We spoke about superposition, and we showed how, when you have two states that are superimposed, the resulting state that is built up doesn't have properties that are intermediate between the two states that you're superimposing. But rather, when you do a measurement, you obtain the result that you would sometimes-- you sometimes obtain the result that you would have with one of the states, and some other times with different probabilities, the result as if you had the other state. So it's a strange kind of way in which things are combined in quantum mechanics.

So the next thing we have to say is a physical assumption that is made here. And it is that if you have a state and you superimpose it to itself, you haven't done anything. So the superposition of a state with itself has no physical import. So we can say this. A physical assumption superimposing a state to itself does not change the physics.

So if I have a state, this is physically equivalent-- I'll write physically equivalent with this symbol-- to the state a plus a, which would be 2 times a. It's physically equivalent to the state minus a. It's physically equivalent to the state ia on anything. It's not equivalent to 0a, because that would be the zero state. So it's physically equivalent as long as you have a non-zero coefficient. All these states are supposed to be physically equivalent.

And that will eventually mean that we sometimes choose a particular one in those collection of states that is one that is convenient to work with. And that will be called a normalized state, a state that satisfies other properties having to do with the norm squared of the state. That will come later. But it's important that the number that is multiplying the physical state of your system has no relevance.

And you could say, well, why all of the sudden you tell us this. Could this be shown to be necessary? Or it's a physical assumption, so can we test it? Does it make some sense? And we can make some sense of this assumption at this level.

And we do it with states of light. So remember, we spoke about photons hitting a polarizer. And we could speak of two independent kind of photons-- photons polarized along the x-axis and a photon polarized along the y-axis. And those are two quantum mechanical states.

Now suppose I decide to superimpose those states to create the most general photon state. I would have an alpha, which is a number here, a complex number, and a beta there. And I would say, OK, here is my most general photon state. And how many parameters does this state have? It has two complex parameters, alpha and beta, and therefore, four real parameters.

And then you think about polarization states, how many parameters they have. And as we'll review in a second, it's well known that photons-- their polarization state can be expressed with just two real parameters. So some counting is not going very well here.

But here comes the help. If the overall coefficient here doesn't matter-- if I can change it, I can multiply everything by 1 over alpha, and therefore get that the state is just the same, physically equivalent to this state, beta over alpha photon y. So all the physics is contained in this state as well.

And if all the physics is contained in that state, I must look how many parameters it has. It still looks like there's two numbers here, but only the ratio appears. So if you call beta over alpha, the number gamma is just one complex parameter. And therefore, thanks to this assumption, you now get that the most general photon polarization state has just one complex parameter, or just equivalently, two real parameters. And that is the correct number.

Indeed, if you have a polarization, a wave that has some polarization, the most general polarization state of a wave is an elliptical polarization. You probably did study a lot about circular polarizations, or maybe you also heard about the elliptical one in which the electric field-- in a circular polarization, the electric field at any point traces a circle. But if you have an elliptical polarization, the electric field traces an ellipse.

And that ellipse has an angle that is one parameter. And for an ellipse, the other-- the size doesn't matter. The size depends just on the magnitude of the electric field. It's not a parameter of the polarization of the wave. Since the size doesn't matter, it's the shape of the ellipse that matters. And that's characterized by the eccentricity or by the ratio a over b of the semi-major axis, so parameters, two parameters, and they are a over b and theta.

So an elliptically polarized wave, which is the most general state of polarization of a wave, has two real parameters. And now, thanks to this physical assumption, we get this right. And this is important because that's something we're going to use all the time, that the overall factor in a wave function does not matter.

So if we have superpositions, I want to emphasize one more thing about superpositions. And for that, I'm going to use spins. So what is spin? Spin is a property of elementary particles that says that actually, even if they're not rotating around some other particle, they have angular momentum. They have intrinsic angular momentum, as if they would be made of a tiny little ball that is spinning.

I say as if because nobody has ever constructed a model of an elementary particle where you can really make it spin and calculate how it works. Somehow, this elementary particle has angular momentum is born. Even if it is a point particle, it has angular momentum, and it's spin.

And spin is very quantum mechanical. And we can't quite understand it without it. So what happens is that you can measure the spin of a particle.

And then if you measure it, you have to decide, however, since angular momentum is a vector, what direction you should use. And suppose you use the z direction to measure the spin of a particle. You may find that the particle has either spin up or the particle has spin down. Spin.

And the spin is the direction of the angular momentum. And that's a funny thing that happens with most matter particles. These are spin 1/2 particles. The spin can be up or it can be down along the z direction that you measure. You measure it, and you never find it's 0 or a little bit. It's just either up with the full magnitude or down with the full magnitude. That is a spin 1/2 particle.

And the state where it is up, we sometimes denote it with an arrow up and call it z because it's up along z. And this would be down, an arrow down along z. If those are possible quantum states, you could build a new quantum state by superposition which would be up along z plus down along z.

Now, if I wish to normalize it, I would put the factor in front of this. I will not talk about normalizations at this moment. They're not so important.

If you are faced with this quantum state-- so suppose you have an electron that is not in this state nor in this state, but is in this state, in a quantum superposition. So you go and you decide to try to measure it. Now, since you cannot predict what that electron is going to be doing-- we cannot predict things in quantum mechanics with certainty-- we, since we're going to do this experiment, avail ourselves of 1,000 copies of this electron, all of them in this peculiar quantum state.

So you have the 1,000 copies, and you start measuring. And you decide to measure the spin in the z direction. And now what do you get? Well, we mentioned last time that you don't get an average, or since this is up and this is down, you get 0. You measure the first particle and you find it up.

Measure the second, up, the third, up, the fourth, down, five, down. And then you get a series of measurements. At the end of the 1,000 particles, you find about 495 up and 505 down, about half and half. And if you did it with 10,000 particles, maybe it would be closer. Eventually, you'll find 50% in this state and find 50% in this state.

And if you think this is strange, which you probably do, well, you could be justified. But here would come Einstein along and would say all this stuff of this superposition is not quite right. You had this 1,000 particles. But actually, those 1,000 particles, half of them were with a spin up and half of them were with a spin down.

So here you have your 1,000 particles, your quantum state, this. But Einsten says, no, let's make an ensemble of 1,000 particles, 500 up, 500 down, and do the same experiment. And the result is going to be the same. So how do you know you really have this as opposed to somebody has given you 1,000 particles, 500 up, 500 down? How can you tell?

And in fact, he would say even more-- whenever Einstein used the word realism to say if I measure a spin and I find it up, it's because before I measured it, the spin was up. It's almost like learning something about an object. If I look at this page and I find the color red, it's because before I looked at it, it was red.

But then in quantum mechanics, that doesn't seem to be the case. The state is this mix. And it was this mix before you measured. And after you measure, it's this. So there is no such thing as you learn by doing one measurement what the state of the particle was.

So we will not resolve Einstein's paradox completely here because we would have to learn more about spins, which you will do soon enough. But here's the catch that actually happens. If, instead of having an ensemble of quantum states, you would have an ensemble of those states that half of them are up and half of them are down, you could now decide to measure the spin of the particle along the x direction.

You take these particles, and you measure along x. And what you will calculate with quantum mechanics later in this course-- if you measure along x, in this state, you will find all of them to be pointing along plus, up along x, all of them. While on this Einstein ensemble of 50% up and 50% down, you would find 50% up along x and 50% down along x.

So there is an experiment that can tell the difference, but you have to look in another direction. And that experiment, of course, can be done. And it's a calculation that can be done, and you can decide whether these quantum states exist. And they really seem to exist.