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**YEN-JIE LEE:**

So what are we going to do today? So today, we're going to continue the discussion, based on what we have learned from the diffraction and also other interesting phenomena. And we are going to make connections to quantum mechanics and discuss in greater detail about this connection. And also, if time allows, we are going to cover information about gravitational waves as well.

So last time, we have discussed diffraction pattern coming from a laser beam. And we discussed about resolution. And you can see that this is the graph we see, last time already in the lecture. Basically, all the point light source coming through a single slit is going to be doing interference to each other. So basically, you see interesting pattern, which you have a central peak. And the total constructing for interference happens at the center of the screen.

And at some point, you have a deep-- which is actually where total destructive interference actually happened. And we were able to understand this with mathematics, which we learned in 8.03. Another interesting result we find is that, if we shoot a laser beam to the moon-- by now, you should be able to conclude that it's not going to be a point on the moon. Instead, it's going to be a spot as large as the whole Missouri state. So that's actually another interesting result we found from that discussion last time.

And finally, we are able to put together all the things we have learned from the last few lectures. Basically, you can have, at the same time, the effect of multi-slit interference and also the effect of a single slit diffraction-- all then put together. Then you have this complicated but also beautiful pattern, which you will be able to observe on the screen.

And basically, the point is that you are going to have, for example, in this case, five-slit interference pattern. But that is actually modulated. The intensity is modulated by the pattern from a single-slit diffraction.

OK, so that is actually what we have learned from the last lecture. Coming back to the original question, why do we study 8.03? The reason is that we would like to understand-- we would

like to hear from that universe. So we cannot even recognize the universe, without using waves and vibrations. So that's the course in which we have been doing here. And we have seen waves of matter.

So for example, here, we have this water wave generator. You can see water waves. We have this Bell Lab machine. Because you can see the coupled oscillator-- a multiple coupled oscillator. And they are doing their job together to form beautiful waves. And you can see those beautiful results from there.

Last time and also including the last few lectures, we have been using this laser to produce interesting interference patterns and the diffraction pattern. So that's the second kind of wave, which we encounter. The first kind is the waves of matters. The second kind is waves of what? Waves of vector fields.

It's not matter anymore. Because this field, this oscillation field, can also travel through vacuum. So that is actually the second kind of wave, which we should learn for 8.03.

They provide a pretty adequate description of the nature, description of the phenomenon, which we can actually measure, and see from the experiment we went over. So what I would like to say in the lecture today is that there are two kinds of completely different waves, which we haven't talked about.

The first one is the probability density wave, which I will cover that in a few moments. The second one is gravitational waves. This is actually a space-time distortion coming from the motion of massive objects. And we would like to see what we can also learn from there, using the existing knowledge of which we have learned from matter waves and the vector-field waves. And that they are, actually, pretty similar to each other if you look at their behavior.

So the first thing which I would like to discuss is light. So far, what is light? Light is like electromagnetic waves. So that is actually what we have learned so far from 8.02 and 8.03. So they are like waves. They are waves, all right.

On the other hand, in the 20th century, there are many, many crises going on. So the first thing which is happening is photoelectric effect. So this experimental result was actually first discovered by Hertz. So he found that, if you want to kick one electron out of some material or some charged material, it is easier if you use a high-frequency light compared to low-frequency light. So that's the issue-- really strange.

Because based on waves and also the intensity formula-- while we really care, intensity is proportional to the square of the electric field amplitude. So that's actually what we learned from the previous lectures. But what Hertz was saying is that the frequency also matters. OK, so that's really a bit strange. And Einstein actually came in and explained this photoelectric effect.

So what we found is that this effect can be explained by viewing the light of small quantas. And light is actually not like waves any more, but like quantas-- the small quantas-- discrete ones-- with energy proportional to the frequency. All right? And basically, the energy of those little quanta-- or we now call them photons-- is actually equal to  $h$ , which is some constant-- relate the frequency and the energy of the quanta.

And with this explanation, the view is that the photons are like particles. And therefore, he can actually explain that, OK, if I measure the kinetic energy, the maxima kinetic energy of the electrons, which are kicked out from this photoelectric effect experiment. I call it K-max.

What you are going to get is a formula like this. The maxima kinetic energy of electrons will be equal to  $h \nu - \phi$ .  $\phi$  is actually some kind of threshold, which you need or, say, some kind of energy, which you need to overcome to kick one electron out of the material.

So you can see from here that, if the frequency of the light is too low, then this will never work. Because the maximum kinetic energy will be below 0. Therefore, you cannot kick out a electron. But if the frequency is high, as shown by Hertz experiment, it is possible. And also, you can create energetic electron, have them kicked out of the material.

And this is actually verified by experiment. And that's essentially why Einstein should get the Nobel Prize in 1921. So let me tell you what the feeling here-- the feeling here is that, now, things are really becoming more and more interesting. Because first of all, we see really well that the electromagnetic wave describes the behavior of the light. We see diffraction. We see interference. All those things can be explained by the beautiful mathematics, which we have been employing to explain all those results.

But here, you can see that, at the same time, the photons are very good tool or very good viewpoint to explain the photoelectric effect. So that's actually really surprising. Because that means you have, suddenly, both kinds of explanation of light, which should be describing the same thing. So now, one idea is to describe them by waves. The other idea is to describe

them by particles.

And also, at the same time, as we actually discussed in the previous lecture, we can do a two-slit experiment with billiard balls or with bullets. Suppose we do this experiment. We fire the balls or bullets through some source. And we have some kind of two-slit set up here, so that the bullets or balls can actually pass through.

And then we were wondering, what will be the pattern which we see from the detector in the right-hand side. And basically, what we see is the following. So basically, we have a distribution of the balls coming from slit number one. And we have another distribution of balls, which is actually coming from slit number two. And I can call it P-1 and P-2, which is actually the probability density distribution of the experimental result.

And the final result, or say, if you just look at the distribution of balls without separating the balls from slit number one and slit number two, basically you get some distribution like this, which is a superpositional P-1 and the P-2 distributions. All of these things doesn't surprise anybody.

The experiment one, which we perform-- now, instead of billiard balls, we perform the same experiment with electrons. And again, we have all those electrons pass through this two-slit two-slit experiment. As we see before, basically, again, we can actually separate electrons from the first slit, which I called I-1, and the intensity of electrons coming from the second slit, which I called I-2.

Then, basically, if you can identify and make sure that the electron is coming from one of the slits, you are going to get distribution like this. OK? And of course, I can always write these I-1 as a sine wave function,  $\psi_1^2$ .

And I can always write this I-2, which is the intensity as a function of position of the electron coming from the second slit-- I can always write it as  $\psi_2^2$ . In the case of light, it's actually just proportional to the electric field. So if you accept this, and now you actually don't measure where the electron actually pass through when the electron from the source passed through this experiment-- we now don't measure if it passed through one or two.

Then, the pattern becomes like this. You have something like this which, essentially, is very, very similar to the two-slit interference experimental result of the laser experiment. As you see from this experimental result, you see some kind of pattern. You have the peak. You have the

valley, as a function of position in the detector.

And we can actually call this  $I_{1-2}$ , which is actually when you don't measure or you don't identify which slit the electron goes through, then you have the intensity, which is actually called  $I_{1-2}$ . And what we actually found is that  $I_{1-2}$  is actually not  $I_1$  plus  $I_2$ . What we found is that  $I_{1-2}$  is actually  $\psi_1$  plus  $\psi_2$  squared.

So basically, we see interference pattern. And this will be equal to  $I_1$  plus  $I_2$  plus  $2$  squared root of  $I_1$  plus  $I_2$  and the cosine delta, where the delta is coming from the path length difference. So based on this experimental result, this is actually really surprising. First of all, electrons are arriving like a particle, right? Because we can see from this slide, if you look at the upper left figure, you see doo-doo-doo.

Every time, you have something hitting the screen. And what is actually left over? It's a single hit on the screen. Therefore, the electrons are arriving like a particle, producing a hit in the detector. On the other hand, what we are saying here is that, before they hit the screen, they are behaving like a wave. It has interference with itself, like a wave.

All right, so is electron a particle? Or is the electron a wave? Strange. The answer is electron is actually neither of them in reality. So how about we actually do some more experiments to convince ourselves what is actually really going on.

So what we could do-- as I actually mentioned. I can still have the electronic gun. I have electron source in the left-hand side. Again, I have this two-slit experiment here. Then what I'm going to do is to produce a light source here. OK, I put a light source there to shine the whole experiment.

Then, I was wondering what is going to happen to the distribution. So if I first close the lower slit, slit number two, and only measure the intensity coming front slit number 1, then this is the distribution I get, which is  $I_1$ . If I close the upper one and open only the lower one, I get a distribution which is  $I_2$ .

And this light source, essentially interacting with the electron-- when electrons pass through this slit, what is going to happen is that you'll see some scattered light coming from the slit. Therefore, you can know which slit the electron actually passed through. And if I do this experiment result-- if I block one of the slits, this is the distribution,  $I_1$  and  $I_2$  and now I'm going to open both slits, and you will see light coming out of the slit when the electron pass slit

number one or slit number two.

You can identify which slit for all the electrons passing through this experiment. And the resulting distribution of electrons on the screen is actually like this. It's actually like I-1 plus I-2. There will be no interference. Why is that? Now, this because you know very precisely which slit the electron actually passed through, this experiment.

And also, we can say that, huh, the electrons are actually disturbed. Therefore, they now behave like bullets or like billiard balls. So this is actually a bit strange. Maybe it is because the intensity of the light is too large. Therefore, it's changing the behavior of the electron.

So what are we going to do now? Experimental result number three is to lower the intensity of the light. So what will happen if I lower the intensity of the light source so that we would like to see the behavior, as a function of intensity?

So at some point, we will find that some of the electrons are not heated by a photon. Or say, there will be no scattered light of the electron when it passes through the experiment. Because the intensity of the light is too small. And we already know, from photoelectric experiments, light is, essentially, also like quanta.

So when the intensity is low enough, the effectiveness of the light source decreases. Then the experimental result would be like what? Can somebody actually give a guess? Is that going to be like experimental result number one? Or is that going to be like experimental result number two? Anybody want to get--

**AUDIENCE:** [INAUDIBLE]

**YEN-JIE LEE:** Very good. So the result-- as I mentioned before, sometimes the electrons are detected by the light source. Sometimes the electrons are lucky. They pass through without getting heated by a photon. Therefore, you cannot know which slit, actually, this electron went through.

Therefore, the experimental result is that, if I just lower the intensity of the light source, then what I'm going to get is a mixture of experimental result number one and the experimental result number two. When I have the intensity low enough-- going to really, really low intensity limit-- the result will become experimental result number one. Because then you are not really impacting the position of the electrons.

Finally, you can actually suggest something else. So OK, now I have low intensity of light. One

way to lower the intensity is like what I was saying. The rate of the photon emission-- I can make it lower and lower. All right? There's another way to do this.

Experimental result number four-- so what will happen if I use this formula  $E = h \nu$  equal to  $h c / \lambda$ ? Because  $\nu$  is actually just  $c / \lambda$ . What will happen if I, instead of lowering the rate of photon emission, I lower the energy of individual photons?

How do I do that? What I could do is to lower the frequency of the electromagnetic wave of the light source or, say, increase the wavelengths of the light source. OK, this is very nice.

Because now, I can keep this in, right? So that I make sure all the electrons are bothered by the photon. Because I can emit very high rate. But at the same time, I can also lower the intensity, so that the intensity is very, very low.

Can anybody guess what is going to happen? With a result like experimental result number one, when I go to extremely low intensity? Or my result will be like experimental result number two? Because each electrons are bothered, are heated, by the emission from the light source.

Anybody want to try? Just guess, no? One or two? Or a mixture of them?

**AUDIENCE:** Be like two.

**YEN-JIE LEE:** The guess is that it's going to be like two, which was actually well-motivated. Very good try. What essentially, happens is that-- OK, I can say, oh! Each electron are bothered by many, many photons. So those are disturbed. Therefore, it has to look like experimental result number two.

The answer may surprise you. The answer is that, if I have the limit  $\lambda$  goes to infinity,  $\nu$  goes to 0, what is going to happen is that, no matter how high frequency of photon emission I have, I am going to get the result of experimental number one. Why is that?

Now, this is because when the wavelengths of the electromagnetic wave or the photon is going to infinity, that means you cannot resolve which slit, actually, the electron goes through. Because if I draw the wavelengths here, it's going to be like this. If you observe some kind of scattered light from the electron, it could come from both slits, because the wavelength is too long.

So if you go to infinity, then it's like you have a constant electric field there. It doesn't really actually help you to identify which slit the electron actually passed through. So therefore, the

interference pattern reappears. So this is really crazy thing, if you look at all these four experimental results.

The conclusion from these four imaginary experiments is that it is not yet possible to tell the position of the electron and also, at the same time, do not disturb it. If you were able to tell the position of the electron, then there would be no interference pattern. On the other hand, if your experimental set-up have no ability at all to tell if the electron's coming from slit number one compared to slit number two, then you are going to get interference pattern.

There's another thing which I would like to make connection to the Uncertainty Principle, which we actually learn from waves and vibrations. So we have learned that Heisenberg's Uncertainty Principle-- this is essentially purity coming from the property of the wave, if you actually remember the deviation which we have done in the previous lecture.

So what is this uncertainty principle telling us? Is that the standard deviation of the position times the standard deviation of the momentum is going to be greater or equal to  $\hbar$  over 2 some constant. And how do we actually understand this from the electron experiment.

That is actually highly related to the single-slit experiment with electrons. So what we could do now is to have a fifth experiment. I have electron source here. And I have a single slit. And the width of the slit is capital D. And we were wondering, what is going to happen? What will be recorded by the smoke detector in the right-hand side?

And by now, it should not surprise you that this would give you some kind of diffraction pattern, which you say should be very similar to what we actually observe with laser experiments. So you can see that. The electron-- the momentum-- now, I would like to actually define my coordinate system. The vertical direction, pointing upward, is my x direction.

So now, take a look at this experimental result. So what this is actually telling us is the following. We know the position of the electron to a accuracy of the width of this slit, which is D. So that is actually telling you about the uncertainty over the position in the x direction.

Now, this electron goes through. And they actually hit the screen. And each electron is having a single path. If I look at one of the paths, the upper one, what I'm getting is that there must be a momentum quintessentially in the x direction, when this electronic goes through the slit and hits the screen.

One interesting thing we learned from the deviation from last time is that, if I just look at the



slide here-- if I look at the left-hand side slide-- if I have a very small slit--  $D$  is small-- what does that correspond to? That corresponds to  $\Delta x$  goes to very small value case. We have a small  $\Delta x$  value. You are really sure where is the electron at some instant of time, when it passed through the experiment.

Then what is going to happen? If you look at the right-hand side, the distribution on the screen, you have a wide distribution. The central maximum peak will be very wide. So what does that mean? That means you have a wide distribution of momentum in the  $x$  direction.

Therefore, that will give you that is actually consistent with what we've actually written here, Heisenberg's Principle.  $\Delta x$  times  $\Delta p$  will be greater or equal to some value. On the other hand, if I increase the width of the slit, the  $D$  is now large. As you are making a  $D$  larger and larger, what is happening is that the central peak, the width, is actually going to be narrower and narrower.

Now, this is actually also consistent with what we have learned from Heisenberg's Uncertainty Principle. When  $\Delta x$  become even larger, then the uncertainty or, say, the distribution of the momentum in the  $x$  direction, becomes smaller. So now, we actually also understand a little bit more about what the single diffraction actually means. And this issue is really closely connected to the Uncertainty Principle Heisenberg actually proposed.

And if we use the mathematics which we learned from last time, the  $C$  function is going to be proportional to integration from  $-\frac{D}{2}$  to  $\frac{D}{2}$ .  $dx$  exponential as  $ikx$  times  $x$ . And if I have  $D$  goes to infinity, which means that you have an infinitely wide slit-- based on the formula which we have derived last time-- basically, we will see that  $C$  function is a function of  $kx$ . It's going to become a delta function.

And this delta function is  $\delta(kx)$ . So that means, if you have absolutely no idea about the position of the electron, you are going to get very, very precise information about-- the momentum in the  $x$  direction is actually going to be equal to 0. Because it's a delta function. It's only nonzero at  $kx$ , which is the directional propagation equal to 0.

Any questions so far? OK, so from those experimental results, we've found that the probability of getting heat on the screen is proportional to  $\psi^2$ , if I only have the first lead there. That means the probability,  $p$ , is proportional to  $\psi^2$ .

And this is actually probably one of the most crazy results in the physics we learned so far. In

some sense, it's kind of sad as well. Why sad? This means that, OK, I can calculate those wave functions. And the probability of getting an outcome at a specific position is proportional to this wave function squared.

But I feel, maybe, demotivated, right? Because originally, we are like god. You can predict-- OK, I have this thing, this object. And I have force. And then it goes like-- oh-- like this way. I can calculate the trajectory of this chalk thing all over the place, as a function of time. And I know what is going to happen. I have the full control of all the objects which I have in my hand in my experiment.

But now, quantum mechanics or this experimental result tells me that we can only predict the probability, the odd, instead of the outcome. You see my point? I can only pretend the wave function, the distribution of the wave function. And the probability of getting a result here is proportional to the wave function squared.

But I cannot predict the outcome before I do the experiment. That's really a big change in your view or, they say, in our current understanding of the physics. You can say that, well, maybe Yen-Jie's not working hard enough. Maybe all those electrons which are emitted from the electron source already made up their mind where this electron is going to.

For example, electron number one is doing this-- rrrrr-- and going to here. And electron number two already made up his mind. He's just going to do this. And the electron number three is-- uh-- maybe do this-- vwooo-do-do-do. And then all those trajectories are already determined. And they are heating variables which Yen-Jie doesn't know. Therefore, he screwed this up and said, oh, come on. We can only predict the probability.

But the thing is that, from the experimental result number two, experimental result number three and number four, you can see that the electrons cannot make up their mind when they are emitted. Because when they got heated by that light-- electrons cannot know in advance that it is going to be heated by a light.

And the light can be a very, very mild, very, very small energy. So that it should not affect the predetermined path of the electron. Do you get this? So that doesn't makes sense. So it is not because Yen-Jie is not trying hard. It is really because nobody can really tell before the experimental result is actually shown or the measurement is already done.

If you can find any case, maybe you will win another 100 Nobel Prize. Because you are

showing that the whole understanding of quantum mechanics is not correct, really. Please tell me when you actually have done this experiment. I will be very proud of you, for sure.

So now, we are entering a position to discuss this result. So now, actually, we can also make use of this understanding and predict what would be the particle probability distribution in a potential well. Suppose I have an experiment, which I have a well, where I have potential goes to infinity in the left-hand side or right-hand side edge of this well.

And I will define my coordinate system so that the well is actually equal to 0, or  $x$  equal to  $L$ . So by now, when you see this, this looks really familiar to you. In the center part, you have some kind of translation symmetry. And the boundary-- those are forbidden regions. You cannot actually have particles there, because the potential is infinity.

Therefore, this is actually giving you boundary conditions of the wave function, describing the state of the particle inside this box. So the boundary condition would be  $\psi = 0$ . It will be equal to 0, because it's actually at the left-hand side edge of the well, where you have infinite potential.

And also, you can have  $\psi = 0$  at  $x = L$ . This will be equal to 0, because the right-hand side edge, you also have infinitely high potential. Therefore, when you see this, your immediate reaction will be, how do I know what is this? This is actually  $\psi = 0$ .

The solution to this problem must be something like  $\psi = A \sin kx$  should be the normal mode of this system. And it's going to be  $A \sin kmx$ , where  $km$  will be equal to  $m\pi$  divided by  $L$ , where  $m$  is a number. It can be 1, 2, 3-- it goes to infinity, right? By far, you have actually learned all these practical calculations from the previous examples we had.

Therefore, what would be the  $\psi = A \sin kmx$  as a function of time? Then what I am going to get is  $A \sin kmx e^{-i\omega t}$ . Of course, we can also really plot all those results.

So for example,  $n$  is equal to 1. Basically, what you are getting is like this. Doesn't surprise you. This is actually  $\psi$  as a function of  $x$ . And  $n$  equal to 2-- this will correspond to the situation where you have one node in the middle, et cetera, et cetera. You can't have many, many higher  $m$  value solutions.

And what would be the probability to find the particle in a specific location? That's why we mentioned before, the probability is proportional to a wave function squared. Therefore, the probability will be proportional to  $\sin^2 kmx$ . And what we are going to get is like this.

If I plot  $m$  equal to 1, using  $P$  as a function of  $x$ , the probability of getting a particle at a specific place, if we are looking at the situation in normal mode number one, that is like this. You are much more likely to find a particle in the middle of the box. On the other hand, we can also plot the probability as function of  $x$ , where  $m$  equals to 2  $k$ 's.

If we are actually operating in a second normal mode, then basically you have some distribution-- looks like this. In this situation, it is forbidden-- or say, there's zero probability you will find the particle in the middle of the box, et cetera, et cetera. You can actually calculate all those corresponding probability distributions as a function of  $m$  value and as a function of position.

OK? Sounds like a very good story. But there's something missing, right? What is actually missing? You have the normal modes. You have the  $k$ - $m$ . What is missing is the wave equation. The wave equation is missing, right?

You don't have the dispersion relation. This solution is incomplete. You don't know what is actually the  $\omega$  value. Because you don't have dispersion relation. So what is actually the wave equation for the quantum mechanics?

So it is actually Schrodinger's equation. So Feynman once commented on the origin of the Schrodinger's Equation. It's from where? It's not possible to derive it from anything you know. It's just coming out of the might Schrodinger, actually. So there's no reason. And it works. That's the beautiful part.

So what is, actually, this equation? So this is the equation Schrodinger actually writes down. It's like  $i \hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \nabla^2 \psi + V \psi$ . And this will be equal to minus  $\hbar^2$  squared over  $2m$  partial square, partial  $x$  square, plus  $V \psi$ .

So this is really nice. And it works and matches with experimental results. And now, I have already the normal mode. I can plug that into this equation to see what I can actually learn from there. So what I am going to do is to plug in  $\psi_m(x)$  into this equation, to get the dispersion relation.

So what this issue, the dispersion relation. So here, I have  $\frac{\partial}{\partial t}$ . So I extract one  $\omega$  minus  $i$   $\omega$  out of this. Then, basically, I get  $\hbar \omega - \frac{\hbar^2 k^2}{2m} = V$  in the  $\psi_m(x)$ . OK, plugging in  $\psi_m$  into this equation and see what happens. Then this will be equal to-- I also know that, in the middle of the box,  $V$ , essentially, the potential-- the potential is 0 inside

the box. The potential is infinity at the edge of the box.

Therefore, I can safely ignore this term, to be equal to 0. So you have a free path to go inside the box. And what, essentially, this term-- this term will give you minus  $\hbar^2$  over  $2m$ . OK, I have a double differential of  $x$ . And therefore, I get, basically, minus  $k^2$ , right? Because it's  $\psi''$ , right?

So basically, I get minus  $k^2$ . So therefore, I cancel this minus sign. I have  $k^2$  squared out of this calculation. And I still have  $\psi = A \sin(kx)$ . I can cancel this too. Then, what I'm getting is that  $\omega = \hbar k^2 / 2m$ . This-- essentially, dispersion relation of the wave function.

De Broglie proposed that wavelengths of the matter wave, essentially, highly related to the momentum of the matter. So basically, he propose that  $p$ , the momentum of the particle is actually equal to  $\hbar k$ , where  $k$  is the wave number of the matter wave. If you accept de Broglie's interpretation, basically what we are getting is something really, really interesting.

If we put together this dispersion relation and the de Broglie's interpretation of matter wave-- what I am going to do is to calculate the group velocity of this dispersion relation. So I can now calculate-- group velocity,  $V_g$ , will be equal to  $d\omega / dk$ . And I know that  $\omega$  is equal to  $\hbar k^2 / 2m$ .

This is essentially equal to  $\hbar k / m$ . Everybody is following? OK. And this is equal to what? This is equal to  $p / m$ , if I use de Broglie's matter wave. Therefore, you have  $p$  equal to  $m$  times  $v_g$ . Wow! Look at this. What are we getting here?

What we are getting here is that the group velocity of the wave equation of the waves is actually the classical velocity,  $p$  equal to  $m$  times  $v$ . Now, everything actually is becoming more and more clear. We know and we learned already, from 8.03, what is the meaning of group velocity.

The meaning of the group velocity is the speed of the propagation of a wave package, right? Remember our discussion before about a AM radio? So what is actually the speed of propagation of a wave packet is the group velocity. So now we have solved the problem-- why electron can be a particle, at the same time, also like waves.

It's essentially described by wave functions. The classical behavior we see on the electron is because it is, as you described, by these wave packages. It's pretty localized. And the motion

of this wave package in a free space is actually the speed of the propagation-- is the group velocity. Therefore, there is no contradiction between the classical calculation and the wave interpretation of the electron.

So that really surprised me very much. And you can see that, given the dispersion relation, also, this is a rather dynamical result. The real part of the wave function is actually blue. And the imaginary part is actually red. It's actually oscillating up and down. And the oscillation frequency, by now, you know is governed by that dispersion relation.

OK, now actually, everything seems to make sense now-- really, really, very cool. So on the other hand, we also have to live with probability density. So you cannot tell the exact position of a particle any more. You cannot tell the exact outcome of an experiment anymore.

And that is actually to do with this interpretation. And all of those phenomena, at a very, very small scale, is actually described by quantum mechanics, which we will learn some more in 8.04. And also, in the future, you will be governed by the quantum field theory, which is actually a father future. And what is actually the life living with quantum mechanics and quantum field theory?

So this is a very simple--

[LAUGHTER]

--Lagrangian of the standard model. And it describes everything except the gravity. OK? And it's really simple. It's called Standard Model. And look at this part. This is governing the Higgs decay to Z boson. And the experimental result-- we don't really know what is the mass of the Higgs. It's a missing observable before.

And on the other hand, as a particle physicist or as a high energy nuclear physicists, I have no idea about what will happen in the next collision. Why is that? You know the reason now, right? Because we cannot predict the exact outcome of our experiment. It's all governed by wave functions.

Therefore, what we are doing is the following. We are doing the brute force. So we collide like crazy-- collide, collide, collide, collide like crazy, until something interesting pops out. That's actually what we're really doing as a particle physicist.

And this is your beautiful event from proton-proton collisions at the Large Hadron Collider-- is

a Higgs to the boson event. And one of the Z bosons becomes the two red lines. It's actually the two muons. And the other decays to electrons, which are detected by the kilometer as the two blue things there.

As a high-energy nuclear physicist, I am interested in the production of quark-gluon plasma from lead ion collisions. So I am now putting together two ions, have them collide. And I hope that, by chance, I can deposit a huge amount of energy in a very small volume.

And then I would like to see this crazy matter, actually, gradually expand and become a lot of particles. And I study those particles to understand, what would be the nature of this material, which starts to exist in the very early part of the whole universe history? Just one microsecond after the Big Bang, we have the whole universe filled by this crazy material. And we are creating this in the experiment.

And we will only be able to hope that, OK, by chance, I have the collision happen. By chance, I have a very high-density environment. Somehow, multiple quanta decide to scatter on each other. And they deposit the energy in a very small volume. And then we collect all those spectacular events to study the properties of all those little Big Bangs. So that is actually the consequence of this wave function interpretation.

So now, coming back to the Standard Model, this really simple one, you can't see, that is a theory of almost everything, except the gravity. Really sad. So if you can actually put them all together, then you will also win the Nobel Prize. And giving you all those ideas, so that I can have a very good student winning the Nobel Prize. Of course you will.

And now, I would like to discuss and use the remaining, maybe, 10 minutes to discuss with you the gravity. So here is actually something related to gravity. So Einstein actually predicted that the distortion of the space-time generated by objects can travel through the space.

I don't have the derivation here, because it would take another, maybe, two hours to do this. But I would like to ask you to trust me. This is actually a result coming from general relativity. And you can see that we can actually generate gravitational waves.

And I can actually do the generation here, like this and rotating. I'm generating gravitational waves. And that student in the back is also generating. Yes, you are. Yeah, you are generating. Everybody's generating. Ah, you are also generating. Yeah, very good.

But the problem is that the space-time distortion is really small for people who are not very massive, like me. So that's a problem. So I can generate. I'm doing the demo here. But it doesn't help. You cannot really detect them. And even Einstein himself thinks it's impossible to detect them, maybe, in our lifetime.

And what would be the outcome of the calculation? The outcome of the calculation-- if you have gravitational wave passing toward you-- so what it does is the following. So basically, the space is distorted in a way such that it first expands in this direction and then expanding the other direction, perpendicular to the original distortion.

And if you put a ray of particles, the circular array of particles, and look at what is going on, when the gravitational wave pass through it, it pass through the array in this direction, what you are going to get is effect like that. Of course, this is actually highly exaggerated in this set-up. You don't really see this kind of sizable distortion when Yen-Jie is dancing around.

OK, so how about we actually visualize this thing. The problem is that we cannot really see the space distortion. But what we can, as you see, is the light which actually pass through those little distortions. So this is a stimulation from LIGO Collaboration. They are simulating the merging of the two massive black holes. And you can see that they are rotating with respect to each other. They are radiating energy out of this two-body system.

Let's take a look at this again. So this is actually a simulation of the event observed by LIGO. So both black holes have a mass roughly 30 times of our sun. It's very massive. And they are rotating with respect to each other. And that generates space-time distortion. And you can see that the space-time distortion stops after they merge each other.

And we were hoping that we can detect those. How crazy is that? So how do we detect them? Actually, you already have the knowledge to design the experiment to detect this kind of effect. So remember, the effect of the gravitational wave is like this. So you have distortion like this.

What we actually-- MIT and Cal Tech and the many other collaborators designed the LIGO experiment. Are what is, actually, LIGO? It's a Laser Interferometer Gravitational wave Observatory. It is actually always good to have a very good name of your experiment. So this is actually LIGO.

So what, actually, it does is the following. So basically, it emits a laser. And you split the laser into two pieces. And there were mirrors in the very far end, reflect those lasers. And they come



together. And there is a photo-detector, which detects the interference pattern of these two optical path lengths.

Wow! Sounds familiar to you, right? Hey, you already know how to explain this to your friends already. Really cool. And in order to have redundant management-- for example, if you only have a single experiment, maybe one graduate student is like, oh, doing dancing next to a detector. Then you see some fake signal. And that's not going to be helpful.

So what it does is that-- basically, we have two experiments. One is actually in Hanford. The other one is actually in Livingston. And they are actually 3,000 kilometers away from each other. So that there should not be any correlation between the signal coming from a earthquake or dancing of the graduate students or whatever. So they can you use that to suppress any coincidence which is actually not related to gravitational waves.

So what does this do? So now, we have, oh, the knowledge to actually explain this phenomena. So what this does is the following. So you meet the laser. And when the gravitational waves come in, then it does this space-time distortion. And then the interference pattern of the waves going through different optical path lengths is going to change.

So you can see that, originally, the experiment is designed so that you have complete cancellation. But when the gravitational waves is hitting the side, you will be able to see that. Really, you have constructive interference at some point, because of the movement of the mirrors. And those mirrors are really, really far away from the sources. Each of them is, like, four kilometers away from the mirror.

And due to the incredible precision which were achieved by this experiment, we will be able to detect this signal of gravitational waves. So you can see, again, from here-- so basically, when the gravitational wave comes in, first you split the light source into two pieces, have them hit the mirror, which is actually four kilometers far away from each other.

And they come back. And initially, the experiment is designed so that, very precisely, there will be no amplitude detected by the photo-detector. And when the gravitational waves come in, you actually really change the length between a splitter and the mirror. Therefore, you see light, constructive interference even, from the photo-detector. So that is really cool.

And this is actually the experimental result. Look at this. You actually can see that light actually achieve the sensitivity. The gravitational wave was first observed on September 14, 2015. And

the LIGO is actually announcing that February 11-- earlier this year.

And what we actually see from here is that this is actually-- as I mentioned to you-- there are two measurements, two sides. The left hand side is actually the measurement from Hanford. And the other one is actually the measurement from Livingston. So they are two different curves.

And they all have almost exactly the same pattern. Of course, there's time has actually shifted, because they are in different places on the earth. They are 3,000 kilometers apart. So therefore, there will be a shift in time. And this is actually a time-shifted result. And you can see, also, the calculation below, which is actually what you should expect if you have the merger of the two massive black holes.

So you can see that they are actually rotating with respect to each other. One of them is actually 29 times larger than the mass of the sun. And the other one is 36 times larger than the mass of our sun. It's not precision. And they generate, theoretically, this kind of pattern. And this is actually really detected by both LIGO experiments, which lay at 3,000 kilometers apart from each other.

So I think this is really a historical moment-- that we actually were very lucky to live in this moment. What does that mean? That means we have a new way to really hear about what the universe is actually trying to tell us. We have a new way to detect phenomena, which is actually really happening very, very far away from the Earth.

How far is, actually, this event? This event, according to calculation, is actually something like 1.3 billion light years away from the Earth. And we can detect that. And we even know the mass of the two black holes. Wow!

What does that mean? This is really crazy to me and really exciting, because we are opening up-- OK, I have two ears. And it's opening up another ear in my brain. And that is actually the way to hear the gravitational wave with the experiment we've performed on Earth.

So I hope, until now, I have convinced you that this is really not the end of vibration of waves, which is actually the end of 8.03. Instead, this is actually just the beginning. You have a lot more to explore when you take general or special relativity course. You have a lot, really, more to explore when you take quantum mechanics.

And I hope you really enjoy the content of this course. Personally, I really enjoy that very

much. I love this course. And I hope you also love it and understand something from my lecture. And thank you very much. And the next time, we are going to have a review of all concepts we have learned from 8.03 next Tuesday. Thank you.

[APPLAUSE]

Thank you very much.