Quantum Physics I (8.04) Spring 2016
Assignment 7

MIT Physics Department — *Due Friday April 8, 2016*
April 1, 2016 — *12:00 noon*

Reading: Griffiths sections 2.5 and 2.3.

Problem Set 7

1. **Two delta functions** [15 points]

   Consider a particle of mass m moving in a one-dimensional double well potential

   $$V(x) = -g\delta(x-a) - g\delta(x+a)\,, \qquad g > 0\,.$$

   (a) Find transcendental equations for the bound state energy eigenvalues of the system. Plot the energy levels in units of $\hbar^2/(ma^2)$ as a function of the dimensionless parameter $\lambda \equiv mag/\hbar^2$. Explain the features of the plot.

   (b) In the limit of large separation $2a$ between the wells find a simple formula for the splitting between the ground state and the first excited state.

2. **Sketching wavefunctions.** Griffiths 2.47, p. 87. [10 points]

   In this problem you should try to figure out intuitively how the solutions look. It is a good idea then to check your intuition with the shooting method and the setup of the H_2^+ ion.

3. **Harmonic oscillators beyond the turning points** [10 points]

   For the simple harmonic oscillator energy eigenstates with $n = 0, 1,$ and 2, calculate the probability that the coordinate x takes a value greater than the amplitude of a classical oscillator of the same energy.

4. **Harmonic oscillator computations** [15 points]

   (a) Calculate the expectation value of x^4 on the energy eigenstate with number n.

   (b) Calculate Δx and Δp on the energy eigenstate with number n. What is the value of the product $\Delta x \Delta p$?

(c) Consider the polynomials $H_n(\xi)$ defined by the generating function

$$e^{-s^2+2s\xi} \;=\; \sum_{n=0}^{\infty} H_n(\xi)\frac{s^n}{n!}\,.$$

Verify that $H_n(\xi) = (2\xi)^n + \ldots$ where the dots represent terms with lower powers of ξ. Show that the polynomials $H_n(\xi)$ so defined satisfy the Hermite differential equation:

$$H_n'' - 2\xi H_n' + 2nH_n \;=\; 0\,.$$

5. **Harmonic oscillator and a wall.** Griffiths Problem 2.42. p. 86. [5 points]

6. **Harmonic oscillator oscillating!** [10 points]

A particle of mass m in a harmonic oscillator with frequency ω has an initial, time zero wavefunction

$$\Psi(x,0) \;=\; \frac{1}{\sqrt{2}}\big(\varphi_0(x) + \varphi_1(x)\big)\,,$$

where φ_0 and φ_1 are the normalized eigenstates of the Hamiltonian with number eigenvalue zero and one, respectively.

(a) Write down $\Psi(x,t)$ and $|\Psi(x,t)|^2$. You may leave your expressions in terms of φ_0 and φ_1.

(b) Find $\langle x\rangle$ as a function of time. What is the amplitude of this oscillation and what is its frequency?

(c) Find $\langle p\rangle$ as a function of time.

(d) Show that for *any* harmonic oscillator state, the probability distribution $|\Psi(x,t)|^2$ is equal to $|\Psi(x,t+T)|^2$ for $T = \frac{2\pi}{\omega}$.