MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.07: Electromagnetism II — September 22, 2012
Prof. Alan Guth

PROBLEM SET 3

DUE DATE: Friday, September 28, 2012. Either hand it in at the lecture, or by 6:00 pm in the 8.07 homework boxes.

READING ASSIGNMENT: Chapter 3 of Griffiths: *Special Techniques*, Secs. 3.1–3.3.

PROBLEM 1: SPHERES AND IMAGE CHARGES *(10 points)*

Griffiths Problem 3.8 (p. 126).

PROBLEM 2: IMAGE CHARGES WITH A PLANE AND HEMISPHERICAL BULGE *(15 points)*

Consider a conducting plane that occupies the x-y plane of a coordinate system, but with the circular disk $x^2+y^2 < a^2$ removed. The circular disk is replaced by a conducting hemisphere of radius a, described by the equation

$$x^2 + y^2 + z^2 = a^2 \; , \qquad z > 0 \; . \tag{2.1}$$

A charge q is placed on the z-axis at $(0, 0, z_0)$, with $z_0 > a$. Find a suitable set of image charges for this configuration. Show that the charge is attracted toward the plate with a force

$$|\vec{F}| = \frac{1}{4\pi\epsilon_0}\left[\frac{q^2}{4z^2} + \frac{4q^2a^3z^3}{(z^4 - a^4)^2}\right] \; . \tag{2.2}$$

PROBLEM 3: IMAGES FOR A CONDUCTING CYLINDER *(15 points)*

This problem is based on Problem 2.11 of Jackson: **Classical Electrodynamics***, 3rd edition.*

A line of charge with linear charge density λ is placed parallel to, and at a distance R away from, the axis of a conducting cylinder of radius b held at fixed voltage so that the potential vanishes at infinite distance from the cylinder.

(a) Find the magnitude and position of the image charge(s).

(b) Find the potential V_0 of the cylinder in terms of R, b, and λ.

PROBLEM 4: CAPACITANCE OF A SINGLE CONDUCTOR *(20 points)*

(a) Consider a single conductor, and define its capacitance by $Q = CV$, where Q is the charge on the conductor, and V is the potential of the conductor defined so that $V = 0$ at infinity. Show that C can be expressed as

$$C = \frac{\epsilon_0}{V_0^2} \int_{\mathcal{V}} |\vec{\nabla} V|^2 \, \mathrm{d}^3 x \ , \tag{4.1}$$

where $\mathcal{V}$ is the space outside the conductor, and $V(\vec{r})$ is the solution for the potential when the conductor is held at $V = V_0$.

(b) Show that the true capacitance C is always less than or equal to the quantity

$$C[\Psi(\vec{r})] = \frac{\epsilon_0}{V_0^2} \int_{\mathcal{V}} |\nabla \Psi|^2 \mathrm{d}^3 x \ , \tag{4.2}$$

where $\Psi(\vec{r})$ is any trial function satisfying the boundary condition $\Psi = V_0$ at the conductor, and $\Psi = 0$ at infinity. (Note that Ψ is *not* required to satisfy Laplace's equation, or any other equation.)

(c) Prove that the capacitance C' of a conductor with surface S' is smaller than the capacitance C of a conductor whose surface S encloses S'.

(d) Use part (c) to find upper and lower limits for the capacitance of a conducting cube of side a. Write your answer in the form: $\alpha(4\pi\epsilon_0 a) < C_{\text{cube}} < \beta(4\pi\epsilon_0 a)$ and find the constants α and β. A numerical calculation* gives $C \simeq 0.661(4\pi\epsilon_0 a)$. Compare this answer with your limits.

PROBLEM 5: LAPLACE'S EQUATION IN A BOX *(15 points)*

Griffiths Problem 3.15 (p. 136).

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* C.-O. Hwang and M. Mascagni, *Journal of Applied Physics* **95**, 3798 (2004).