k, that means I have delta x times delta p greater or equal to h bar over 2. That is exactly the uncertainty principle, which was actually introduced by Heisenberg. And what is actually the meaning of this?

So if we describe all those particles we see by quantum mechanical waves, if I have momentum p, now it corresponds to a wave function, with wave number k. And the constant, which is associated with p and the k is the Planck constant.

So this means that if I measure one particle really, really precisely in a position, due to the nature of wave function that means I will not have a lot of information about the momentum of that particle. And where this uncertainty principle is coming from, it's coming from purely the mathematics related to waves. As you can see there, there's really nothing to do with quantum so far. Quantum I'm saying actually only goes in after we prove the uncertainty principle, delta omega times delta t. You can cannot have a very precise frequency and a very precise position in a coordinated space over time at the same time.

And that actually has direct consequence. That means if you are considered in quantum mechanics, that is essentially the limitation which will be posted, the uncertainty principle. So we will take a five minute break. And we come back and we take a look at 2-3 dimensional waves. And let me know if you have any questions.

So welcome, back everybody. So before we actually moved to 2-3 dimensional waves, we will discuss a very interesting topic, which is related to the dispersion relation of the light actually. So if you use spatial relativity, basically you can relate energy to momentum and the mass. So E square will be equal to a p square c square plus m square c to the 4. And you actually interpret light as a photon, then basically E is actually equal-- to the energy of the photon will be equal to h bar times omega.

So we are actually really going really forward a bit. Because maybe some of you actually haven't seen this before. But if you just believe what I have said, basically you can actually divide everything from the first formula, which is the spatial relativity formula, by h bar square. Then you will be able to derive and arrive the second formula, which is omega square equal to c square k square plus omega 0 square.

And the omega 0 is actually defined as mc square over h bar, just for simplicity. So if we look at this equation, this is essentially a dispersion relation. Now you have seen this so many times. And this omega square equal to c square k square plus omega 0 square, this formula is actually reminding you that this is actually a dispersion relation.

So what I mean by a photon having mass here? That means the m term in this special relativity formula is not 0. Therefore omega 0 will be non-zero. What is going to happen? That means the space of velocity of light is going to be different. It depends on what value of k you choose.

That's kind of interesting. Because that means light with different frequency or different wavelengths is going to be traveling through the vacuum at different speeds, if that's true. Everybody get it? Very good.

So how do we actually test this? So that means I need a light source, which are very, very far away from earth. Then I would like to measure the delta t as a function of frequency, for example, and analyzing. So how do we do that?

So this is actually possible if you actually use a natural light source, which is the pulsar. So what is actually a pulsar? So what we actually use, essentially a millisecond pulsar. So those are actually coming from rapidly rotating neutron stars, and that those rotating neutron stars will emit pulses of radiation like x-ray and radio waves, at regular intervals. Because it's essentially rotating, rotating, rotating again and again.

Based on this movie, basically what it's showing here is a very old neutron star. It's actually in a binary system. And this neutron star can absorb the material from the other partner. So that actually is-- the rotation speed actually increased. And finally at the speed of a millisecond per turn.

So this actually really happened. And we can actually observe this. And if we are lucky, the earth is essentially somehow in a spatial direction such that the emitting radio wave actually pointing from the pulsar to earth, then I can see the pulsar, the amplitude of the light from pulsar essentially changing rapidly as a function of time.

And another very good news is that typically those pulsars are really far away. For example, in this example, pulsar B1937+21, this is essentially a pulsar with rotation period of just 1.6 milliseconds. And this is actually something which is really happening really far away from the Earth, which essentially is 16,000 light years away. And that we can actually observe this. This is actually pretty close to Sagitta, and you can actually see this pulsar.

And how does that actually associate with the original question we were posting? The original question is, does the light with different frequency travel at different speed. And this is essentially a very nice tool. Right? Because it is emitting the radio wave. And now I can just measure the spectra as a function of time. And I will be able to see if we actually can observe different speed. Because we know the rotation in the world, and et cetera. And it also emits a wide spectra of the frequency, the light frequency. Therefore, I can use this as a light source far, far away from the Earth, to see what will happen.

So somebody actually did this measurement, and this is that what they found. They found a non-zero omega 0. A non-zero omega 0 was found. So that means the mass will be 1.3 times 10 to the minus 49 gram. That sounds really small. But it's not small at all. That's actually destroying the whole understanding of light.

What is going on? So we are in trouble. So after all this discussion, et cetera, and also other measurements which are sensitive to photon mass, they actually threw out this possible contribution. This is essentially is just simply too large based on, for example, measurement of magnetic field in the galaxy, et cetera. It doesn't really work.

So what essentially is really happening? The explanation is that the path from the pulsar to the earth it's really not vacuum. There are a lot of-- not a lot, but we have very few or very dilute electrons, very diluted free electrons all over the place. And that will change the frequency and the speed of light slightly. Therefore you observe the interesting-- observe the effect. And we are going to actually also talk about how the material actually changes the behavior of the electromagnetic wave in the coming lectures. I hope you find this interesting. Any questions?

All right. So we are going to move on. So far what we have been discussing is always 1-dimensional waves. So for example, a string, and also the sound save in a tube, et cetera. We always discuss things which are in one dimension. But we are actually not one dimensional animal. We are 3-dimensional And of course, for example, these objects the surface is 2-dimensional So there are many, many things which are more than one dimension.

So can-- the question that I'm trying to ask is, can we actually understand this kind of object, and how actually to understand those objects and how do we actually derive the normal amounts, and how do we actually write down the general solution, which describes a 2-dimensional or a 3-dimensional wave. That's actually the next topic which I would like to discuss.

So that's actually gets started with a plate like this. So basically that plate is actually a 2-dimensional. And assuming that this plate is infinitely long, for a moment, very, very long. So what does that mean? This means that if I define my x and y-coordinate, which is actually used to describe the position of a specific point on this plate, then basically you will see that they are beautiful symmetries, which you can actually identify from this simple example.

What is actually the symmetry which we can identify? Can anybody help me with that?

AUDIENCE: x and y.

YEN-JIE LEE: Yeah. So yeah, x and y are symmetric, yes. And the other function of x, what kind of symmetry to you have?

AUDIENCE: Reflection.

YEN-JIE LEE: Yeah. Also reflection, and what I'm looking for is if I change x and change y, what kind of symmetry do you have?

AUDIENCE: Translation.

YEN-JIE LEE: Translation symmetry. Well, all of you are correct. But what I am trying to focus on now is the translation symmetry. So if I use translation symmetry, what I'm going to get is that I can already know the functional form of the normal mode. Because essentially if it's translation symmetric, as a function of x, it's translation symmetric as a function of y. Then I can say is in the x direction will be proportional to exponential iKxX. K underscore x is essentially the wave number associated with the wave in the x direction.

So that's essentially one consequence which we actually learned from the discussion of symmetry. And in the y direction, I can conclude also that the normal mode will be proportional to exponential iKy times Y. Therefore, I already know what will be the function form of the normal mode of this highly symmetric system.

What is that? The psi xy will be equal to A times exponential iKx times X, exponential iKyY. So you can see that. And also I need to take the real part. Something like this will be possible in normal mode.

Therefore without going into detail basically, we will see that the expected behavior of psi as a function of x and y will be something like a sine Kx times x, sin Ky times y. So that's actually the kind of normal mode, which we will expect based on the argument of translation symmetry.

And of course if I now go back from infinitely long system to a finite system, then you can use the boundary condition to determine what would be the K value, Kx value, and allow the Kx value and allow the Ky value using boundary conditions.

So actually without doing any calculation, we can already find that, so now if I have a plate with finite size, basically you expect that I can have some kind of normal mode, which this is the amplitude, a projection in the x direction, it can be a sine function. And that it can become 0 at the left-hand side edge and the right-hand side edge.

And in the y direction it has to be also some kind of sine wave as a function of y. And of course it goes to 0 at the edge. Because if those are actually the fixed boundary, for example. And if those are actually not fixed boundary, then you expect that-- like open-end solution. So you expect that the distribution will be more like a cosine function for the first normal mode.

And if you look at this, the structure of this kind of solution, it looks really complicated. Because you have x direction and you also have y direction. Both of them are actually sine functions. And how do we actually visualize this kind of sine function? And here is a demonstration, which I have prepared. It's really a 2-dimensional plate.

And as you can see that under this plate, I have a loudspeaker which actually produces a sound wave to try to excite one of the normal mode. And the one I am going to do is to turn on this loud speaker. You can hear the sound. And I would like to see the normal mode. But it's very hard to see that, without doing anything. Because it's vibrating, but its so fast that it is really very difficult to see it.

So what I am going to do is to pour some sand on the surface, and see what is going to happen. And if we look at this, I am putting sand on it. And you can see that, there is something happening. If I change the frequency to one of the normal mode frequencies, you can see that now we are reaching some kind of resonance and exciting one of the normal mode. And you can see that the sand actually it doesn't like to stay on some of the plate. Because it's vibrating like crazy and it's not very comfortable to sit there.

So the sand, where will the sand actually sit? They will set at the place where you don't have any vibration. Because what we are talking here, is essentially some kind of sine wave times sine wave or cosine wave times cosine wave. That means there will be nodes on the plate. And those are 2-dimensional nodes. In the 1-dimensional case, we are talking about nodes, it's actually the place where you have zero amplitude.

And now I have cosine times cosine. Therefore, there will be a complicated pattern appearing which is essentially the place the plate is not actually moving at all as a function of time. And you can see that now I can actually excite one with the normal node. And you can see a really beautiful pattern. And allow me to do this and increase the frequency. So that if we see if I can excite another normal mode.

Look at what is happening. So now you see that the number of lines actually increased. So this is actually so-called Chladni figures. Basically those figures are actually produced by this trying to excite one of the normal mode. And basically the sand will be collected in the nodal lines. And you can see that this higher frequency input sound wave. You can excite the higher frequency in normal mode.

And of course I can continue to increase and see what happens. Now I'm increasing the frequency even higher and higher. You can see that now the sound is actually rather loud. And I am actually putting more sand. You can see that there are more and more patterns. Because now I am increasing the frequency, so that actually the higher frequency normal modes are excited. And you will expect more nodes for higher frequency ones. And now I can even go even higher to see if I find success. It's not easy now. Look.

Probably this is a very good way to design the pattern of your t-shirt. OK. So how do we actually understand all those patterns? And we have already started. This is actually something related to cosine and sine multiplied to each other. And the next time we are going to do a more detailed calculation and show you a few more demos and see what we can actually learn from the 2-dimensional case.

Thank you very much. I hope you enjoyed the lecture today. And if you have any questions, let me know. And you can actually come forward and play with those demos if you want.