8.321 Quantum Theory-I Fall 2017

Prob Set 9

1. **Wigner's theorem**

   Prove Wigner's theorem: any symmetry transformation is represented by a unitary or an anti-unitary operator. More precisely any linear (or anti-linear) operator that preserves all measurement probabilities

   $$|\langle \alpha' | \beta' \rangle|^2 = |\langle \alpha | \beta \rangle|^2 \tag{1}$$

   is unitary. Here $|\alpha'\rangle = U|\alpha\rangle, |\beta'\rangle = U|\beta\rangle$. (Remider: an operator is called anti linear if it can be represented as $A' = KA$ where A is linear and K is complex conjugation).

2. Show that the inversion operator which takes $\vec{x} \to -\vec{x}$ does not commute with the translation operator. What are the consequences for the spectrum of a free particle?

3. **Practice with angular momentum algebra**

   (a) Starting from the angular momentum commutation relations

   $$[J_i, J_j] = i\hbar\epsilon_{ijk}J_k \tag{2}$$

   show that

   $$[\vec{J}^2, J_i] = 0 \tag{3}$$

   where $\vec{J}^2 = \sum_{i=1}^{3} J_i J_i$.

   (b) Next prove a more general result. For any vector operator V_i, show that $\vec{V}^2$ commutes with all the J_i.

(c) Show by explicit calculation that the three components of the orbital angular momentum $\vec{L} = \vec{x} \times \vec{p}$ satisfy the angular momentum commutation relations.

4. **Sakurai 3.5**

5. **Sakurai 3.14**