

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.07: Electromagnetism II
Prof. Alan Guth

October 24, 2012

PROBLEM SET 6 REVISED*

DUE DATE: Friday, October 26, 2012. Either hand it in at the lecture, or by 6:00 pm in the 8.07 homework boxes.

READING ASSIGNMENT: Chapter 4 of Griffiths: *Electric Fields in Matter*. For lack of time we are skipping the susceptibility tensor, discussed on p. 184 of Griffiths, so we are discussing only the case of a linear, isotropic medium, for which we can assume that $\vec{P} = \epsilon_0 \chi_e \vec{E}$.

CREDIT: This problem set has 80 points of credit.

PROBLEM 1: A SIMPLE ATOMIC MODEL OF POLARIZABILITY (10 points)

Griffiths, Problem 4.2 (p. 163). Assume that the electron cloud is rigid, and that in response to the applied electric field it moves relative to the point proton, until electrostatic equilibrium is established.

PROBLEM 2: CAVITIES IN DIELECTRIC MEDIA (15 points)

Griffiths, Problem 4.16 (p. 177).

PROBLEM 3: A POINT CHARGE AT THE CENTER OF A SPHERICAL DIELECTRIC (10 points)

Griffiths, Problem 4.32 (p. 198).

PROBLEM 4: FORCES, TORQUES, AND ANGULAR MOMENTUM CONSERVATION WITH DIPOLE-DIPOLE INTERACTIONS (15 points)

Consider a system of two ideal electric dipoles. The first is located at the origin, and points along the z -axis, so

$$\vec{p}_1 = p_1 \hat{e}_z . \quad (4.1)$$

The second dipole lies along the positive x -axis at

$$\vec{r} = r \hat{e}_x . \quad (4.2)$$

* In Problem 1, I inserted a preposition “to,” which was missing in the original October 21, 2012 version. In Problem 6, the phrase “with side larger than $2a$ ” was replaced by “with side larger than $2R$.”

and is oriented along the x -axis,

$$\vec{p}_2 = p_2 \hat{e}_x . \quad (4.3)$$

- (a) Calculate the force \vec{F}_2 on dipole 2 caused by the field of dipole 1, and the force \vec{F}_1 on dipole 1 due to the field of dipole 2. Are these forces equal in magnitude and opposite in direction, as described by Newton's third law of motion? Do the forces point along the line joining the two dipoles?
- (b) In part (a) you should have found that the forces are equal and opposite, but that they do not point along the line joining the particles. But this raises a question about the conservation of angular momentum, since the standard proof of the conservation of angular momentum from Newton's laws relies on the "strong form" of Newton's third law, which requires the force to point along the line of centers. Under that assumption, one argues that for an isolated two-particle system, $\vec{F}_2 = -\vec{F}_1$ and the total torque is

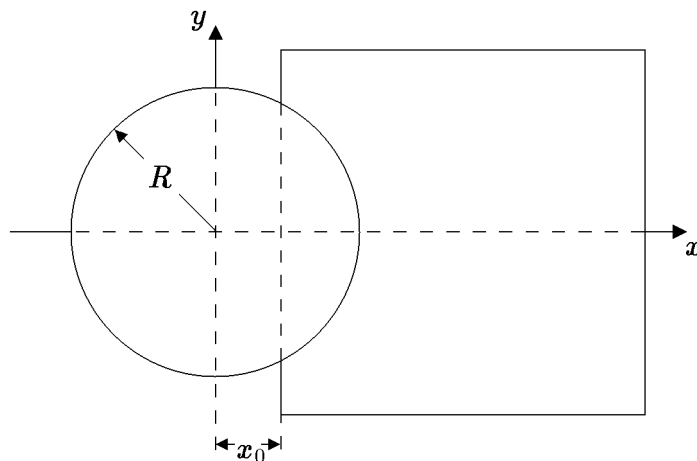
$$\begin{aligned} \vec{\tau} &= \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 \\ &= (\vec{r}_1 - \vec{r}_2) \times \vec{F}_1 = \vec{0} , \end{aligned} \quad (4.4)$$

since the cross product between parallel vectors is zero. Show, however, that the conservation of angular momentum is nonetheless valid in this case, if one includes the torques that each dipole experiences due to the field of the other, $\vec{\tau} = \vec{p} \times \vec{E}$. Calculate the total torque on the system, for example about the origin, and show that it vanishes.

PROBLEM 5: CONCENTRIC SPHERICAL CAPACITOR, HALF-FILLED WITH DIELECTRIC (*15 points*)

Problem 4.10 of J.D. Jackson, Classical Electrodynamics, 3rd Edition (John Wiley & Sons, 1999).

PROBLEM 6: FORCE ON A DIELECTRIC SLAB PART WAY INSIDE A CIRCULAR CAPACITOR (15 points)



Consider a circular capacitor, consisting of two thin circular disks, of radius R , parallel to the x - y plane. Both are centered on the z -axis, one at $z = +\frac{1}{2}d$ and the other at $z = -\frac{1}{2}d$. The two disks are held at a potential difference V_0 . A square sheet of dielectric, with thickness just a shade smaller than d and with side larger than $2R$, is placed part way between the two disks, as shown in the diagram. The left-hand edge of the square is at $x = x_0$. Calculate the force on the slab as a function of x_0 .

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