[SQUEAKING]

[RUSTLING]

[CLICKING]

PROFESSOR: So let's start. So let me just clarify one question, which was asked last time. So when we calculate the total cross-section for, say, for this e plus e minus to mu plus, mu minus, then we find that there's a funny fact -- yeah, let me just write down-- so then there's a factor like this. m prime square E squared divided by m square e square and then times something else. OK, so times something else.

And so there's funny feature that this factor seems to blow up when the E is equal to m. So if you decrease E to the value of m-- and then this seemingly blows up, OK. But of course this never happens in the real situation because this mass is much larger than this mass.

So the upstairs will go to 0 before this blows up. So that's why people never cared about it, OK, including myself. But you can ask the question-- suppose the electron is more massive than the muon and then you will reach the 0s of this first before you reach the 0 of this. And then you will see something blows up. OK, then it's curious.

There's a very simple mathematical reason for this. And the reason is that the cross section is defined-- remember, the cross section is defined to be defined to be the divided 1 over the flux. And the flux is proportional to 1 -- it is the density times the velocity.

And the density is the same. It's just one particle per volume. And then the velocity goes to 0. This precisely comes when the velocity goes to 0 because when the velocity becomes very, very slow, then your flux become very slow and small. Then this becomes big.

So mathematically, that's the reason why this becomes very big, OK, when the velocity goes to 0. But still it is a little bit funny, I should admit. So when the velocity equal to 0, of course, it's unphysical because when the velocity is 0, they just never scatter, OK.

So this divergence is never an issue. But you can ask the question why physically somehow when you decrease the v and somehow the cross section should become bigger and bigger? And the mathematically it's due to the flux just due to the way the cross section is defined.

But I also mentioned before that the cross section is supposed to measure the effective area of the interaction. And then why should that depend on the velocity, OK? And so this aspect that I don't have a very good explanation-- but this is the mathematical reason for it. OK, good. Yeah, so this is just clarify the question over the last time.

OK, so then we talk about crossing symmetry. So let's consider two process. One is the e plus e minus goes to mu plus mu minus as we just discussed with the Feynman diagram going like this. So this is the-- OK, so this is the e minus e plus. This is mu minus mu plus, OK.

So there's also another process. Let's consider another process, which is e minus-- with mu minus going to e minus plus mu minus. And the Feynman diagram for this is given by this. OK, so this is the e minus and mu minus are at the initial state now. And this is e minus. And this is mu minus.

OK, again, only one diagram contributes. OK. So if you compare a and b, you can see that b-- the diagram for b is essentially the diagram for a if you view it sideways. OK, if you view it sideways-- and then you see this goes to that. And then this becomes final state. And the final state coming out e plus essentially becomes e minus. And this just becomes that, OK.

So the difference between these two is that the e plus-- so e plus in initial state of a then goes to e minus in final state of b, OK, in final state of B. Similarly, the mu plus in the final state of a then become the mu minus in the initial state of b.

Yeah, so essentially, mu plus-- essentially, you take the e plus going to the other side become e minus and then take-- mu plus going to this side become mu minus, OK. So essentially you just exchange that. So now let's label the quantum numbers.

Let's call this p1, r1. So this is-- call it p2, r2. And call this one-- Call this k1, s1. So these are the polarization. And this is k2, s2. And now similarly I label here by p1, r1. Label this one by p2, r2 bar. So this is for the antiparticle. And so this one would be k1, s1. And then this will be k2, s2 bar, OK.

So now if we look at this map, so it's like in the process A, we take this to be initial state, p2, r2. If we replace it by minus k2, s2-- OK, minus k2 means that the-- yeah, so this one e plus now becomes the final state.

So this is the-- so from the momentum direction, this is the going in. And this will be going out. So we need to change the sign. OK, we to change the sign and, similarly, the k2 for s2 for k2, s2 bar and then go to minus p2, r2.

If you make this placement, then we get the process b, OK. You get the process b. So you just make the replacement. And then the process in a will go through the process in b. So suppose you forget about those polarizations. If we consider how we're talking about the scalar particles-- so there's no polarization. And then this just trivially-- so for scalar, then we just have trivially--

We can just do the replacement. You can trivially see that the amplitude for the process a with p1 minus k2 and the k1 minus p2 then will be just equal to the amplitude for the process b with p1, p2, k1, k2. OK, so we just rename your momentum of your process a. And then you will just get essentially the amplitude for two p because the only thing you need to do is just exchange the name of the momentum, OK. So then we'll be trivially the same.

But for fermions, we should worry about-- also look at the wave function associated with external legs. OK, so in a-- let me just write down those wave function explicitly. So in a, so the p2, r2 bar are associated with the wave function is to be v bar r2, p2. OK, so this is for the e plus.

And the k2, s2 bar are related to v s2, k2. OK, so this is related to the mu plus. OK, so according to our previous rule-- but in b the wave function for these two-- so the k2, s2, so the corresponding one is-- the one you want to exchange is u s2 bar, k2. This is corresponding to e minus and the p2, r2. And that's corresponding to u r2, p2, mu minus.

OK. So now you see even when you make the label change-- suppose you make the label replacement from here to replace by that. You're not going to change the wave function from v to u. OK, so the wave function actually is different. So similarly this does not-- so this does not work if you have fermions, OK, if I have fermions, because the function changes. It changes from v to u, in this case here again also v to u, OK.

But actually this is another problem if we consider the unpolarized spin sum actually still works, OK, for unpolarized. So if you have an unpolarized situation-- remember, we need to sum over all the spins. And consider M squared, sum of all the spins, OK. so we will involve this y sum with itself.

For example, when you sum of all the r2s, then we will have something like this. So, for example, we need to sum over r2, then we will have a combination like v, r2, p2-- that's the calculation we did before-- and v, r2, bar, p2, OK. And then this one will give you just minus i p2, slash, plus m of the electron, OK.

And now you can just do the replacement. Let's just do what was written -- the p2 replace it by minus k2. So if you do the replacement, then that goes to minus i, slash, k2 plus still the electron mass, OK. And then this is the same as the sum s2, u, s2, k2, u, s2, bar, k2, OK.

So despite that the wave function are different, OK-- so the wave function. But after you do the spin sum, OK, they actually get the same answer, OK. So this is the same as the minus this one. OK, so in this case after you do the spin sum-- so each replacement of momentum-- so in this case just give us a minus sign, OK, when you calculate this spin sum, m squared.

But now since we need to replace two of them, OK-- so 2 minus sine is a positive sign. So we conclude that the sum over spin Ma squared evaluated at p1 minus k2 and k1 minus p2 is the same as the sum over spin, the amplitude for b squared divided p1, p2, k1, k2, OK. So there's a simple relation between the amplitude square when you do the spin sum, and we just need to rename the momentum, OK. Rename the momentum.

Good? Any questions on this? So for unpolarized amplitude-- so we still have this very nice relation between these two process, OK. If you calculated one of them, then you don't need to calculate the other. You can just immediately get the answer. You just change a few momenta, change a couple momenta. But if you consider the polarized amplitude--

So the story is now is much simpler. Now you do have a problem because now the wavefunction are different, OK. But nevertheless, you can choose a different basis, OK. By choosing appropriate basis, you can still directly relate -- equate the amplitudes, OK.

You can still recreate the amplitude. OK, so, yeah, I will not go into there, OK. But it's possible, say, you choose a basis of v2 and the u2 and u and then somehow you can relate them to each other, OK. So this relation between the amplitude of a and b-- so this is called crossing symmetry.

Yeah, let me just call this star. So star is called crossing symmetry. So calling it symmetry is actually a misnomer because this is not a symmetry, OK. This is just a relation between the amplitudes of different process. OK, so this is not a symmetry. So the symmetry should be put as quotes, OK.

So this just express the relation of one amplitude to the relation of another amplitude. Yeah, just relation between the amplitude of one process to the amplitude of another process. And the relation come from a fundamental fact, come from a very simple fact--

OK, so if we consider this fermionic field, psi-- so psi can play two role. It can either annihilate-- remember psi is called a plus b dagger, OK. It's called a plus b dagger. So you can either annihilate an initial particle-- so that's the a part-- or can create an initial antiparticle or create, sorry, a final-- sorry, an antiparticle.

OK, so let me just elaborate. So if we have a, when you act on this side-- when you act on this side, you only have-- if you have a particle in the initial state, then a can annihilate that particle. But if a acts on the left, act on the left, a becomes a dagger. And then that creates an antiparticle.

Yeah, so in such a process, X plus a goes to Y. And then you can just-- so this relates to-- the X and Y are some combination, some combination of particles. And then this just relates to-- X goes to Y plus a bar, OK.

So here just X and Y, some collection of particles. Yeah, a's just some particle, OK OK, so just confirm that, OK. Good. Any questions on this? Yes. e minus all mu. Yeah, just say you have one field for e. You have one field for mu. Yes?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Sorry?

AUDIENCE: [INAUDIBLE]

PROFESSOR: Sorry, say it again.

AUDIENCE: The process, like for y, can we think of it as-- first, X to Y and a bar then a bar, like annihilate or do something like that?

PROFESSOR: Yeah, heuristically you may-- but it's simpler than that. Yeah, it's simpler than that. Yes?

AUDIENCE: Just regarding energy conservation. So if you can keep doing that can't we just say that like out of nothing comes Y plus a bar plus X bar.

PROFESSOR: Yeah, so that's why you have to change the momentum. So some momentum for this process. Corresponding to some momentum for that process. Yeah, we have to switch the initial and final momentum. Yeah, it's not to say these two processes are the same. It's just that this process for one set of momentum have the same amplitude as the other process for some other set of momentum. Yeah, there's a relation between them.

Other questions? OK, so now let's consider another important process called Compton scattering. So Compton scattering played a very important role in the early days of physics and in showing that actually this is one of the early experiments to show that the macroscopic world is governed by quantum mechanics rather than classical mechanics.

So now let's consider how to calculate the physics for this process. So Compton scattering is a process you have a photon hit the electron. And then you get another photon. And then you get another electron, OK. And so we can draw the Feynman diagram for it.

So one Feynman diagrams is the following. Let's just imagine you have a fermionic trajectory. So you can imagine at some point there's a photon coming in. OK, so this is the electron line. So the electron line, the arrow is the charge line. So the photon line, the arrow should be understood just as a momentum, OK. And then you emit another photon as a final state, OK.

So this is the simplest Feynman diagram for this process. But actually there are two diagrams because you can also-- for this electron line, you can also first emit a photon and then absorb a photon can also have that, OK. So you have two. And these two processes are not the same, OK.

So now let me put some label on the diagram. So, again, the fermion is p1, r1. So electron, initial state, and let's put its final state to be k1, s1. And the photon, let's put its initial state to be p2, alpha. So alpha now is a polarization for the photon.

And so call it alpha 1. And then this one is called k2, alpha 2. And then label this index to be nu and this index to be mu, OK, because the photon carry a vector index because the polarization carry a vector index. So there's mu here.

So I always imagine the momentum is coming in for the initial state and the momentum come out for the final states, OK. And for this diagram, it's the same thing. So I have p1. So I label the p1, r1. So this is the p2 alpha 1. And so this is k1, s1.

And this is k2, alpha 2. The only difference is the role of mu and nu is switched. So for the out leg is mu. So mu now is here. And this one is nu here, OK. So nu is associated polarization of the incoming photon. And the mu is the polarization associated with the outgoing photon.

So this is my Feynman diagram. So this process compared to the one we considered, this e plus, e minus 1. So these have some new elements. So that's why this is a good example to look at. There are two new elements.

First, in this example, we have-- so this propogator is a fermionic one. So now we have a intermediate fermionic propagator, OK. And the momentum for this one-- so let's call it q1 equal to k1 plus k2. So this is p1 plus p2.

But this one will have a different momentum. So here let's call it q2. So what's the momentum for this one? If I draw the momentum going to the up? Yeah, so let me put it here. So what would be the momentum this one? Let's go with q2.

Can you read the momentum of this intermediate line from the diagram? Yeah, it's p1 minus k2 because we have p1 coming in and then k2 come out. So that's the momentum, OK. So in this one, you have p1. And then you have the p2 coming in. And so this is p1 plus p2.

OK, so the new element is that now we have a fermionic propagator as an intermediate state. And the second new element is now we have photon in the external state. OK, so this is a new element compared to this e plus, e minus example.

But still using our previous rule, we can immediately write down the amplitude, OK. I think I may not have enough space. Yeah, let me try-- so let me start from here. So the amplitude, again, the i times the-- this i is not important.

But nevertheless, let me just write it down. So the amplitude is given by-- so we follow-- you see we follow the fermionic line. So here there's one fermionic line, OK. And so we should follow that fermionic line. And then we also have the photon polarization.

OK, so let's first write down the photon polarization. So for this photon final state, then we just have epsilon, mu, alpha 2, star, so k2, OK. So, remember, for the photon in the final state, we need to put the star. And also, remember, the mu is associated with the photon in the final state.

And then we have epsilon nu for the initial state of the photon, alpha 1, then k1. OK, so this is the photon one, OK, photon polarization factor. And now we need to write down the-- so these two factors are the same for both processes. OK, for both processes we are not changing the external photon state, OK. So these two factors are the same, OK.

And now the rest-- so now let's look at this diagram and follow this fermionic line. OK, so we start with here and then going backwards. So we just have u, bar, s1, k1 for this electron in the final state. And then we have fermionic-- and then we have this vertex should be minus ie, gamma mu.

All this order is important because they are all matrices, OK. They all have spinner indices. And so this is a matrix. And then we have this fermionic propagator, which is minus 1, i q1 slash. So I just write down the fermionic propagator minus m plus i epsilon, OK.

So I did the final state, intermediate propagator, and then I have another vertex. And now it's nu. And then I have the final. OK, so that's that diagram. OK, that's the diagram. And now for this diagram, we do the same thing. So the final state, again, is the same. It's the u, s1, bar, k1.

But now here you have ie gamma nu. So now the order changed, OK. And then you have the photon-- then you have this fermionic propagator, which is i q2 slash plus m minus epsilon. And then you have minus i e gamma mu from here. And then you have the final state u r1, p1, OK. Yes?

AUDIENCE: Where did you get these two Feynman diagrams, by doing [INAUDIBLE] which one gets absorbed or-- but if we were working with photons. It doesn't it matter. This seems like the real difference is not the kinematic difference of which one gets absorbed. It's the fact that we have this vector that describes with this polarization through the photons, right?

PROFESSOR: No, these are just two inequivalent Feynman diagrams, inequivalent.

AUDIENCE: But that's because of mu and nu, because they don't-- when you were writing down the Feynman diagram.

PROFESSOR: No, no, no, even not for-- even they are not vectors, still they are inequivalent diagrams. Because the momentum here is different. The momentum here is different. So they're still an inequivalent diagram. OK? So these are the amplitude. So again, we look at unpolarized cross section.

So unpolarized cross section. So let's look at the unpolarized situation. OK, so, again, we need to average over the initial spin and sum over the final spin. So here now the difference is now we need to average also over sum over the index for alpha and for alpha 1 and alpha 2.

So alpha 1 and alpha 2, they only have two polarization. So, again, when you average over the alpha 1, you get a factor of 1/2, OK. So still we get just a factor of 1 quarter sum over all the spins of M squared. So this is just the same as sum over, say, alpha 1, alpha 2, r1, r2, OK, M squared.

And each index take value 1 and 2, OK, because the photon also only have two polarizations. So now I will not do a complete calculation of this. OK, so you just square it. So you use the trick. And you just square it. And then you just try to calculate it.

And now you calculate this guy. Now we need to use some more tricks, OK. So I will just explain what are the new tricks needed to compute this guy. And we will not actually do a calculation, OK. I'll do a general calculation. So the new elements we need to use-- so a few new tricks in order to do this, to do the calculation.

So the first trick is how you treat this fermionic propagator. OK, so how you treat the fermionic propagator. So we can just rewrite-- yeah, so this is the inverse of a matrix. OK, so we can just do the-- we say maybe has a propagator like this, i k slash, plus m, plus epsilon.

This will not worry about epsilon because the downstairs neighbor 0-- so this is the same as i k slash plus m, k squared plus m squared, OK. It's just because of the familiar relation we used before that ik slash plus m and ik slash minus m is equal to minus k squared minus m squared.

OK, so you can just-- then the inverse of this matrix is just given by this matrix. Sorry, I think here should be minus sign. The inverse of this matrix is given by that matrix, OK. So then you can use this to simplify. Then you can use to simplify that expression a little bit, OK.

So this is the first thing used. And the second thing used-- is when doing the spin sum. So for fermions, for spinors we use the same trick as before as the one we used for the last example.

But for photons, there are some new elements, OK. So now let's discuss how we treat the photons. So now I treat photons. And the way to treat the photons actually involves some important physics. So let me explain a little bit how we do that.

OK. So now if you look at the structure of this amplitude-- OK, so let's focus on one of the photons. So let's look at the final photon, OK. So the polarization has the following form. Yeah, this just look at one of the photons. It doesn't matter. Say, let's look at this one, initial photon.

And then the amplitude-- then you have this. Then the whole amplitude is scalar. OK, so the amplitude then has the structure. They have epsilon nu star -- nu alpha -- times M nu. OK, just say this is the polarization. And the rest I just call it M nu because the index has to be contracted, OK.

So now I do the spin sum. So if I now some of nu to the spin sum over the alpha, and then I just have alpha equal to 1, 2 to the spin sum relevant for the alpha. I just have alpha 1, 2. And then I have epsilon nu alpha star, alpha nu alpha, alpha mu star, alpha nu, alpha, and then I have M mu star and have M nu. OK, I just take the square of it.

So now to do the spin sum, we looked at this object. OK, I need to look at this object. So do you remember what is the sum of this object? Yeah, exactly, the transverse projector. OK, because of the physical photon polarization is projected to the transverse space, OK. So this gives you the transverse projector.

But now I claim-- OK, so if you do the alpha 1, 2, mu, alpha star, nu, alpha-- so this gives you the transverse projector. But now the claim-- but transverse projector is a little bit awkward to work with, OK. But the claim, you said actually I can replace the sum of alpha equal to 1, 2 by actually sum over alpha for all polarizations, OK.

So now when I sum of all polarizations-- so if I sum of all alpha, then now do what is this? Do you remember what is this? You just get eta mu nu. OK, and this one is much simpler. So now the claim-- OK, so now the claim is that we can just simply replace this sum by eta mu nu. OK, we can replace this by eta mu nu.

Yeah, so this is the claim. OK, so this amounts to the following. So this is equivalent to the following. So if you look at the difference between these two-- so now if you look at the difference between the alpha equal to sum over 1, 2 and the alpha sum over all here-- so the difference in other words-- we claim that sum alpha equal to 0 and 3 epsilon alpha star mu and alpha nu alpha M mu M nu to be 0, OK.

So the difference between the 2 is just from sum 0 to 3, OK. So in other words-- so if I write down this explicitly-- OK, so this corresponding to-- so this corresponding to 0, epsilon mu, 0, M mu squared is equal to epsilon mu 3 M mu squared, OK.

So this claim, you see equivalent to this claim. OK, so these two will cancel each other. The equal here is because the signature, the zeroth component, there's a minus sign, OK. So it tells you that the sum of these two actually will cancel each other, OK.

So now let me remind you how we choose the-- yeah, now let me try to prove this fact. OK, turns out this fact is actually very important. It contains very important physics here, OK. So do you have any questions before I do that? OK, good.

So now let me try to show this is true, OK. So let me just remind you of a convention is that epsilon mu 0 is equal to just 0, just 1, 0 and mu 3 is equal to 0 and then in the direction of the k. OK, it's in the direction of the k. OK, and the epsilon mu 1, 2 will be orthogonal to both of them, OK. Be orthogonal to both of them, OK.

So now to see this equation, let me call this equation star, star. So let's consider just a general physical process. We can actually make a general statement, OK, not just restricted to that particular example we have here. So let's just consider some general process.

So you have a bunch of initial states and some final states, OK. But imagine one of the initial state is a photon. It's the one we are interested here, this polarization k alpha, polarization k alpha. And so now the amplitude just become mu and then this is the same as the-- you have some final state.

And then you have some initial state. But within the initial state, there's a k alpha state, OK. Then the k alpha state corresponding to the photon. But now, remember, which we discuss in your homework, OK, so that's the purpose to put it in your homework.

The final state for k alpha, which defined by transverse polarization, it's only a representative in the equivalence class of states. And within this equivalence class, they are related by these called null states, OK. So we can shift it by our null state and the physics will be the same.

So, now, remember the last state-- it's like a gauge transformation. So the last state corresponding to-- and this process corresponding to you change your initial-- you change your polarization to that corresponding to a null state. And null state, the feature is that it's polarization is proportional to the momentum.

OK, so that's how we are-- remember, we showed that-- because this is a gauge transformation. OK, this is like a gauge transformation. And we also discussed the reason this is a equivalence class because when you shift by a null state, the overlap between the null state to any state is 0, OK, to any state is 0. And so you can shift by a null state.

So the fact that you can shift this by a null state, your physics is the same. So now you see this equation 2 here. So this implies that M mu k mu must be zero. k mu must be 0, OK.

And this is a very important identity. This is called the Ward identity because this is a very important feature, which can simplify your calculation a lot, OK. So the M mu must satisfy the feature that when it contract with k mu you get 0.

So now remember the k mu, you just equal to-- so this is on-shell external state, or minus. OK, this is the on-shell external state. And then this is just given by-- if I take a factor k out and this is given by minus 1 and then k-- and this is a fact I think you also used in your pset-- and this is the-- and this thing is just the same as the difference between the zeroth polarization vector and the third one from what we wrote here.

I think I get the sign. Yeah, the sign does not matter very much here. I mean, just make sure. Yeah, for my sign this would be-- yeah, it's minus epsilon. Yeah, that's right. And then this equation just implies that epsilon mu 0 M mu is equal to epsilon mu 3 M mu.

So that tells you that epsilon mu 0 M mu squared is equal to epsilon mu 3 M mu squared. OK, so this is the one we wanted to prove. So this is the one we wanted to prove. So this simplifies life a lot. So now we show that we can actually make this replacement because the 0 and the 3 component add together become actually a null vector. And that actually does not contribute.

So now this amplitude we have two external photons. So we can write it as-- OK, so for simplicity-- so for our amplitude, so back to Compton case, to the Compton story. And then the amplitude then have the form-- so we have 2.

So I have epsilon mu, alpha 2, star, and epsilon mu alpha 1. And then this will multiply some T mu nu. OK, so I call the rest T mu nu. OK. So now when I do the spin sum, M squared, then I can just use this twice for each epsilon.

And then I just get-- essentially I just get-- let me just write one more step. I will get alpha 1, alpha 2 equal to 1 and 2. Then I have epsilon mu alpha 2 star, epsilon lambda alpha 2. So epsilon nu, alpha 1, epsilon rho, alpha one star.

And then I have T mu nu, T lambda rho star. OK, just the square of this guy. And now I can replace this by eta mu lambda, I can replace by eta mu rho. OK, and then this just become equal to eta mu, lambda, eta mu, rho, T mu, nu, T lambda, rho, OK.

So this just makes life much easier, OK. Makes life much easier. Yeah, so essentially these are the two important tricks which one need to do in the photon in this Compton case compared with this story. And once you have these-- and this part will be just the spin sum for the fermions.

And then that we can just use the same trick as we did last time, OK. So we will not repeat that, OK. So now let me just write down your final answer, OK. And now we can just write down the final answer. So before writing down the final answer, let me just define the frame.

So for the Compton scattering, it's often convenient to consider the reat frame of the electron. So we consider photon because there's no rest frame for photon, OK. So it's often convenient to consider the rest frame of the electron. So let's just consider the picture.

When you consider the rest frame of the electron, -- so this is often the way roughly often also experiment is done. OK, so you can essentially consider the, say, electron in the matter, which they don't have much velocity. And then you can have a photon come in.

So imagine you have an electron here and you have a photon come in. OK, you have a photon come in. So let's call this, the incoming axis, to be the z-axis. OK, and then after scattering the photon, say will be scattered into this direction. OK, so let's call this direction theta. OK.

So in this setup-- so then the momentum of the electron is just the m, 0, OK, just the mass of the momentum. Or I labeled the momentum. And the momentum of the photon, incoming photon, p2, would be just omega 0, 0, omega. OK, so only have the momentum in the z direction.

Then the final momentum of the electron is k mu, k1. So this can be something-- OK, so let's not worry about it. Then for the photon-- so photon momentum, final photon momentum-- so let's call it-- yeah, so this is k2. OK, so k2 we can parameterize it by omega prime.

And then omega prime then going into the spatial direction, the unit vector in the spatial direction for k2-- let's call it n. OK, so this is the direction of n, which is the theta angle of respect to the z direction, OK. And then the k1 just can be obtained by momentum conservation from these quantities, OK.

So k1 would be just p1 plus p2 minus k2. So from the fact that the electron have a static mass minus m squared, OK. And then this relation-- then the p1 plus p2 minus k2 should satisfy the constraint that this should be equal to minus m squared, OK.

It should satisfy the constraint -- from this equation, you can actually solve omega prime in terms of omega. OK, so we will not write down this equation explicitly. You can easily check yourself. You can easily check yourself. So you find the omega prime is equal to omega divided by 1 plus omega m and the 1 minus cosine theta.

OK, so this is the final frequency of the photon, expressed in terms of initial frequency of the photon and the mass of the electron and then this scattering angle, theta. So now let me just write down the final answer, with this set up.

And then you find in the rest frame of the electron, the differential cross section can be written. So again the phi direction would be symmetric, OK. And so this only worry about the theta direction. So this can be written as pi alpha square, so alpha against the fine structure constant divided by m squared and then given by omega prime, the final momentum frequency for the photon divided by omega squared and omega prime divided by omega and omega, omega prime minus sine squared.

OK, so this is the final answer. OK, so this is the final answer for the Compton scattering and the cross section. OK, so this formula actually contains a lot of physics, contains a lot of physics. So let me just describe some of the physics here.

So let's first consider the regime that the photons have very low energy. OK, just low energy photons, shining some light, shining on the-- so suppose the photon have very low frequency. They're much smaller than the electron mass. So now if you look at this formula-- so if the initial photon is much, much smaller than the mass and then this factor is approximately 0, then you find in this regime that omega prime is actually approximately equal to the omega.

OK, so you find in this regime the frequency actually does not change when the photon scatter away from the electron, OK. And then now omega prime equal to omega. So this factor becomes 1. This factor become 1 and 1 minus sine theta equal to cosine square theta.

And this becomes one. And there you find then d sigma, d cosine theta just equal to pi alpha squared divided by m squared then plus 1 cosine squared theta. OK, so this is a famous formula, more than 100 years ago or even more. Because this is the classical-- so this is called the Thompson cross-section.

OK, so these are Thomson cross-section derived from the classical electrodynamics. OK, and so this is a result already known in the 19th century from the classical electrodynamics. So how people derived this formula in the classical electrodynamics-- so in classical electrodynamics, you do it this way.

So in classical electrodynamics-- so light is believed to be a wave, electromagnetic wave, OK. So essentially, it's an electromagnetic wave. So this electron, or this electric field, so with a polarization, and then the magnitude. And then it's a wave, OK. It's a plane wave.

OK. So from the classical electrodynamics, the light is just a wave, OK. When you shine the light on the material, the light will scatter. Under the physical description of the light scatter is the following. So this is an oscillating electric field.

So if you have a charged particle like electron, then it will oscillate because it has a charge under such an E. So you can approximate the electron in the matter just by a forced harmonic oscillator. OK, it's the forced oscillation. It's a forced oscillation.

So you should have learned in 8.03 that under a forced oscillation, essentially the electron velocity electron will have acceleration essentially given by the given by this. Yeah, let me just-- proportional through the exponential i omega T plus i kx. OK, essentially just the force acceleration were like this.

And then from classical electrodynamics because this is accelerated motion. This is accelerated charge. And accelerated oscillators in the classical electrodynamics will emit waves. OK, we emit electromagnetic wave. So this will radiate. So this will lead to radiation with frequency omega.

So essentially controlled by this acceleration, OK. So this is the classical result in the classical electrodynamics that such a oscillator will emit the light with frequency omega. So that's the scattering process from the classical point of view. OK. From a classical point of view, drive the oscillator, the oscillator will emit, OK.

And if you go from this process, then you precisely find this formula, OK. You precisely find this formula, OK. So in this case-- so regarding this, the value of omega, the omega prime, the emitted photon always-- so in this story, omega prime always equal to omega, OK, because it's a driven oscillation, OK.

So classically, you always have this. So you always have elastic scattering. So the photon frequency does not change. And then the equation-- so let me call this equation 1 and the equation 1 holds. OK, yeah, and then equal this a and b. And equation 1 holds.

So the classical, very robust prediction is that you always have elastic scattering when you shine light on this Compton. And then the equation, you have this Thomson cross-section. But then now in the early part of the 20th century, when Compton did this experiment-- and then he said-- then he observed actually the frequency can be smaller, OK.

So in this formula, generically omega prime is smaller than omega. OK, but in quantum mechanics-- But in quantum mechanics, we see that omega prime is generically smaller than omega big because this is 1 plus a factor, which is greater than 1, OK.

And in particular, this deviation will become more obvious when omega become comparable to m when omega become comparable to m. Yeah, so this is the quantum mechanics. So you will always have inelastic scattering. So this makes sense because just for momentum conservation in actual-- if some photon hit the electron, electron needs to move.

The electron need to move. Then, of course, the energy will increase. And then the energy will increase, in the omega prime of course, have to be smaller than omega, OK. So in the rest frame, an electron always gain energy. So that's why in quantum mechanics always inelastic scattering, OK.

And then the star under this equation 1 no longer holds OK. So in the early days-- so in the early days, this just cannot be explained using classical electrodynamics. OK, so this is decisive evidence that actually photon-- the light behaves very different quantum mechanically from classical wave, OK.

In fact, this simple fact can be just explained by treating a photon as a particle. OK, so this is the decisive support that the photon actually behave like a particle, OK, behave like a particle. So leads to the particle picture of photon, OK. OK, so we are out of time soon. So let me just mention one simple application of this.

So in particular, let me just mention two small things. So one thing is that if omega is much, much greater than m, then omega prime divided by n becomes very small. OK, so this actually becomes much, much smaller than 1, except for theta equal to zero, OK.

So if the initial photon momentum energy is very big, then actually most of the energy will go to the electron, OK. And then the final frequency is actually become much, much smaller than initial frequency. This is one remark. And another remark-- so this is in the electron rest frame.

But in a different frame, you will actually get a very different answer. So now let's imagine we have a frame-- So in this frame the electron just sits here. The photon comes in the z direction. Now, imagine we boost the system in the negative z direction, the negative z direction.

So the result of this boost is that the photon energy becomes very small. And the electron energy will become bigger and bigger when you increase the boost, OK, because the boost is opposite to the direction of the photon so that you boost more. The photon energy will become smaller and smaller, OK.

So now imagine you go to a frame, which photons have a very small energy compared to electron. And then this is a very-- then in that frame will be a very fast electron heat of a very low energy photon. OK, so this is called the inverse Compton scattering.

OK, so in this case, then the energy in-- then the energy in the-- initial energy in the electron then can be transferred into the photon. Yeah, you have a very fast, very high energy electron hit a a low energy photon. Then you just give a big kick to the photon. And the photon can have very high energy.

So this is a very simple effect. But it can have very important astrophysical applications. So this is called the important application in this called the Sunyaev--Zeldovich effect. So essentially they have the simple observation like this.

So in the universe, we have the-- we have microwave background radiation. OK, the microwave background radiation, the photon is very low energy because the temperature is very low. But around a galaxy-- so if you're near the galaxy center, so near the galaxy cluster-- and the galaxy cluster, some of the electrons, they can have very high energy.

And when they scatter and the microwave background photon and then they can give those microwave photon a big kick, and then they will-- then the photon will get a very high energy. So then by looking at the photon spectrum in the sky, look for this kind of hotspot. And then this is a way to detect the galaxy cluster. OK, then you can just use this inverse Compton scattering that detects the location of the galaxy cluster. So it's a very cool-- it's a very cool application.