[SQUEAKING]

[RUSTLING]

[CLICKING]

HONG LIU: OK. Good. So last lecture, we talked about the property of this propagator, which is defined to be x, x prime which is also the same as phi x, phi x prime here. OK? So this object, we discussed. So conceptually, it can be considered-- so you have two interpretations.

So one is from here. So this roughly can be heuristically interpreted as the transition amplitude for a particle from x to x prime. OK, so heuristically, you can think of it, say, if you have a point-- say, if you have a space-time point, x prime has another points x. And then you ask, what's the transition amplitude from x prime to x? OK, so one way to interpret this object is that.

And as I emphasize, this is only heuristically, because this does not really describe a generally localized state. It's only a approximately localized state, OK? And similarly, the second interpretation is the correlation function.

It describes correlations of phi between, say, x. Yeah, just correlations, square correlations between the value of phi at x prime and the value of phi at x. OK, so this is the interpretation most convenient, for example, from condensed matter perspective if your phi, say, describes average spins.

OK. So similarly, the G minus, which is defined to be in the opposite order, also have two interpretations. So this is like a transition from x to x prime. Yeah, so if I draw x and if I still draw x that way, so it's like a transition amplitude like this, from x to x prime. OK?

So you can similarly interpret the other using this language, for example, this Feynman, which is theta G plus x, x prime plus x minus. And then this means that when t is greater than t prime, say, for now, let's take this to be the direction of time going up. And now if the x has a later time than x prime, and then describe the transition from x prime to x. OK?

But if the x prime has a larger time, and x has a smaller time, and then describe the opposite direction from x to x prime. But in both cases, you always go from the smaller time to bigger time. OK? And so we often call this, also GF, the time-ordered correlation function. So the GF is also called-- so it's called the Feynman-Green functions. It's also called the time-ordered correlation function.

And then we wrote down the expression. So we calculate the expression explicitly for G plus. And we also almost finished the expression for-- yeah, similarly, you can do retarded et cetera. And also, so any questions on this? Yes?

AUDIENCE: So on the set, we showed that for the complex scalar field, phi x phi x' is 0.

HONG LIU: Yeah, yeah.

AUDIENCE: So what does that mean? There's no commutator?

HONG LIU: Yeah, yeah, yeah, because of the charge conservation. And you cannot propagate from one particle to the other particle. So because phi, when it acts on the 0, generates, say, a particle. But when phi acts on the left, it actually generates the antiparticle. So a particle cannot become a antiparticle, right?

So only when phi and the phi dagger, and then it's from just the particle. Yeah. Other questions? OK, good. So we also discussed the-- yeah, so we wrote down this formula for R, A, F, say, in coordinate space. So x prime can be written as a Fourier transform.

I think it's k squared plus m squared. And let me just check the sign. I think it's i or minus i. Yeah, minus i. So right, OK? And then we mentioned that this factor actually has a pole. In the downstairs, these have the form omega squared-- minus omega squared plus omega k squared if you write it explicitly.

And then this quantity becomes singular at-- so singular at omega equal to plus-minus omega k. OK? And the singularity lies on the integration contour if we consider a complex omega plane. And so here, one of the integral is the omega. And then the omega go along the real axis. So the integration goes along the real axis.

And here is minus omega k, here is omega k. So you have singularity on the integration contour. And then different ways of going around the singularity then give you different choice of retarded and advanced and the Feynman.

And which one to choose based on-- which control you should choose, whether you should go around, go above, or go below the singularity. And depends on the prescription for each of them. So we discussed that for G R, which is proportional to theta, t minus t prime, then that means we should-- yeah, so let me just draw the singularity.

So that means you actually need to change your contour to go above the singularity, to go above the both singularity. OK, so that will guarantee you to proportional to theta minus theta prime when you do the integral-- when you do the integral, the omega integral using the contours and using the contours.

And similarly, for G A, which is proportional to theta t prime and t minus t prime-- in order to guarantee that, then you have to take your control to be below the singularity, be below the singularity. So this is omega, real omega axis. And for the Feynman-- so the Feynman is half and a half.

So there's a theta t minus t prime part-- there's a theta t prime minus t part. OK, if you try to check that-- so we're not going to detail it there. Here we'll give you a exercise for yourself.

So if you want to satisfy that kind of condition when t greater than t prime gives you G plus and when t smaller than t prime gives you G minus, when you do the contour integration, you have to do the following way. So for this one, you go below, for this one you go from up. OK, you go up.

And then there's another way, which is G F tilde, which is the opposite for G F in terms of time-ordering. And then you will do the opposite way over here. OK, so altogether, there are four different ways you can go around the singularity that give you four different kind of functions. OK, they give you four different functions.

So for those of you who are not familiar with complex analysis or the contour integrals, try to refresh yourself a little bit. Because the complex analysis is definitely a very, very powerful tool. To be very familiar with it is very important.

OK, so it's allowing-- each time, if you have to specifier your contour like this-- so if each time you have to specify a contour like this, it's not convenient. And the physicists developed a more convenient way to treat all of them in one shot. OK, so instead of going around the contour like this, we just try to move the singularity.

OK so we keep the contour to be fixed, but we slightly move the singularity. OK, for example, for the retarded, this is the same if I just slightly move the singularity by a tiny bit below the real axis. So if my integral contour is like this, then this is equivalent to that. If this is just a tiny part, then this one won't affect your answer.

OK, and similarly here, in this case, we just move both singularity above a little bit. And in this case, I move one singularity above, move one singularity down, in which case I don't have to change my contour, just slightly change my singularity. And this can be achieved by so-called epsilon prescription.

So a convenient trick is in the-- for example, in the retarded case, instead of this-- so we always do the integral with the same contour. So we don't change the contour, but we slightly change the integration to be omega plus i epsilon squared plus omega k squared. OK, so the downstairs previously was minus omega squared plus omega k squared.

But now, I add a tiny-- so epsilon is infinitesimal. It's positive and infinitesimal. So you see, by adding such a tiny piece to here, when you solve this singularity-- so it's now omega plus i epsilon equal to plus minus omega. And then move the epsilon to the other side, and then the solution becomes minus i epsilon.

And then because epsilon is positive, then this is slightly in the lower half plane. And similarly, for the advanced, you just do a minus, OK? And then both singularity will move up.

And for the Feynman-- so the trick is you can do the same thing. Yeah. Sorry, I still have i k. So this is the minus i. So I just do k squared plus m squared, the whole thing minus i epsilon, rather than change the omega, and the whole thing minus i epsilon.

And you can check yourself. When you solve the omega using this, this is the whole thing minus i epsilon, in this case. So for retarded and advanced, you put it inside the omega squared. But here, you put it other side, OK?

And then you can check when you solve for omega. Actually, you get this situation. One move up-- it will actually move up and the positive move down. OK? Yes.

AUDIENCE: Can you outline how I would convince myself that these two pictures are equivalent? Are they going-- moving, integrating with the contour that jumps out over or under these poles for--

HONG LIU: Yeah, yeah. Yeah, because from the contour point of view, from the control integration point of view, the precise shape of the contour is not important. It only matters what singularity it encloses. So as far as it encloses these two singularity, and then the story is the same.

But since I changed the singularity location only by epsilon-- and in the end, I will take this epsilon go to 0-- and then the value will be the same. Other questions? And so this is a very, very simple, but very powerful trick. This makes things much more convenient when you do many manipulations so that you don't have to constantly worry about the contour, the precise contour you take.

Any other questions? Good. Good. So let me just give some final remarks. So this almost concludes our discussion of the free field theory. So let me just give some final remarks.

So as I mentioned, the G plus and the G R and the retarded have many applications. Saying, in condensed matter. So you deal with them-- you deal with them all the time. And then this Feynman propagator, we will see it plays a very important role in next chapter, which we will start in the next few minutes. So the G F will play a very important role.

And so in addition to these two points-- so these are the correlation function of two points, OK? Just the x and x prime. You can, in principle, consider, say, more general correlation functions. Say, you can say, consider an arbitrary state.

Say, for example, you can just consider some state psi, and then you can consider phi x1, phi x2, say, phi xn. So this is some n-point function. Yeah, what's normally called n-point function, some general state, psi.

Psi does not have to be the vacuum. So general state-- remember, general state psi can all be built from a and a dagger. And the phi can also be written this way, in a dagger. So if you give me a state-- so I can, in principle, calculate this for any correlation function. OK, so this theory is fully solved.

And then no matter what you want me to compute using what we have developed so far, you can now compute them. So this theory, everything is computable. And so now, let me just mention one simple fact regarding the vacuum correlation function.

So let's consider the n-point function of vacuum. And then you can easily convince yourself-- OK, you can easily convince yourself this factorize into sums of product of two-point functions.

So this is simple because a and a dagger, they have to be paired. Because any a dagger creates something out of 0 has to be annihilated by a somewhere later. So that means all the a and a dagger has to be paired. And then that means that they have to reduce to all the two-point functions.

OK, so you can check explicitly. So for example, the simplest is the four-point function. So if you consider phi x, OK? So you can easily convince yourself that the a dagger-- when a dagger of this creates on this, and then that can be, say, annihilated by any of them.

And so you can have, say-- and similarly, a dagger created by this one have to be annihilated by some later. So you have to always pair them, OK? So you can show that this just reduces to all possible pairings. So you can pair 3 and 4, or pair 1 and 4 and 2 and 3, or 1 and 3 and 2 and 4. OK, you just write down all possible pairings. OK, just write down all possible pairings.

So in free theory-- so in this free theory, any point functions in the vacuum, they just reduce to the two-point function. If you know the two-point function, you know everything. OK, you know everything. So this, I will give you as an exercise to check yourself.

And then the final remark is related to the problem 3, I think, in your Pset. So in addition to phi, we can also consider more complicated operators. We can consider, say, so-called composite operators. Means that we take the product of phi, of which is momentum.

So we can have-- say, we can consider phi squared, phi cubed, say, the Hamiltonian density, the stress tensor. So these are all involving some product of phis. Or its canonical momentum. And so these are normally called the composite operators.

And as you should have learned from your P set problem 3 that those operators often are not well defined. Because when you multiply them at the same point, you will get a singularity. You will get singular behavior. You will get singular behavior. And the really to renormalize them, et cetera. OK?

And so such kind of divergent behavior is generic in quantum field theory. And one of the most important parts of quantum field theory is to find sensible ways to renormalize those divergent quantities in the physically sensible way. And the problem 3, it's the first example you encounter.

And later, when we deal with interacting theories, there are more complicated examples, et cetera. OK, so do you have any questions? Yes?

AUDIENCE: So I'm a little confused on how to interpret this n-point correlation function. Because for the two-point, we have this heuristic condition amplitude.

HONG LIU: Right.

AUDIENCE: But how do you interpret this, especially with theories where it's not a free scalar field where you can't decompose them like this--

HONG LIU: Right.

AUDIENCE: --by being expressed in terms of ladder operators?

HONG LIU: Yeah, yeah. So just heuristically, you can imagine-- say, you can imagine these correlations. So, say, you make measurements at different points. And you study some kind of joint amplitude of them. Yeah, so this is one way to interpret this quantity.

AUDIENCE: Like, the value of the field?

HONG LIU: Yeah, yeah. For example, if you imagine they are spin operators. And say, phi denotes some kind of spin, and then you can imagine you measure them at different space time points. And you look for their correlations. Yeah.

Yeah, so this is the one example. But later, we will see-- we will actually almost see immediately-- yeah, just today, I think we will reach there. And you will see this actually is related to the scattering amplitudes when we talk about interacting series. Yeah.

Other questions? OK, good. Good. So try to enjoy the problem 3 in your Pset. And after doing it, really read the problem again, OK? Read the problem again. That problem contains the things which-- yeah, just the simplest situation for the divergences you will see in quantum field theory. And to be prepared for that is important.

Yeah, even though we will not directly use the result of the problem 3 in the immediate future, but I think it's important as part of your education to get used to the infinities and the divergences in quantum field theory. OK, good. Other questions? Yes?

AUDIENCE: So I guess over here, everything on the left hand side is, in terms of these Green's functions, which could have been solved classically without doing any quantization. So is there a reason why the correlator and the quantized field theory is the same as the correlator in the the classical?

HONG LIU: Yeah, just because it's a very simple-- this is a very simple theory. Yeah. Yeah. Yeah, but even in this case, it's actually-- yeah, it depends on how you interpret the behavior of those functions. Even though this form, you might say, oh, maybe we could anticipate this. Say, if you have a, say, Gaussian statistical system, you will anticipate this. Just say you only have Gaussian fluctuations-- you only have Gaussian correlations and nothing else.

But the specific form of the functions, they do encode the quantum physics. And even though this factorized form is like in-- yeah, say, like in the Gaussian statistical theory. Yeah. Good? OK, so so far, we discussed the simplest free theory, just the theory of free scalar particles.

They don't have spin. They have spin 0, so they don't have any space-time component. And they have some mass, relativistic particles. And later, we will talk about electrons, which have spin 1/2, and talk about photons, which have spin 1.

And before talking about them, let's talk a little bit about interactions. Because so far, this theory is absolutely boring. So even though this theory illustrates this very important conceptual point that somehow when you quantize the fields you can get particles. OK, you can get arbitrary number of particles. So it's a connection between the field and the particles.

But just in terms of theory of particles, this series is absolutely boring. There's nothing-- the particles don't do anything. They just go straight, which is Newton's First Law. They just go straight like Newton's First Law.

And so now, let's try to add a little bit fun to introduce a little bit interactions. And you will see that when you introduce a little bit interactions, the story becomes much richer. So now, let's talk about interactions.

It turns out to describe interactions, a very powerful approach is the path integral. OK, so we will also discuss the path integral. So we will use the path integral approach to describe how to treat interactions.

And yeah, so first, let's just say a few words about some general remark on interacting theories. So previously, we consider theories with a Lagrangian density of the following form. And we find it's a free particle. It's a theory of free particles.

And now, we can easily make this theory to be of interacting particles if we just add some higher order powers. So for example, one of the simplest power is phi to the power of 4. So yeah, you can also add phi to the power of 3, but phi to the power of 3 is a little bit sick. It's because the phi cubed doesn't have a definite sign.

I don't have a definite sign, and so phi cubed, say, does not have a well defined energy. So your energy can go arbitrarily negative. And so this is the simplest interacting theory with a well defined energy. So we will write this as L0, which is the free theory part, which we can see there, and then the interacting part, OK? This lambda factorial, phi fourth.

And so this lambda, essentially heuristic, this lambda should characterize the importance of this term. So if lambda goes to 0, of course, then this term goes away, and then this term is not important. And when lambda becomes bigger, then this term becomes more and more important. And so we call the lambda to be the coupling, which characterizes the importance of this term.

So now, first, let's follow what we were doing before for the general quantization procedure. We tried to write down the classical equation of motion, and then we quantized-- and then we solved the most general classical equations, the most general solutions for the classical equations. And then we promote those solutions to operators, right? Yeah, so let's try to do this one.

So now, when you write down the equation of motion-- so the equation of motion is very simple. You just get partial square phi, m square phi. So this is the free theory part. And now, you have a nonlinear part, the cube, OK?

So now, you have a nonlinear equation. So who can solve this equation? Any volunteers? It turns out actually this equation, nobody has been able to solve it so far. OK? So we don't know how to solve it.

And so our previous strategy, even though sounds very powerful and immediately it breaks down, OK? Because rely on you can actually find the solutions to this equation of motion. And yeah, so we don't know how to solve this.

But heuristically, you can imagine that this, indeed, should be a theory of interacting particles. Because previously, when you don't have this term, you just have a linear equation, then the phi is not doing anything. OK, now, you have a phi cubed term.

So that means that the different phis can now doing something together, OK? So when different phi's doing something together, by definition, that's called interactions. OK, and that's called interactions.

And so just heuristically, this should already-- this should give you a theory of interacting particles. OK. So we will talk about how to treat such a theory. Since this cannot be solved exactly, we can only try to solve it approximately.

So the only way we know how to solve it approximately so far is to imagine the lambda is small. And then you just expand the lambda perturbatively. Just treat the lambda as a small parameter, you just expand the lambda. And so I will outline this procedure in a little bit.

But before that, let me just make some general remarks, which apply to general interacting theories, which this is just one of the examples. So actually figuring out interactions from experiments-- it's actually one of the main tasks of the particle physics and many areas of condensed matter physics. OK, and so for example, we build LHC at CERN and you collide these protons together, billions of dollars. The whole purpose is to figure out the interactions between the particles. Between the elementary particles and to verify them, et cetera.

And similarly, with many condensed matter experiments. So now, I'll ask you a simple question. So what's the most powerful way, experimentally, to probe the interactions between particles? What is the word to describe that kind of experiment? What?

AUDIENCE: Scattering.

HONG LIU: Yeah, scattering. So you should already have learned this in quantum mechanics. So scattering is the key thing, OK? So that's essentially the universal approach we have been using for more than 100 years, starting from Rutherford, who first shoot the helium atom at the-- who first shooting electrons at the helium atom. OK?

No, no, no, no, no. Shoot the alpha particles at the-- alpha particles at atoms. And so the scattering-- and then, so you have Rutherford, and then you have the so-called deep inelastic scattering, DIS, which you figured out that the proton actually has substructure, which leads to the discovery of quarks. And actually, MIT played a key role, Friedman and Kendall, in the DIS experiment, which figured out the structure-- which discovered that the quarks were-- yeah, they played the key role in that.

So and still, many, many interactions and the particles were discovered this way. And so the basic idea of scattering-- so essentially, it's very simple. You just collide a bunch of particles, OK? And just examine the outcome.

OK, so from the outcome, you try to deduce what are the interactions between those particles? OK, so that's what the experiment-- scattering experiment about. OK, so from here, you deduce the interaction, which has always been very successful.

So in condensed matter, you have neutron scattering, et cetera. X-ray, you can use X-ray and the neutrons to shine on your samples. And one of the most important observables for the scattering experiment is what? Again, this is a simple question of quantum mechanics.

AUDIENCE: Momentum?

HONG LIU: Hmm?

AUDIENCE: Momentum?

HONG LIU: Sorry?

AUDIENCE: Momentum?

HONG LIU: No, momentum is not that--

AUDIENCE: And cross section. Cross section.

HONG LIU: Cross section is close, but there's a more fancy word.

AUDIENCE: [INAUDIBLE]

HONG LIU: Hmm?

AUDIENCE: Differential cross section?

HONG LIU: That's fancier, but not fancy enough. It's called S-matrix.

AUDIENCE: No, I said S-matrix, but I didn't say it loud enough.

HONG LIU: Oh, OK. Yeah, just be brave. Be braver next time. So one of the key elements-- yeah, so one of the key observables for scattering is a so-called the S-matrix. OK, to define S-matrix, we need a little bit of mathematical idealization, just as always in physics.

So the mathematical idealization, in order to define S-matrix, is the following. So at t equal to infinity, t equal to minus infinity, prepare initial state-- yeah, initial state may be the initial state as localized, say, wave packet infinitely far apart. So this is the mathematical idealization part, OK? We want them to be infinitely far apart.

But you aim them, OK? So you prepare them spatially infinitely far apart, but you aim of them-- you prepare their initial momentum so that they can come together at some point. You aim them. And then they will come together at some point, and then they will scatter. They will interact with each other.

And then they will form some final product, and then the final product will then run away from each other, OK? They will have velocity momentum. They will run away.

And then you just wait. Wait until the products are far apart, then you match with them. So mathematically, this is called t equals infinity-- yeah, of course, t equal to plus-minus infinity is also a mathematical idealization. And the final particles are also far apart.

Again, infinitely far apart. Because they are all relativistic particles and they all move. And even with slightly momentum difference, eventually, they will be-- if you wait long enough time, then they will be-- so this final path are very important, these two, because this means we can neglect the interactions. OK, as t goes to plus-minus infinity.

OK, so with this set up, and then t equals plus-minus infinity, they are very far away. And then you can neglect the interactions. This is important. Then we can identify each particle, because when they are together, they all interacting with each other, it's not easy to cleanly identify each particles and discuss their properties.

And so now, we can neglect the interactions. So the initial and final states, and then a collection of free particles. And they can be described by free theory.

So this is very important because remember, we cannot solve this kind of interacting theory exactly. So now, but we can reliably almost to infinitely accuracy if you talk about the initial state and the final state. And we can talk about them as precisely as we can, as far as we go to t going to plus-minus infinity. Yes?

AUDIENCE: So how do you know adding a phi to the fourth term leads to interactions? Just because its the simplest term that [INAUDIBLE].

HONG LIU: Yeah, just in principle, any nonlinear term will lead to interactions. Yeah, the intuition is that anything beyond the linear level, you will have different phi's coming together. So whether it's phi cubed, or phi four, so higher powers or exponentials. Yeah, just anything which is not linear phi's have to do something with each other, and that just means interaction. Yeah. Yes?

AUDIENCE: Why are you doing the interaction from the minus one half m squared phi squared term?

HONG LIU: Sorry, say it again.

AUDIENCE: So we have--

HONG LIU: Oh yeah, yeah, yeah. Yeah, this does not give you interactions because this just-- remember, when you do equation of motion, that still gives you a linear term.

AUDIENCE: Oh, OK.

HONG LIU: Linear term just give you a mass.

AUDIENCE: [INAUDIBLE].

HONG LIU: Yeah yeah, yeah. Right, right. Yeah, yeah, yeah, yeah, yeah. Yeah, in the Lagrangian, the quadratic term, a free term. So since above the quadratic term gives you the interactions. Yeah. Yes. Yes?

AUDIENCE: If I know what the interaction potential is to the particles, like gluon or something like that, how can I translate that into--

HONG LIU: Yeah, yeah, yeah. Here you can not translate. Here, I'm just giving you the intuition that somehow when you add such kind of nonlinear term that should lead to some kind of interactions. But to understand what precisely kind of interactions, you have to look at physical observables. So that's why we need to look at the S-matrix.

Even for this theory, we want to say, suppose we have this theory, but in real life, how do we see in the real experiment? Yeah, if this theory can be realized in the real experiment, what it looks like? And the only way to see it is through this S-matrix. Yeah, yeah. And so this S-matrix, and then yeah, we can deduce back. Good? OK, good?

So this is important because this makes sure that even when we're unable to solve the theory exactly, we can still talk about initial and final state exactly. OK? At least we can define our problem precisely. So let's denote the collection of initial particle to be alpha, the momentum of initial particles.

So essentially, this characterizes your initial state. And because here we're talking about scalar particles. They don't have spin, so the momentum, they are only quantum numbers. And say I use beta to describe their final momentum. I call them p 1 prime, say p n. So the initial number of particles and final number of particles don't have to be the same.

And then the scattering process is just to go from alpha to beta. OK, you start a bunch of particles at t equal to minus infinity with those momentum, and then at t plus infinity, you observe some other particles with those momentum, OK? And then we want to understand the transition amplitude between them. So that's why the experiment matches.

And from here, of course, you get the cross section, et cetera. But this is a more fundamental object. So to write this precisely-- so essentially, we are looking at, say, in the Heisenberg picture, this is my final state. And I look at the evolution from minus infinity to plus infinity, start from initial state alpha.

So this is the evolution operator of your quantum system. And so sometimes, we also write this as the-- so this is in the Schrodinger picture. And in the Heisenberg picture, we write it as this.

So now, this means that this is the state defined at the minus infinity, this is the state that's defined as plus infinity. So this is the notation in the Heisenberg picture. And this is the notation in the Schrodinger picture.

OK, so this object we often denote. So you can worry-- you can run alpha and the beta over all possible initial states and all possible final states. And then that forms a big matrix, in fact infinite times infinite matrix. And so this is our S-matrix.

So essentially, all your secret of interactions are encoded in here. Yes?

AUDIENCE: Why is it an infinite dimensional matrix?

HONG LIU: Yeah, because you can-- yeah, so you have a value for any choice of alpha and the beta, right? And so any choice of alpha and the beta is one matrix element. But you have infinite possible choice of initial momentum.

Yeah, you can have an arbitrary number of particles, you can have a arbitrary momentum. And similarly, in principle, for the final state. Yeah.

AUDIENCE: And all this is one kind of particle? Or how do you know?

HONG LIU: Yeah. Yeah, here, for this particular problem, it's one kind of particle. But this definition is very general. This is a very good question. It doesn't matter whether you have one kind of particle, two kinds of particle, even an infinite number of particles. Yeah. An infinite species of particles.

So here, even for one kind the particle, I have infinite possible choices because I have infinite possible choice of all those momentums. I can choose k equal to 1, 2, 3, to infinity. And also, the value of those k can all change. P can change.

So yeah. So this is the infinite times infinite matrix. OK, so essentially, this S-- this is essentially just a matrix element of this evolution operator. OK, essentially, this is the matrix element of the evolution operator. You can see it from here explicitly from minus infinity to plus infinity. OK, good?

So it's convenient-- so let me just say a few things about the S-matrix. So it's convenient-- so when the interaction is weak-- so when the interaction is weak, you can imagine, most of the time, the particles, they don't interact with each other, OK? So when you do the scattering, you put a bunch of particles scattering-- if the interaction is very weak, then they don't actually interact very much.

So we expect-- so when the S equals to identity, then that means there's no interaction at all. OK, for free theory, it will be just the identity. So now, if you're seeing that the interaction is weak, and then it's convenient, we write the S-matrix in the following form, i T, and this includes the interaction part. OK, captures interacting effect. OK? Good?

And by definition-- yeah, any questions on this? Yes?

AUDIENCE: So I was interpreting the elements of the S-matrix as sort of the probability to end up in a certain state from the initial state.

HONG LIU: Yeah. Yeah, S-matrix is the amplitude, the probability you have to square it.

AUDIENCE: Oh, yes.

HONG LIU: Yeah.

AUDIENCE: But then why does it make sense that if it's a weak interaction, that it would-- like, as the limit goes to no interaction, the S-matrix would be normal at the identity?

HONG LIU: Sorry, say it again.

AUDIENCE: So why do we see that for weak interactions, it's the identity?

HONG LIU: Yeah, if there's no interaction, it will be identity.

AUDIENCE: But why does that make sense? Because that's just saying the amplitude is 1 to end up in a certain state?

HONG LIU: Yeah, just you always go back to-- the state does not change. What it means is that-- it means delta alpha beta. Because alpha must equal to beta.

AUDIENCE: Oh, I see. I see.

HONG LIU: Yeah, what it means is just delta alpha beta, right? So it means the alpha equals to beta.

AUDIENCE: Thanks.

HONG LIU: Yeah, yeah. Yes?

AUDIENCE: Is S a Hermitian Matrix?

HONG LIU: Yeah, we will talk about that. It's not Hermitian it's unitary. Yeah. Yes?

AUDIENCE: Does the phase of the entries in S carry any physical consequences?

HONG LIU: Yeah, that's a good question. So if you can see the particular amplitude when you square it, of course, the phase don't. But in principle, you can design the different-- you can design experiments in the different things they can interfere. Yeah. Yeah, so in principle, they contain physical information. Yeah. Yes?

AUDIENCE: Why is this entry complex?

HONG LIU: Or just convenience.

AUDIENCE: OK.

HONG LIU: Yeah, just for convenience. Yeah.

AUDIENCE: Yeah, if alpha was a particle at minus x infinity, beta was the same particle at plus x infinity?

HONG LIU: Sorry, say it again.

AUDIENCE: If my alpha to beta was just particle here goes to particle there, is that captured by S equals 1?

HONG LIU: Yeah. Yeah, yeah.

AUDIENCE: OK.

HONG LIU: Just say in the free theory, if you start with a bunch of particles at p 1 and p k then you end with the same p 1 p k Yeah. Yeah. Yeah. OK? So due to translation symmetry-- so we expect energy momentum conservation. OK, so yeah, so this, if you write it in terms of the component rotation, so this is delta beta alpha plus i T beta alpha. OK, and i just purely convention. i is purely convention, just for convenience.

So due to the momentum conservation-- so this corresponding to the interaction part. So in this part, we expect alpha not equal to beta because you have interactions. But the total momentum must be conserved.

So the sum of here must be the sum of here. Just the total momentum must be conserved. So that means this scene must contain a delta function between the total initial momentum and the total final momentum.

And so we can just convenient to extract-- to separate that delta function. OK, so convenient to write i T beta alpha equal to i, you separate out this delta function. Because we know the delta function must be there, so p alpha minus p beta. So p alpha and p beta means total momentum for alpha and the beta. And then, you have M alpha beta.

So this is the nontrivial part. So this part is trivial. You expect from kinematics. And so this M alpha beta is called the scattering amplitude. So now some properties of the S-matrix because you see that S-matrix is essentially just a matrix element-- essentially just the matrix of the U. And the U is a unitary matrix, because it's a evolution operator.

OK, so that means that the S-matrix is unitary since U-- this is a unitary operator, so that means that S is a unitary matrix. Also, you can show-- somebody be running out of time because today, we have to end at 5:40. And so for any symmetry of the Hamiltonian-- for any symmetry of the Hamiltonian, that means that if you have some Q-- yeah, if you have some, say, unitary operator, which commutes with the Hamiltonian.

OK, so remember all the symmetries, the interaction are generated by unitary operators. OK, so suppose we have some unitary, which commutes with the Hamiltonian. Then, you can easily show yourself-- so I'll leave this exercise to yourself.

So lambda is a unitary operator generating some symmetries. OK, so for example, for the Lorentz transformation would be like this e to the i omega M, which you have done in your homework. And then it will satisfy-- then when you act initial state and the final state by this lambda, and then the matrix should be the same.

So you can easily prove this by using the commute. OK? Yes?

AUDIENCE: Is another equivalent way of saying that like S and lambda also commute?

HONG LIU: No. No, no, no. No, they don't. Yeah. Yeah, that's the statement, right? This is a statement. So as S-matrix element. Yeah. Yeah. Yes, it's a statement that the lambda commute with U.

AUDIENCE: OK, OK.

HONG LIU: Yeah, lambda commute with U and S is the matrix element of U.

AUDIENCE: OK, OK. Thank you.

HONG LIU: Yeah, yeah, yeah. OK, so this is the key object we want to calculate in the interacting theory. So now, without proof-- because the proof requires something going beyond what we discussed so far. So let me just code this LSZ theorem, which said that the M alpha beta, this scattering amplitude, can be obtained. So we will discuss how you obtain in nature from such correlation functions, time-ordered correlation functions.

So this omega is the vacuum for the full interaction theory. OK, so this is the vacuum. So I write omega to distinguish it from 0, which is the free theory we discussed earlier. And this time-ordering means that you order the operator according to their time. So whichever time is bigger sits earlier, OK?

So this is the object, key object, if you want to calculate the M alpha beta. And you have to calculate this object first. OK, and so I will stop here. And so next time, we will say a few more words about how the general approach to treat this kind of theory, and then we will start talking about the path integral.

So have people studied the path integral before? So you are a expert of path integral or not quite?

AUDIENCE: Expert is a strong word.

AUDIENCE: I know we've heard of it.

HONG LIU: OK. Yeah, so I will review the path integral of-- yeah, I will actually-- so I will actually introduce the path integral from scratch, but I will do it a little bit faster than ordinary quantum mechanical class. And even for people who have not seen it before, I think you should be able to follow with a little bit of effort after the class. OK.

AUDIENCE: Good point.