* EXAM 2

11/13 11:30 - 1:00

* Review

Snell's Law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

slides

* Do we need a review section

(Yes from the students)

Set up poll in Piazza

Polarization :

Another way to "add more dimension" !

If we choose our coordinate system such that the wave is going in the z-direction

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Then:

complex vector

$$\vec{E}(z,t) = \mathrm{Re}\left[\vec{\psi_0} \cdot e^{i(kz-\omega t)}\right]$$

where $\vec{\psi_0} = \psi_1 \hat{x} + \psi_2 \hat{y}$

⇒ Can be understood as superposition of two (EM) waves!

$$\psi_1 = A_1 e^{i\phi_1} \qquad \psi_2 = A_2 e^{i\phi_2}$$

Or sometimes we write it as

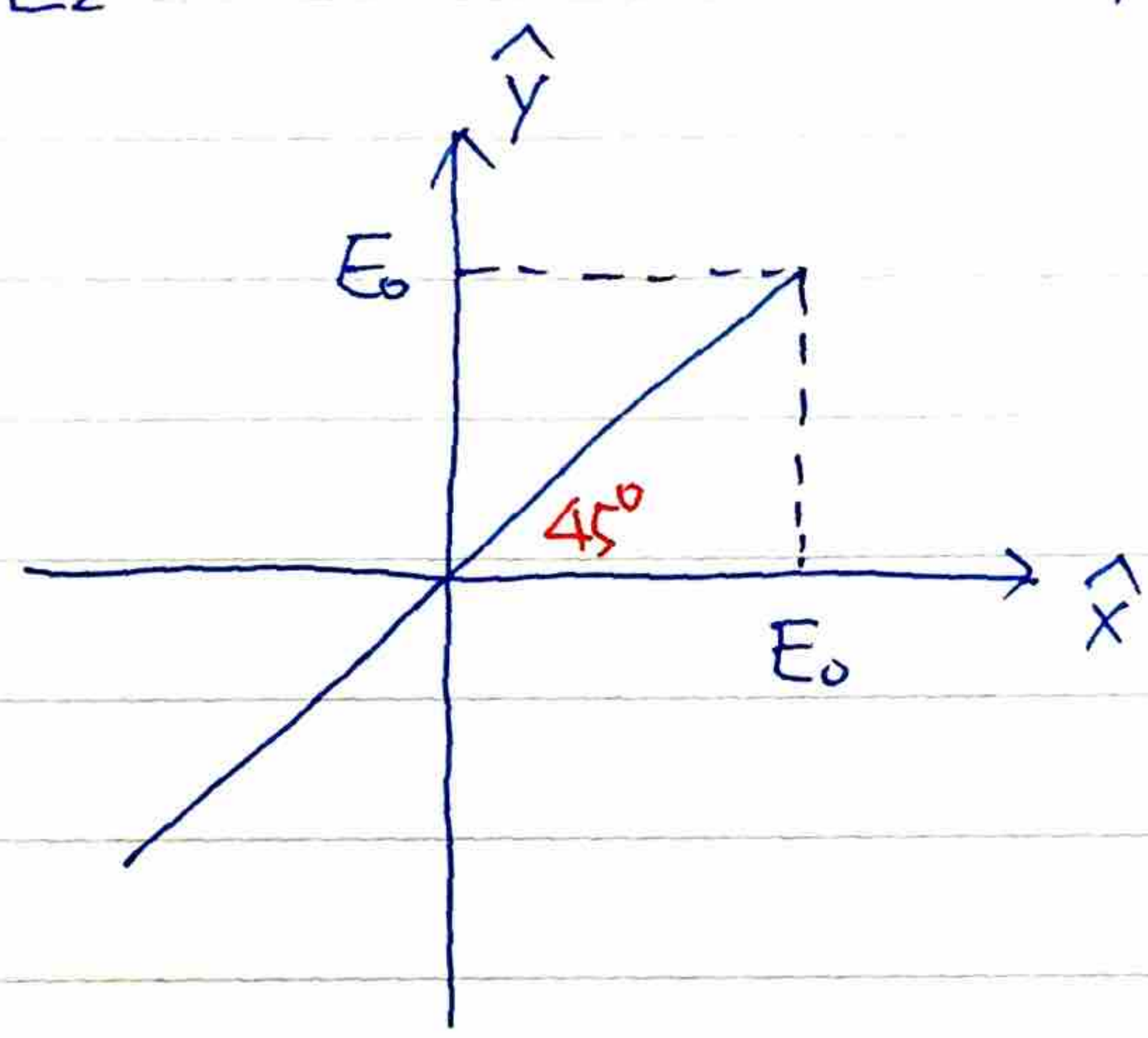
$$E = \mathrm{Re}\left(Z \cdot e^{i(kz-\omega t)}\right)$$

$$Z = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

(1) If we add two waves with no phase difference

$$\vec{E}_1 = E_0 \cos(kz - \omega t)\,\hat{x}$$

$$\vec{E}_2 = E_0 \cos(kz - \omega t)\,\hat{y}$$



$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

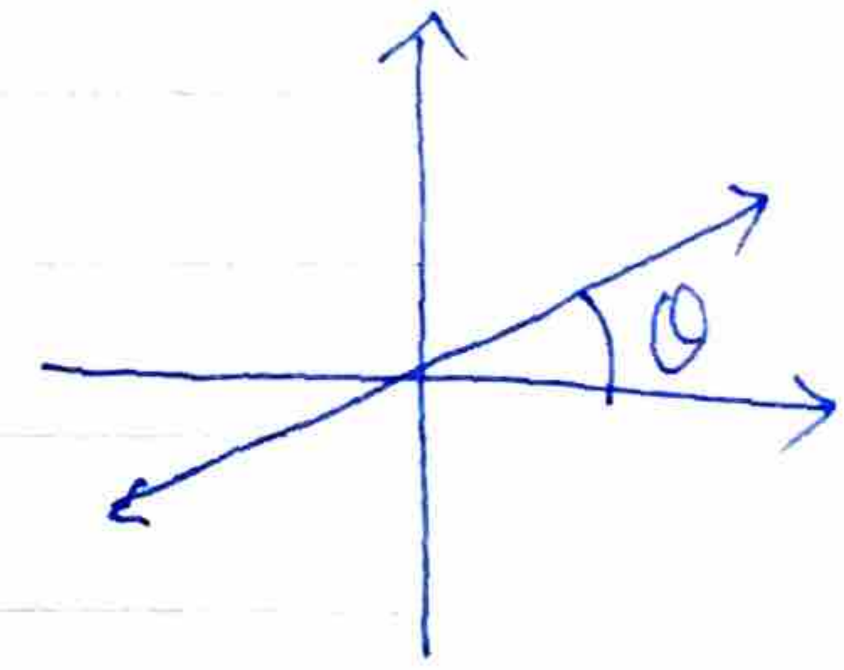
Write it in matrix notation:

$$\vec{E} = \mathrm{Re}\left((E_0 \hat{x} + E_0 \hat{y})\, e^{i(kz-\omega t)} \right)$$

$$\mathbb{E} = \mathrm{Re}\left(\begin{pmatrix} E_0 \\ E_0 \end{pmatrix} e^{i(kz-\omega t)} \right)$$

$$Z = E_0 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Linearly polarized !



Other examples:

$$Z = E_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad Z = E_0 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad Z = E_0 \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

(2) If we add two wave with the same amplitude but a phase difference of $\pi/2$

$$\vec{E}_1 = E_0 \cos(kz - \omega t)\,\hat{x}$$

$$\vec{E}_2 = E_0 \sin(kz - \omega t)\,\hat{y}$$

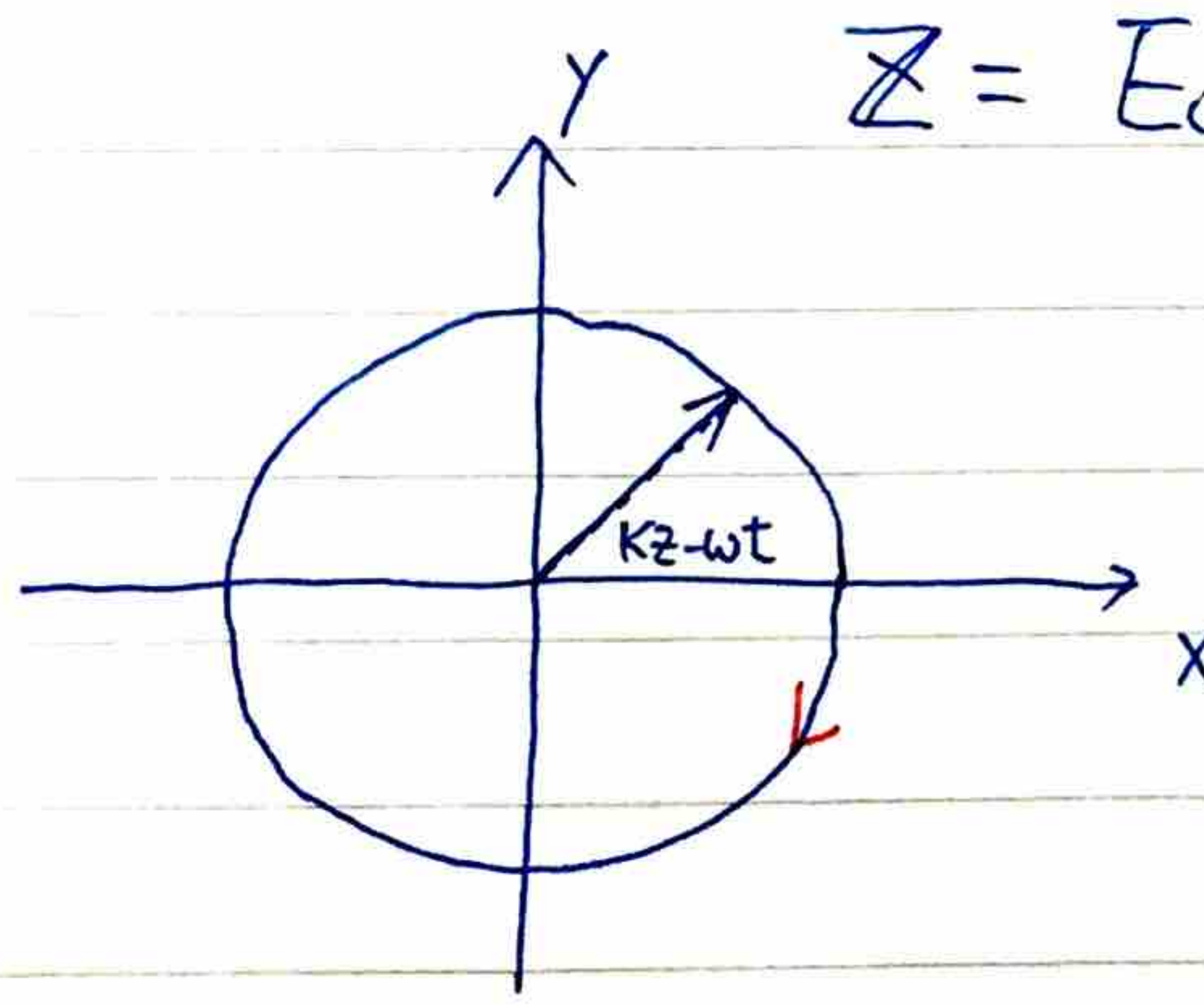
$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= E_0 \cos\left(kz - \omega t - \tfrac{\pi}{2}\right)\hat{y}$$

$$\vec{E} = \mathrm{Re}\left[\left(E_0\hat{x} + E_0 \underbrace{e^{-i\frac{\pi}{2}}}_{-i}\hat{y}\right)e^{i(kz-\omega t)}\right]$$

$$\mathbb{E} = \mathrm{Re}\left[E_0\begin{pmatrix}1\\-i\end{pmatrix}e^{i(kz-\omega t)}\right]$$

$$\mathbb{Z} = E_0\begin{pmatrix}1\\-i\end{pmatrix}$$



clockwise "right-handed"

Circularly polarized!

Counter-clockwise : $\mathbb{Z} = E_0\begin{pmatrix}1\\i\end{pmatrix}$

(3) We can also add two waves with different amplitude

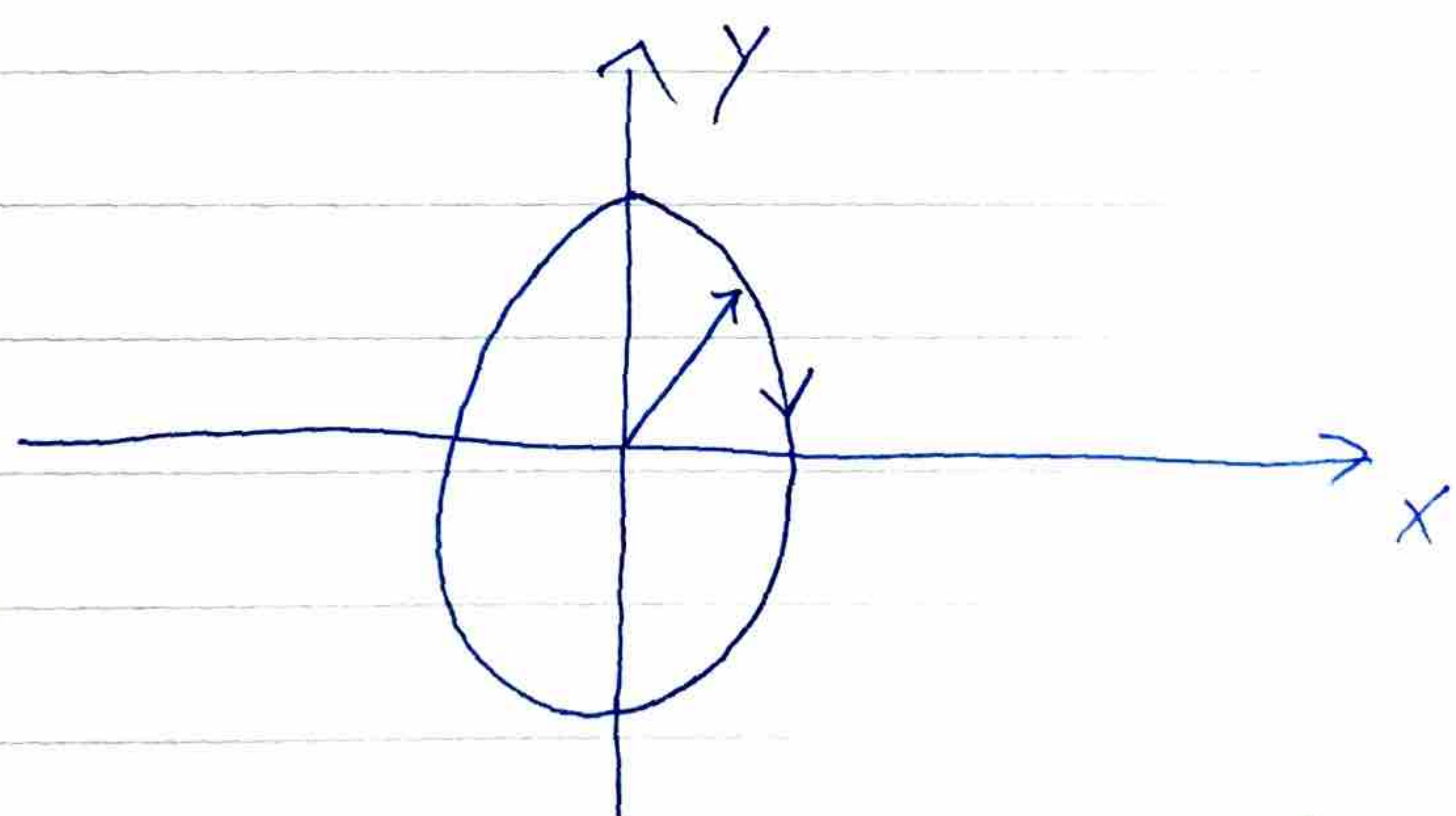
$$\vec{E}_1 = \frac{E_0}{2}\cos(kz-\omega t)\,\hat{x}$$

$$\vec{E}_2 = E_0 \sin(kz-\omega t)\,\hat{y}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\mathbb{E} = \mathrm{Re}\left(E_0 \begin{pmatrix} \frac{1}{2} \\ -i \end{pmatrix} e^{i(kz-\omega t)} \right)$$

$\Rightarrow$



$$\mathbb{Z} = E_0 \begin{pmatrix} \frac{1}{2} \\ i \end{pmatrix} \qquad \mathbb{Z} = E_0 \begin{pmatrix} A \\ iB \end{pmatrix}$$

$$\mathbb{Z} = E_0 \begin{pmatrix} C \\ -iD \end{pmatrix}$$

" Elliptically polarized "

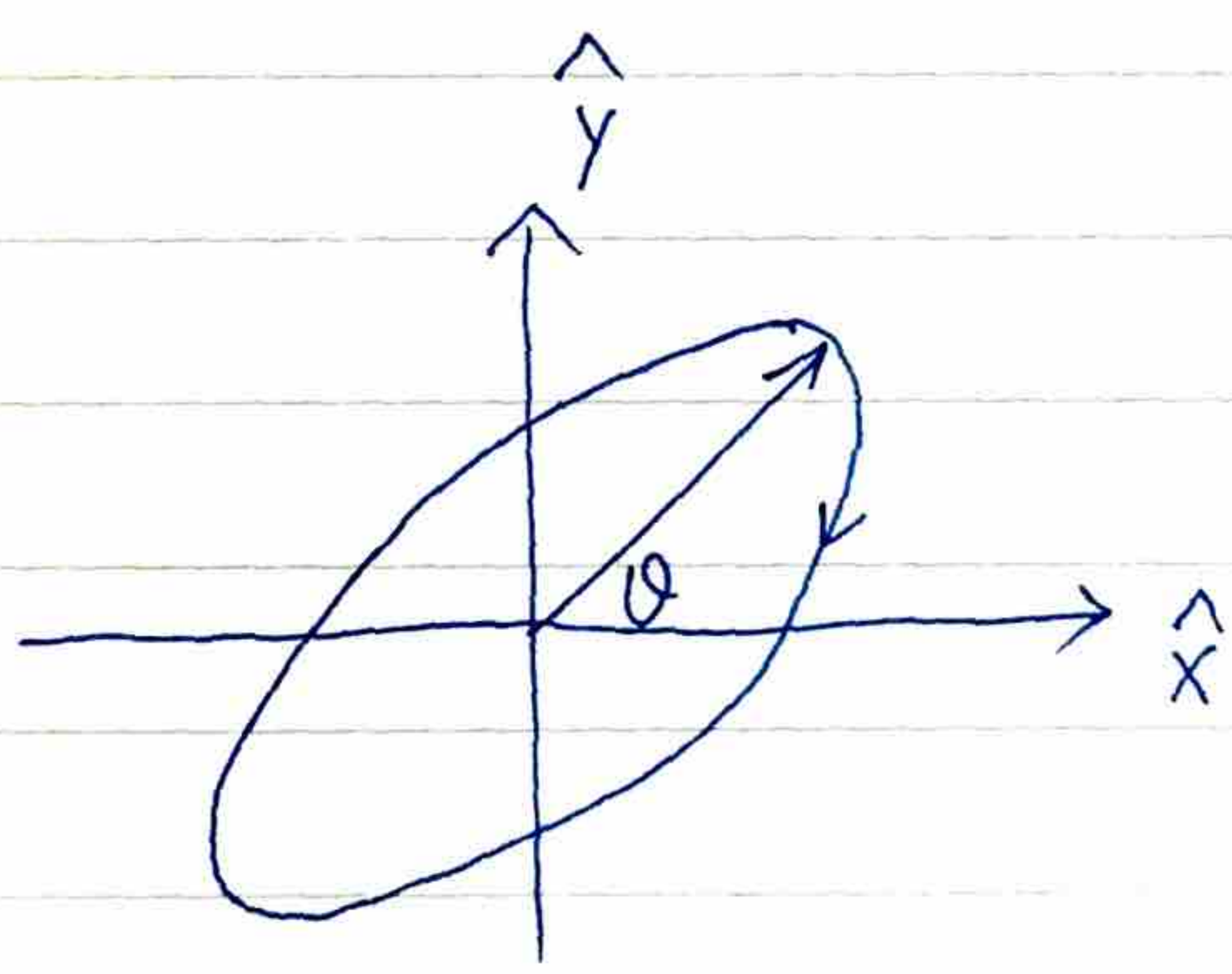
(4) There is another way to produce elliptically polarized EM wave: Phase difference.

$\Delta\phi$ ($\Delta\phi \neq \frac{\pi}{2}, \frac{3\pi}{2}$ otherwise circularly polarized)

Example:

$$\begin{cases} \vec{E}_1 = E_0 \cos(kz-\omega t)\,\hat{x} \\ \vec{E}_2 = E_0 \cos(kz-\omega t+\Delta\phi)\,\hat{y} \end{cases}$$

$\Rightarrow$



Elliptically polarized.

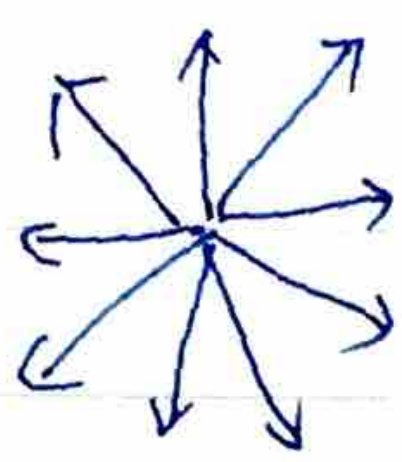
In general: $A \geq |B|$

$$Z = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = e^{i\phi} \begin{pmatrix} A\cos\theta - iB\sin\theta \\ A\sin\theta + iB\cos\theta \end{pmatrix}$$

(5) "Unpolarized" light :

EM waves produced independently by a large number of uncorrelated emitters.

Not



because that gives 0 !

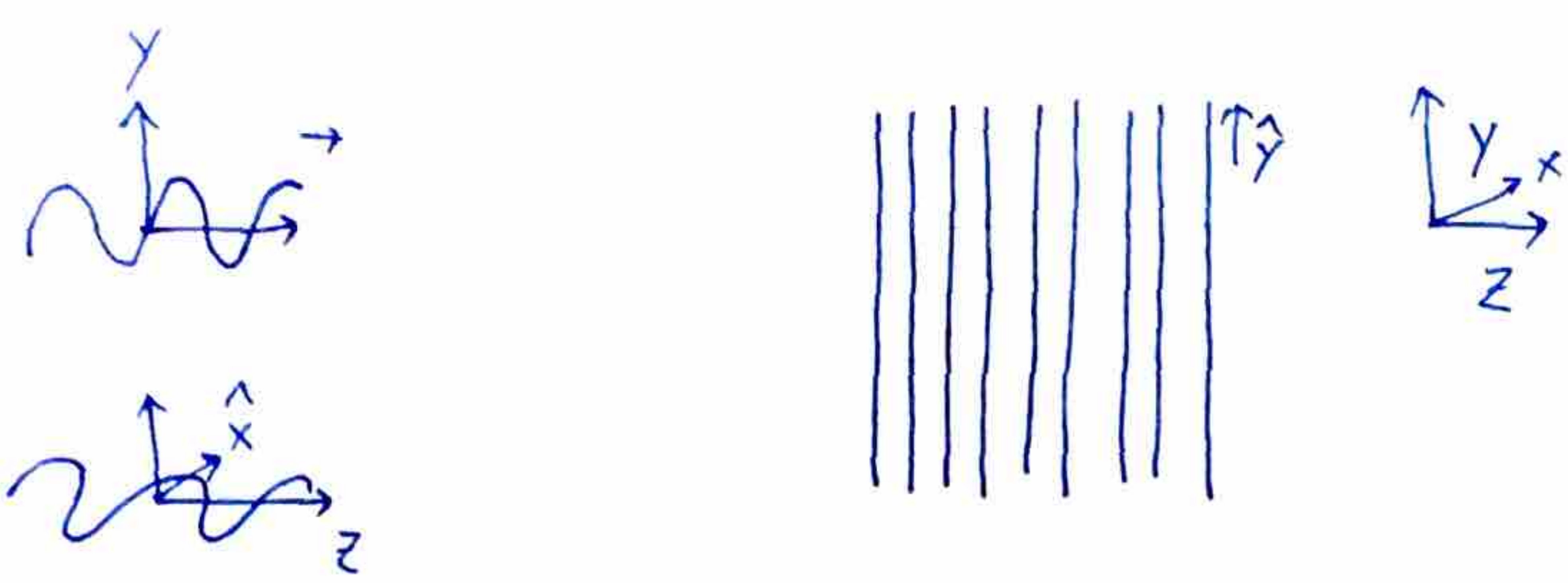
* Emitted at different time !
* With slightly different frequency !

Matrix presentation :

Useful tool.

Polarizer :

Example : grid of metal wires.



(1) If the EM wave is in the $\hat{y}$ direction

⇒ induce movement of electron in the $\hat{y}$ direction
(on the metal wires)

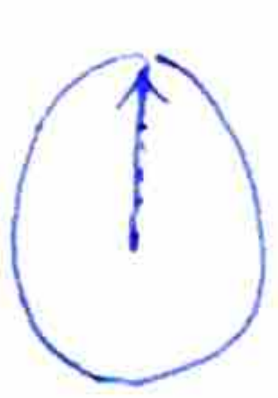
⇒ EM wave is reflected like what we worked on before with metal plate

(2) EM wave in the $\hat{x}$ direction

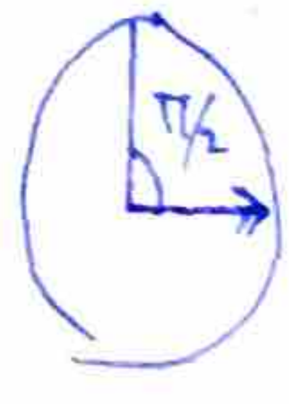
⇒ can't induce movement of electron in the $\hat{x}$

In this case : the "Easy Axis" is $\hat{x}$

$P_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ for polarizer with $\hat{x}$ easy axis



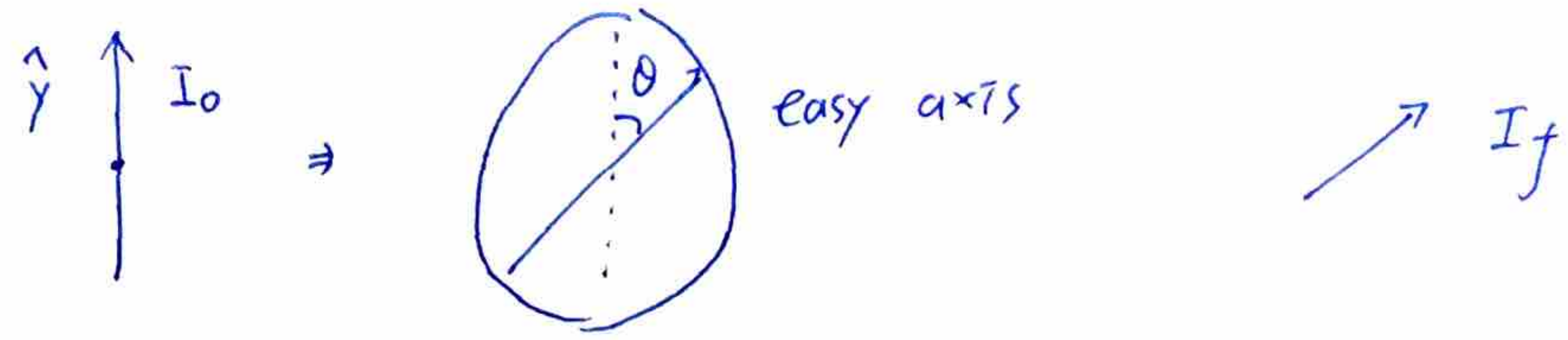
$P_{\pi/2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ for polarizer with $\hat{y}$ easy axis



In general:

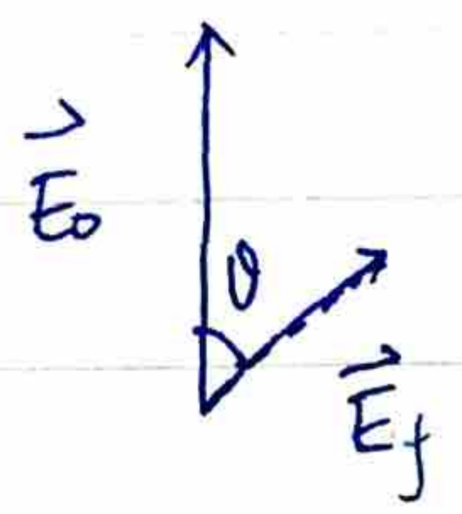
$$P_\theta = \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix} \qquad P_{\pi/4} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Example :



Intensity $\propto \langle \vec{E}^2 \rangle$

After passing through the polarizer:

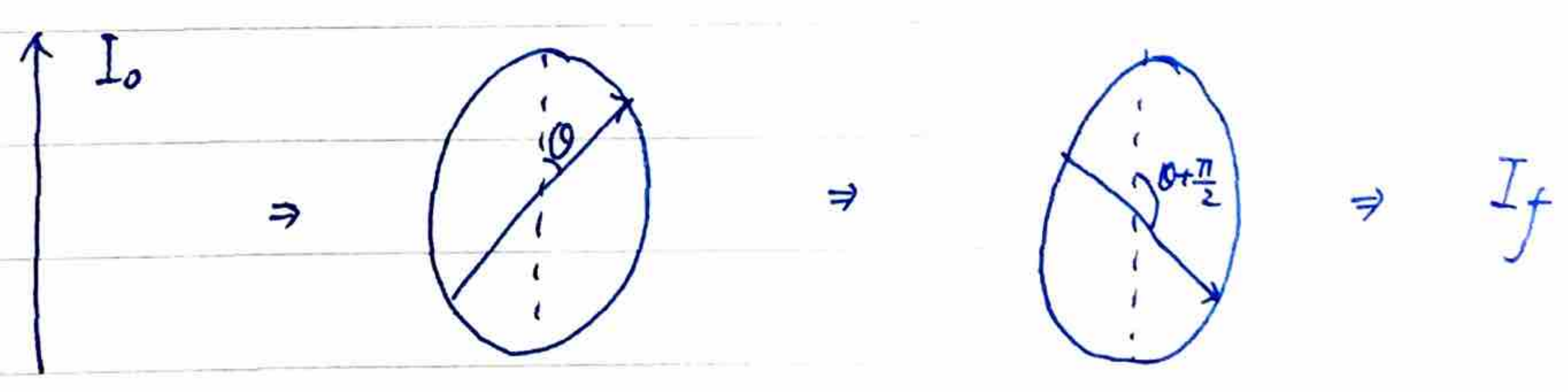


" $\perp$ component " is eliminated

$$\Rightarrow \vec{E}_0 \Rightarrow |\vec{E}_f| = |\vec{E}_0| \cos\theta$$

$$\Rightarrow I_f \propto \langle \vec{E}_f^{\,2} \rangle \Rightarrow I_f = I_0 \cos^2\theta$$

Example:



$I_f = 0$

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