[SQUEAKING]

[RUSTLING]

[CLICKING]

MARKUS KLUTE: Welcome back to 8.20, Special Relativity. So in this section, we want to look again at collisions and study momentum conservation. And so we have this scenario here in which we have two balls colliding with velocities uA and uB, mass mA and mB. And after the collisions there, the mass is changed to mC and mD. And the velocities are uC and uD.

Momentum conservation tells you that the product of the mass and velocities and the sum of the two particles before and after the collisions is the same. But now what happens if you boost the system? If we look at the very same system with the boosted reference frame, and we can just simplify the case here by just considering the x direction.

So the question is if momentum is conserved in a frame s, like the one we're looking here in this picture, is the momentum also conserved in a moving reference frame with relative velocity v? And so you can show this quite easily, that this is actually not the case, right? So you write down the velocities.

The momentum equation is the same as before with boosted velocities. And you find that this is not the case. You can easily show this by, for example, setting the right part of this equation to 0, and see whether or not this equation will hold true in general. And it doesn't.

But in the last section, we introduced the new concept of proper velocity. So how about redefining momentum through proper velocity and just saying mA times proper velocity A plus mB times proper velocity B is equal to mC times proper velocity C, and so on, and see whether or not we can learn something from this equation?

So why don't we write down this very same equation and with proper velocity and see whether or not it's invariant in the Lorentz transformation? OK. Again, a good moment to stop the video and just work out the math by yourself, on your own.

So I did this here. And I'm just doing this for the x component. So we have our proper velocity vector, which is gamma times C, gamma times uX, gamma times uY, and sometimes uZ. And so the x component is the first component. And in the boosted reference frame, the proper velocity of the x component in our boosted frame is gamma times the proper velocity of the particle A, first component, minus beta, upper velocity of A0's component.

And then you can write your equation. And this should be our momentum energy, momentum conservation equation. OK. And the Lorentz transformation? We find this one here.

And so all we need to do now in order to show that this is always true, or true in general, is to reassign, relabel, reorder the individual terms. And I did this here to make this visible. You see everything behind, in this bracket behind the beta, is equal to the equation we had before with the minus sign in between, which means it's 0.

And everything we see on the top of the first component, it also be 0. That's for the boosted reference frame. So we see here, and it just shows this for the x component. You can show this for all components that momentum is conserved.