PROFESSOR: Next is this phenomenon that when you have a wave packet and it moves it can change shape and get distorted. And that is a very nice phenomenon that takes place in general and causes technological complications. And it's conceptually interesting. So let's discuss it.

So it's still wave packets. But now we have to go back and add some time to it. So shape changes. So we had a psi of x and t is equal to 1 over square root of 2 pi phi of k e to the ikx e to the minus i omega of kt. And what did we do with this to analyze how it propagates? We expanded omega of k as omega of k0, which, again, this quantity is centered and peaks around k0, plus k minus k0 times d omega dk at k0 plus 1/2 k minus k0 squared, the second omega, dk squared at k0.

And it might seem that this goes on forever. And what did we do before? We looked at this thing and we did the integral with this term and ignored the next. And with this term, we discovered that the profile moves with this velocity, the group velocity. Now we want to go back and at least get an idea of how this term could change the result. And it would change the result by deforming the shape of the packet. So it is of interest to know, for example, how long you have to wait before your packet gets totally deformed, or how do you evolve a packet.

So we need to recall these derivatives. So the omega vk is the same as de dp by multiplying by h bar. And this you'll remember, was p over m. The edp is p over m and is equal to h bar k over m. So the second omega, dk squared. I must differentiate the first derivative with respect to k. So I differentiate the first derivative with respect k. And now I get just h bar over m, which is quite nice. And the third derivative, the 3 omega, dk cubed, is 0. And therefore, I didn't have to worry about these terms. The series terminates. The Taylor series terminates for this stuff. Yes?

AUDIENCE: The reason this happens is because we're [INAUDIBLE].

PROFESSOR: That's right. So of what is it that we get? Well, this term is roughly then 1/2 k minus k0 squared times h bar over m. And we can go back to the integral that we're trying to do. We don't do it again or not by any means. But just observe what's going on there. And we have an e to the minus i omega of kt that we did take into account. But the term that we're dropping now is a term that is minus i omega of k, well, whatever we have here, 1/2 k minus k0 squared h bar over mt.

That's the phase that we ignored before. But now we'll just say, that we expect, therefore, that the shape doesn't change as long as we can ignore this phase. And this phase would start changing shape of the object. So our statement is going to be that we have no shapes. So let's imagine you started with a packet that sometime t equals 0. And then you let time go by. Well, there's some numbers here and time is increasing. At some point, this phase is going to become unignorable. And it's going to start affecting everything. But we have no shape change, or no appreciable shape change, as long as this quantity is much less than 1. So as long as say, k minus k0 squared h bar over m absolute value of t is much less than 1, no shape change.

Now it's convenient to write it in terms of things that are more familiar. So we should estimate this thing. Now we're doing estimates in a very direct and rough way here. But look, your integrals are around k0. And as you remember, they just extend a little bit because it has some width. So k minus k0, as you do the integral over k, you're basically saying this thing is about the size of the uncertainty in k. So I'll put here delta k squared. Then you'll have h bar t over m much less than 1.

Now h bar times delta k is delta p. So this equation is also of the form delta p squared t over h bar m much less than 1. There's several forms of this equation that is nice. So this is a particularly nice form. So if you know the uncertainty and momentum of your packet, or wave packet, up to what time, you can wait and there's no big deformation of this wave packet. Another thing you can do is involve the uncertainty in x. Because, well, delta p delta x is equal to h bar. So we can do that.

And so with delta p times delta x equal to about h bar, you can write t less than h bar over m over a delta p squared, which would be h squared delta x squared. I think I'm getting it right. Yep, so t much less than m over h bar delta x squared. That's another way you could write this inequality.

There is one way to write the inequality that you can intuitively feel you understand what's happening. And take this form a from a. Write it as delta p t over m is less than h bar over delta p. And h bar over delta p is delta x. So you go delta p over mt much less than delta x. I think this is understandable.

Why does the packet change shape? The reason it changes shape is because the group velocity is not the same for all the frequencies. The packet mostly moves with k0. And we haven't rated the group velocity in k0. But if it would have a definite velocity, we would have a definite momentum. But that's not possible. These things have uncertainty in momentum. And they have uncertainty in k that we use it to write it. So different parts of the wave can move with different velocities, different group velocities. The group velocity you evaluated at k0. But some part of the packet is propagating with group velocities that are near k0 but not exactly there.

So you have a dispersion in the velocity, which is an uncertainty in the velocity or an uncertainty in the momentum. Think, the momentum divided by mass is velocity. So here it is, an uncertainty in the velocity. And if you multiply the uncertainty in the velocity times this time that you can wait, then the change in shape is not much if this product, which is the difference of how one part moves with respect to the other, the difference of relative term, is still smaller than the uncertainty that controls the shape of the packet. So the packet has a delta x.

And as long as this part, the left part of the packet, then the top of the packet, the difference of velocities times the time, it just still compared to delta x is small, then the thing doesn't change much. So I think this is one neat way of seeing what an equation that you sometimes use in this form, sometimes use in in this form-- it's just things that you can use in different ways.

So for example, I can do this a little exercise. If you have delta x equals 10 to the minus 10 meters, that's atomic size for an electron. How long does it remain localized? So you have an electron. And you produce a packet. You localize it to the size of an atom. How long can you wait before this electron is just all over the room? Well, when we say this t, and we say this time, we're basically saying that it's roughly still there. Maybe it grew 20%, 30%. But what's the rough time that you can expect that it stays there?

So in this case, we can use just this formula. And we say the time could be approximately m over h bar delta x squared. It's fun to see the numbers. You would calculate it with mc squared over h bar c times delta x over c, this squared. The answer is about 10 to the minus 16 seconds, not much.

This is a practical issue in accelerators as well. Particle physics accelerators, they concern bunches, a little bunch of protons in the LHC. It's a little cylinder in which the wave functions of the protons are all collimated very thin, short, a couple of centimeters short. And after going around many times around the accelerator, they always have to be compressed and kept back, sent back to shape. Because just of diffusion, these things just propagate. And so it's a rather important thing.