[SQUEAKING]

[RUSTLING]

[CLICKING]

PROFESSOR: Yeah, so I'm Hong, so you can just call me Hong. I'm a theoretical physicist. I work on high energy theory, including string theory, quantum gravity, non equilibrium statistical physics, et cetera, and many different topics. So now let me say a few words regarding the QFT, the quantum field theory itself.

So the goal of this quantum field theory class is to develop concepts, formalism, and techniques for quantum dynamics of fields. Say, I'm sure all of you have studied Maxwell's equations, OK? So Maxwell's equations describe fully classical dynamics of electric and magnetic fields. But we live in the quantum world, and so we should also treat electric and magnetic fields using quantum language, OK?

And then you find that once you do that, and then the concept of photon, once you treat the electric field and the magnetic field quantum mechanically, and then you get the concept of the photon. And the photon now is the fundamental particle, which mediates electromagnetic interactions, OK? So you actually get a completely different physical picture from what you get classically. And yeah, so one goal of the class is for you to appreciate, say, to understand the quantum electrodynamics, OK? And that's actually where we will end this class, is we will discuss the quantum electrodynamics.

And the quantum field theory is also very important for other interactions in nature. So among the four fundamental interactions in nature, three of them are described by quantum field theory completely, OK? And you can also use quantum field theory to describe gravity, to describe quantum gravity. But at the moment, quantum field theory does not offer a complete description of quantum gravity. But still, you can use the quantum field theory techniques for understanding certain questions of quantum gravity.

And also, quantum field theory over the years, even though it's initially developed for particle physics, but over the years has been also found many applications in many, many other branches of physics, in condensed matter, statistical physics, et cetera. And it's fair to say, nowadays, quantum field theory has become a universal language, OK, for theoretical physics, essentially many in all different fields.

And so if you are serious, certainly you need to master this language. And even if you are only interested, say, in atomic physics or statistical physics, the field theory concept will be very important. And for experimentalists to know basic concept of quantum field theory and to have some basic understanding of it should also go a long way to help you to appreciate, say, the most recent theoretical developments and also for help you to communicate with theorists, and yeah.

And so this class is the first of a three-semester sequence. And so this semester will mostly develop the fundamental concepts, and the quantum field theory 2 and 3 will be more about technical development. And say, if your experimentalist, and then you can view this quantum field theory 1 as a stand alone class, which is just enough for you to get the basic idea of quantum theory of fields, which you don't-- yeah, you may not-- it depends on your need and whether you have 2 or 3 or not. Yeah, may depend on your needs.

OK, so under the main topics, I plan to cover are listed in the outline, this document of the outline, which is already on the website. And so I should emphasize that outline is rough, is only a rough roadmap and may change. It depends on the pace. I may change things along the way. And sometimes I change my mind, say, half through the course, somehow I feel-- yeah, anyway. So don't treat it too literally. Any questions about this subject of quantum field theory so far? OK, good.

And also, let me say a few words that the quantum field theory has a reputation of being a very difficult subject, OK? Actually, indeed, myself have suffered a lot when I learned it myself, OK? But with 20/20 hindsight and also from interacting with large number of students through teaching various level of quantum field theory classes in many years, I can assure you now that actually quantum field theory is actually not difficult at all, OK--

[LAUGHTER]

--if you learned it the right way. And of course, the learning thing-- yeah, of course, anything is not difficult if you learned it in the right way. And so in a sense this is empty words, but keep it in mind. Whenever you think it's too hard and there might be-- the reason might not be-- the reason might be you have to change your perspective, OK? You have to change your perspective.

And so quantum field theory, one thing people complain about is that quantum field theory often involve a lot of calculations, and that's true. That's just a fact of life. You cannot avoid it. But that's not what it makes it difficult, OK? Complicated calculations, you can just go through them from one line to the next line to the next line. If you're careful enough, patient enough, you can go through. So the difficulty, I think for most people, of quantum field theory, it's more at a conceptual level. It's because this subject is not a very intuitive subject. It's not something you can just understand just by thinking, OK?

So that's why I emphasized earlier the exercise and really working through it is very important. It's a little bit like quantum mechanics. In quantum mechanics your intuition was developed through examples. By working through many examples, you slowly develop intuition about the quantum mechanics. And if you learn all the lessons, and then you get good feeling about quantum mechanics. And quantum field theory is the same thing. It said, you have to-- some kind of intuition has to be developed, OK? It's not something you can easily-- just like mechanics, which you can in some other subject, maybe if you have very good intuition, you can just imagine it, OK?

Yeah, so to help you develop good intuition about quantum field theory, I can offer you three pieces of advice, OK? So first, yeah, so the first piece of advice is that the quantum field theory is essentially quantum mechanics but dealing with infinite number of degrees of freedom, OK? So in your quantum mechanics class, you always treat with finite number of degrees of freedom. But quantum field theory, the difference for quantum field theory is now you treat an infinite number of degrees of freedom. It turns out that this treating infinite number degrees freedom makes a difference. So more is different and actually, sometimes make conceptual differences. And so that's why sometimes the quantum field theory is unintuitive, OK?

But that said, I found, for many people, including myself when I learned it, for many difficulties you encounter in learning quantum field theory, it's not due to the difficulty in quantum field theory itself. It's actually due to your gap in understanding of quantum mechanics. So whenever you encounter something you don't quite understand in quantum field theory, try to step it back, to say, can I formulate this difficulty in terms of quantum mechanics with only a finite number of degrees of freedom? And often, you find actually your difficulty can already be formulated in quantum mechanics. And then that way then you should be able to just settle it yourself, OK, because we are supposed already to be a master of quantum mechanics, OK?

And certainly, when I learned quantum field theory I was stuck at a certain point for a long time. And then later I realized, just because I didn't understand certain Heisenberg picture of quantum mechanics very well, OK? Somehow I realized when I understood the Heisenberg picture of quantum mechanics well, and those difficulties just went away. And have nothing to do with quantum field theory itself. So that's why, in your first Pset you will get familiar with Heisenberg picture of quantum mechanics, OK? That came from my own experience.

And also, the second point is that quantum field theory deals with formalisms. And sometimes the subject seems very formal, OK? You have a lot of formalisms, OK? But just keep in mind, any formalism in physics, no matter how abstract it is, it was always designed to solve some concrete physical problems and physical questions, very concrete physical questions.

And if you understand what kind of concrete physical questions quantum field theory was designed to solve, then that can give you a very good perspective on those formalism and why people do this why people do that, why people do this trick, why people do that trick because they were invented to solve certain concrete problems, OK? And once you understand the questions, understand the problems, then the formalism becomes much easier to understand.

And the third thing we already said is that in quantum field theory, as in quantum mechanics, intuition was built through experiences, OK, through examples. So when you do your Pset, when you look at the examples in the class, you should always ask yourself afterwards, say, after you have done your Pset problems, always look back at that problem. Say, what did I learn from this problem, OK? And just think through it again. Think through what you learned from that problem again. And that is a very good way to help you to learn from your experiences and to help you develop intuitions, OK?

And so yeah, so also a very important thing you should keep in mind is that in the graduate course like this most of the things should be learned outside the class. So inside the class the purpose is to give you a guide, OK, is to emphasize the conceptual picture and the physical intuition, et cetera. And so sometimes I will leave some details for you to finish in the Pset. And sometimes the Pset will involve problems which I did not, say, fully discuss in lecture, but I want you to work out yourself, OK? And so P set is important part of the learning, even new things, OK, not just to practice something, but it's also a very important part for learning new things. Right. Good?

Also, finally, I would like to make an apology. So you will soon find that notations rotations are used in the lecture are different from the notations in the recommended reading books, OK? So I recommended reading Peskin and Weinberg. And you will find that my notations are actually different from them. Also, the order of presentations are also different from them, OK? I know this is very annoying, but there's just no perfect textbooks.

And there's no perfect set of notations everybody use. And we all use the notations which we find the most convenient to use, OK? And so even though I realize this problem, but I don't have a good resolution, OK? So just keep in mind, the notations in my lecture can be different from the notations in those textbooks, OK?

Good? So do you have any other questions? Good, OK, so if you don't have any other questions, so let's start.

So the Chapter 1 will be about why we consider quantum field theory, OK? So first, we talk a little bit about the classical field theories to set the stage or quantum field theories, OK? And the first important concept is called the principle of locality. So if you remember from your high school days Newtonian mechanics, so in Newtonian mechanics you have action at a distance, OK? For example, if you look at gravity and the gravity is exerted by the sun on the Earth, but they are very far away, OK, and yeah. And the same thing with the Coulomb interactions between the charged particles.

But then in the 19th century, they came from this principle locality. And that's formulated by Faraday around 1830, OK? So the principle of locality said all points-- you actually don't have action at a distance. He said all points in space participate in the physical process, OK? And the effect, so if you have interactions, OK, so effect propagates from points to a neighboring point, OK?

OK, so in this principle locality you don't have action at a distance. So action at a distance is always conveyed-- the action is always conveyed from one point to another point through the propagating in the space, OK? And the fields, the concept of fields is the mathematical device or vehicle that the principle of locality is at work, OK? So this is essentially the device we need to use to realize this principle of locality.

And so the main idea of the field is that we associate each point with each point in space a dynamical variable, OK, or dynamical variables, OK? So for example, so for example, so if you have electric field-- so the electric field is defined for all space, OK? So at each point x, we can introduce an electric field, OK? And then this electric field can also depend on time, and yeah.

So of course, normally we write it this way. E of x, t. And the reason I write it this way is to emphasize that in the definition of the electric field, the space and time actually play a very different role. The space plays the role of a label. So at each point we have electric field, OK? At each point x, we have electric field. And so x here is just a label, OK? And the t is used to describe the evolution, the change of the electric field. So the x and t plays a very different role. And similarly, you can do it for magnetic fields. It's the same thing, OK? And here you should always view x as labels, OK? So the spatial point, which we denote as vector x, is the labels.

And we know that the evolution of electric and magnetic fields are described by Maxwell's equations. So let me just write them down to remind you. So if you have dot v 0 dot E equal to 4 pi rho. So this so-called the differential form of the Maxwell equations. So the reason I'm taking trouble to write them down is to emphasize the following point.

So this set of equations exemplifies perfectly the principle of locality. It's because you see those equations only involve the value of electric fields and the magnetic field at a single point, OK? So if here is at x point, so here is also at x point. You never say x here and here is some other point y. And so this is reflected the principle of locality that everything-- so the effect is propagate from point to point, OK? You just have the derivative of the same point, OK? You don't involve separated points, OK?

So these are local equations. So these are what we call the local equations. And they contain only E and B at the same point and also the charge density and current density at the same point, with finite number of derivatives. So the derivatives are the ones to help you to propagate, OK, because it relates the point to the neighboring point, OK? And so that's how it propagates. It's through the derivatives, OK? So the derivatives are key, OK? The derivatives are key.

And another example with examples of locality, which I will not go into here, which some of you may know, is the Einstein gravity, so Einstein's general relativity. So in Einstein gravity the dynamical variables are so-called spacetime metric. So they are object with two indices, two spacetime indices, OK? And then the Einstein equations are equations for this kind of object. And again, the equations are local equations in the sense that they only depend on the G evaluated at the same point with a finite number of derivatives, OK?

So in Einstein's gravity you no longer have action at a distance, OK? So the effect of the gravity is propagated through the space time, through space time. So in fact, so here we say these two examples exemplifies the principle of locality. In fact, the principle of locality plays a very important role in formulating those equations because, as we will very soon see, when you have principle of locality, you can significantly constrain the theory you can write down. So that's a very, very powerful principle. Yes.

AUDIENCE: Are there popular examples you think we might have seen of nonlocal equations?

PROFESSOR: Yeah, you can have nonlocal equations, but it's believed that the fundamental equations in nature, they are all local, yeah. Yeah, and so far, the equations govern all fundamental interactions. All four different interactions in nature, the equations are local. Other questions? Yes.

AUDIENCE: When you wrote down the Einstein's gravity thing, why did you pick out a particular time component t?

PROFESSOR: Sorry?

AUDIENCE: When you wrote down the G mu nu as a function of x and t, why did you pick at a particular time component?

PROFESSOR: Say it again. I don't quite understand the question, yeah.

AUDIENCE: Sure. You kind of treated the x and t asymmetrically when you write down the equation for Einstein's gravity.

PROFESSOR: Yeah.

AUDIENCE: Is there a particular reason why you wrote it in that way?

PROFESSOR: Oh, no, no, no, here, I just want to emphasize again that, of course, in Einstein's theory these two are treated the same way. But if you think in terms of the fields, they actually have very different physical interpretation. So that's why I write this way. Yeah, it's the same way I write it like here, yeah, yeah, yeah, same reason, just here to emphasize for this-- to emphasize the different role played by x and t. Yeah, but I think you're asking a very good question. So I'm going to mention later in the relativistic series then, of course, then these two become the same, play the very equal role. Does that answer your question?

AUDIENCE: Yeah.

PROFESSOR: OK, good. Good, also yeah, so let me also very quickly mention the different types of fields. OK, so you can have what's called a scalar field, scalar. So these are quantities, which at a given point there's only one value, OK, say, for example, the temperature. So it's just a single quantity defined at the point, for example, the temperature, OK, and et cetera, and maybe other quantities. And then you also have a vector field. And E and B, they're examples of a vector field because at each point you have a vector, OK?

So here I'm using the three-dimensional notations. But in the relativistic series, You. Can also use the four vector notations. Say, for example, the vector potential will have the following form. Say, A mu will be a four-vector, and then the space time I combine them together into a four vector, OK? And so at each point, and then you have a four vector, OK? And so yeah.

And also you have tensor field. So this metric is an example of a tensor field. Here you have two components you have two indices, OK? So you have many, many different components depending on the-- so again, this is a relativistic notation. I can write it as a relativistic notation, OK? So at each point, and now you have some object with two indices, OK? And you can also have-- later we will see you also have something called spinor fields, alpha. And this alpha is some other indices, which we we'll define later, OK? OK, so you can also have so-called spinor and alpha, some other indices, OK?

And as our convention, is that mu is always from 0 to 3, OK? Then when I sometimes I just write x, when I write x just means a four vector, means x mu. You could do Ct and x. So x vector always denotes a spatial vector, OK? And yeah, and then this is the same as Ct xi, OK? So the spatial index is always denoted by i, and yeah. And so partial mu will be the same as 1 over C partial t, and then the derivative, the gradient, on spatial directions.

And say, if you have a four-vector, then again, we have the convention that a 0. Then the A vector, it's the same also as A0 and Ai. I use the i to denote the spatial components. And this is the last time I will write speed of light. So the c I will always take to be 1, and h bar will taken to be 1, OK, just for notational convenience. You have questions?

OK, so now, so with this preparation then we can talk about action principle for classical fields, OK? So first, recall in your classical mechanics, so we introduce the action, which is the integral of your Lagrangian. And Lagrangian is a function of x, this is your variable, and x dot and the time, OK? And so this is a one-dimensional-- yeah, this is the 8.01 one-dimensional particle motion. And you can also introduce momentum, canonical momentum, which is defined by partial L, partial x dot. And then you can also define a Hamiltonian. And Hamiltonian is related to the Lagrangian by p x-dot minus L, from a Legendre transform.

And the equation of motion is obtained by extremize S, OK? So S is considered to be a functional of your trajectory. So whatever trajectory you have, and you extremize this S, and then you get the equation of motion, OK? So now we can generalize to field theory, OK? So now, from principle of locality, so for field theory from principle locality, so the form of the Lagrangian, so the form of the Lagrangian L-- so again, we here, for field theory, we can again define S as a time integral of a Lagrangian. And the form of L is significantly constrained, OK?

To have the following form, so you must have the form L, you could choose a spatial integral. So this is an integration over all spatial directions, OK, so this d^3 x and some thing. So let me first write down the notation, and then I will explain the notation. OK, yeah, maybe just write partial i, OK? So let me just explain notation a little bit.

Well, first, so here I use a shorthand notation to denote the fields. So phi a so will be a function of spatial direction and time is general field, and a label different fields, OK? So this index, a, labels different fields, OK? So for example, a can label different scalar fields. If you have multiple scalar fields, they can label them. And a can also refers to indices, space time indices, like A mu, and also, yeah, et cetera, OK? So a just label whatever fields you have, OK?

And then the second point, you said this L, this script L is a function-- I emphasize here-- is a function of phi a and its derivatives, OK, and its derivatives. So in other words, the L-- so this is a key point, OK? So that this L only depends on the value of phi a and its derivatives at a single point, OK, say x. And then you integrate over all x, OK? So L is called the Lagrangian density.

AUDIENCE: So these are all still local?

PROFESSOR: Sorry?

AUDIENCE: These are all still local?

PROFESSOR: Sorry? Say it again.

AUDIENCE: Still local?

PROFESSOR: Yeah, yeah, yeah, yeah. Yes.

AUDIENCE: Can you even higher order derivatives?

PROFESSOR: Yeah, yeah, we'll mention that. Yeah, we'll mention that. So here I just explained the notation, OK? So now let me just make some remarks on why the Lagrangian must have this form. So first, the principle of locality implies here must only involve a single integral, OK, because the locality does not allow something like this, say, for example, does not allow a term like this, OK, does not allow a term like this, which involving the phi at a different point, OK?

Why? It's because if you have terms like this in your Lagrangian, OK, so later you will-- oh, sorry, L, not L script. So if you Lagrangian have this kind of terms, and then, so as we will describe the equation of motion later, you will see from the equation of motion, then your equation of motion will not be local, OK? So equation of notion will involve the behavior of your field at one point and then influenced by point at some point far away, OK?

And then you will not have local. So the locality significantly constraint because if you give away locality, and then in principle your Lagrangian can be arbitrarily complicated. You can have many integrals as you want, OK? But because of locality, you're only allowed to have such simple integral, one integral of a function, OK? So this is the key. OK, so does not allow, so here is a key point.

So the second point is that we only allow first derivative in time, OK? We only allow first derivative in time. We don't allow the second derivative in time. So the reason is that, again, as you will see equation of motion, if you involve the second derivative in time in your action or in your Lagrangian, and then when you get the equation of motion, you will get equation of motions involving more than two derivatives in time. So this will lead to-- so this implies the equation of motion only contain two derivatives, two time derivatives, OK?

So these constraints come from our experiences. It's the same reason here we only include the first derivative in time, OK? It's because in real life all the experiment is determined by the initial condition. The initial condition you only need to specify the location and the velocity, OK? You don't need to specify more. If the equation of motion involving more than two derivatives, then you need to specify more general initial conditions. And so yeah, so here it's the same thing. We only allow the first derivative in time. But you can, in principle, allow arbitrary number of derivative in spatial direction, OK?

But for simplicity, for the most of the time, as we will see, we will restrict to quantum field theory in special relativity, OK? Means that there will be a relativistic invariant, will be Lorentz invariant. And in Lorentz invariant theory, space and time, they can transform to each other, play equal role. So if you only have single derivative in time, you only have single time in spatial derivatives. So the example we will see they will all have only single derivative in spatial directions, OK? But certainly nonrelativistic systems you can have a higher number of spatial derivatives, OK? Good? Any questions on this? OK, good.

So now, as in classical mechanics, and now we can introduce the canonical momentum, Hamiltonian, et cetera, OK? So here we can introduce so-called the canonical momentum density. The reason we call it density will be clear. So remember, for this phi-- so x is just a label, OK? So you can just view this theory, essentially it has an infinite number of these such kind of. Yeah, it's an unfortunate notation here, we use x as a dynamical variable, OK? But here the x is only a label, OK? Here x is a label.

So here you can just imagine you have infinite number of degrees of freedom. Just each one is labeled by x, OK? So now just imagine here you have many, many x. Just you have some labels for it. And so for each such one we can introduce its momentum, OK? So each phi a we can introduce its canonical momentum, OK, phi_a dot, derivative, OK? Yeah, yeah, let me just-- so defined as the derivative of the Lagrangian density divided by time derivative of phi_a, OK? So this is just the direct generalization of here.

So remember, so this is evaluated at each point, OK, in spatial direction. So this thing will also depend on x and t, OK?

AUDIENCE: Sorry, what symbol is that you're using to represent the momentum?

PROFESSOR: Oh, this is just the capital Pi, capital Pi. OK, and this phi a dot just the time derivative of phi a, OK? And remember, x are the labels, OK? So x does not do anything here. So the reason we call this the Lagrangian density is because-- call it the momentum density is because L is a Lagrangian density, OK, and yeah. And then we can also define the Hamiltonian density the same way, associated with each degree of freedom. So we have to find the script H, which is defined by phi_a dot Pi_a . Again, this is all defined on the same spatial point-- minus L. So this is Langrangian density.

And then the Hamiltonian you just integrate over all space, essentially sum over all the degrees of freedom, OK? Just leave them at that. Yes?

AUDIENCE: Is it, so are we implicitly summing over the little a's for the Hamiltonian?

PROFESSOR: Yeah, exactly. Yeah, yeah, it's just like you sum-- so if you treat the-- yeah, yeah, yeah, yeah, so the a is summed. So I will always-- yeah, I forgot to mention-- good, this is a great question. So I always assume this Einstein convention in the sense that all the repeated indices are assumed to be summed. So no matter how many components are there, you just sum over a. So a is summed here. Good? Other questions? Good?

So now we can talk about equation of motion. So it's the same thing as classical mechanics. So classical field theory is just like classical mechanics. You generalize to infinite number of degrees of freedom. So now with each point in space you associate with some degrees of freedom.

OK, so now let's look at the equation of motion. OK, so again, you just do the variation of the action, extremize the action to be 0, OK? So the action is, again, just defined by the dt over the Lagrangian, OK? So the S is the dt L and then can be written as the four-integral dt and dx of the density, OK? So now let me just write-- let me now just write it in a more relativistic notation, just assuming there's one time and one spatial derivatives, OK? So the mu-- now, yeah, I combine the space and the time derivative together, OK? And you can straightforwardly generalize to, say, involving more spatial derivatives. Good?

So here, so let's just do the variation. So we want 0 equal to delta S. So you want the variation of this S to be 0. So now let's just vary S. So you have d4 x. Now, just remember this Langrangian density is just an ordinary function because of the locality just an ordinary function of phi_a and its derivatives, OK? And we can just do the-variation in a straightforward way. We can just write a partial L, partial phi a and delta phi a plus partial L partial mu phi a and delta partial mu phi a, OK?

And in this case, so the delta is a variation and the partial mu. So these two operations are independent of each other. So you can exchange them, OK? So you can put extra delta inside. So this is the same as partial mu delta phi_a, OK? So now you can integrate by parts of the second term, OK, because here is a little bit awkward because involving the partial mu of delta phi a. So again, here the repeated indices, they're all summed, OK? So you should sum them. So now we can do integration by parts here.

So then you get the d4 x, and you get partial L partial phi a minus partial mu partial L partial partial mu phi a, OK? And then the whole thing, delta phi a and then path boundary terms. So the boundary terms come from you do integration by parts here, OK? You're doing integration by parts here. So the boundary term will be proportional to delta phi a, OK? Yes.

AUDIENCE: So could you define delta?

PROFESSOR: Delta's just the arbitrary variation, yeah, just arbitrary variation. Other questions? OK, so the boundary-- so we always assume we have the boundary conditions, OK, and so that the boundary term vanishes, OK? So here I will not go into to detail. So we always choose delta phi a so that boundary terms vanishes, OK? The term comes from integration by parts, OK? And the second, just remember repeated indices are summed. OK?

And now, then we don't have to worry about boundary terms. So the boundary term vanishes. Then now we just have this has to be 0. But this has to be 0 for any variation. So this delta x mu can be any function of x, OK? Can be any function of x. The only way this can happen is that this prefactor must vanish, OK? So this implies that the delta L partial phi a minus partial mu partial L partial partial mu phi a, [INAUDIBLE], OK? So this is the general equation of motion for classical field theory, OK? Good? Any questions on this?

OK, so from now on, we will make two restrictions, OK? Just for simplicity, also they're the most-- we make two restrictions just for simplicity. Yeah, most of our discussion can be generalized beyond those situations. Yeah, so the first is that we restrict to field theories which are translationally invariant, OK? Means that there's no special point. Means there's no special spacetime point in this theory, OK? If you do experiment here in Boston, it's the same as you do experiment in Washington DC, et cetera, OK? And if your theory is not translation invariant, and then when you do experiment in Boston, then it will be different from when you do it in, say, in New York.

And the second is that we assume it's a Lorentz invariant. So in some condensed matter applications, which you don't have Lorentz symmetry, and then you don't have Lorentz symmetry. But yeah, but similar techniques we are going to talk about can be applied. And we will also elaborate a little bit more on both of these aspects. OK, so now let me give you some simple examples of the classical field series, OK, which satisfy these two condition. Examples, so the first one is the Maxwell theory. So for Maxwell's theory, the dynamical variable, this 4-vector potential A mu, OK? So now I'm using the four-vector notation. So I will often suppress the indices on x. That means this is a 4-vector, OK? And then from the A mu we can define the field strengths. And the E and B then can be obtained from this F mu nu OK, can be obtained from F mu nu And then the action for the E & M then can be obtained by-- OK, that's this, OK?

So this is the action for the E & M. without charged matter, OK? So if you do the variation here, OK, to get the equation of motion, then you get the Maxwell equations without sources, OK? You get the vacuum Maxwell equations. You get the vacuum Maxwell equations. And so you see that this action have the form we mentioned here is corresponding to a local theory, OK, a single integral of space time, OK?

And the second example is Einstein gravity is another classical field theory, OK? So let me just write down the action. So this is written as G Newton. So G is the metric. Yeah, anyway, if you don't know the Einstein theories, OK? OK, so R is a Ricci scalar. And if you don't know the Einstein series, OK. I'm just write it down just to show that this is a local theory involving only a single spacetime integral of some quantities.

And the simplest quantum field theories are called the scalar field theories because-- I think I erased it-- because this involving the vector field, OK? So this is a vector field because at each spacetime point there's a vector. Here, the Einstein gravity, the dynamical variable is a tensor. So it's even something more complicated. So the simplest case is a scalar, which at each spacetime point, you just have a single value quantity, OK?

And so let's just consider the simplest case. Just consider a real value scalar field phi, OK? So this is just take a real value. You can also consider the complex value, OK? But the simplest is just a real value. So at each spacetime point you have a single value, OK? And so that's the dynamical variable.

And so in real life there are many examples of scalar fields. So for example, the Higgs, OK, the Higgs field, which is a celebrated Higgs field, which was discovered a number of years ago, is a real scalar field. And also pions, OK, the pion particle can be considered as excitations of a scalar field. Anyway, so simplest family of a scalar field action is the volume form, OK?

So here I just write this action down. But essentially, you can argue this is the most general action you can write down, which are compatible with locality, OK, with locality and with translation symmetry and the Lorentz symmetry, OK? So here you see the Lorentz indices are contracted. The Lorentz indices in the derivative are contracted. So that guarantees the series Lorentz invariant. And V just some function of phi. V here is just some function of phi.

So this is essentially the simplest theory you can write down based on localities, OK? So say we can take V phi, in general, we can take it to be polynomial, yeah, just some function of phi, OK? So when you have a translation symmetry, means that in this theory if you do experiment it's the same everywhere. That implies that V phi-- so here there's no parameter here. V(phi) then must-- yeah, sorry. All parameters in V(phi) must be constant, OK? They cannot depend on space time, OK, must be constant.

So if you have some parameters depend on the space time, then of course, different space time point will behave differently. And they will not be translation invariant, OK? So let's try to work out this theory a little bit in more detail. So let's find this Lagrangian density-- let's try to find its momentum and the Hamiltonian.

So it's momentum density. Here there's only a single field, so there's no index. So pi x would be take derivative of this. So this is L. So this thing inside the bracket is L, this partial L partial phi dot, OK, time derivative of phi. And so if you look at here, here the time derivative is just phi dot squared, OK, using this four vector notation. And if you expand it, and then the momentum density just phi dot x, OK, because just because partial mu phi partial mu phi is equal to minus phi dot squared plus, OK?

And then you can find the Hamiltonian density. Pi phi dot minus L, and then you find that this is given by-- we express in terms of momentum, OK? And the equation of motion, if you apply here, then you find given by partial square equal to partial V partial phi equal to 0, OK? And again, here I'm using a shorthand notation. Partial square is defined to be partial mu partial mu, OK? So this is the same as minus partial t squared plus v squared, OK?

So here let me also give you some simple examples of the v. In the simplest case-- so in the simplest case, is you take v just, of course, it will be a constant. But a constant does not do anything because the-- yeah, so the simplest case is just v phi equal to linear term. And from time translation symmetry, this f has to be a constant, OK? It cannot be depend on space time. If this depend on space time, and then you will violate the space translation symmetry, OK?

So when you have these, then your equation of motion, given by partial squared equal to f, OK? f is a constant. So in this case-- so essentially you have some kind of thing, which-- some kind of a constant, which is source for this phi. So in this case, we say the system have external force. OK, we call this f external force because if f is nonzero, then phi cannot be 0, OK? Phi has to be nonzero. And so in a sense, the phi will be always be excited. It will always be generate-- nonzero phi will always be generated if s is nonzero, OK? So here we call it the external force.

And so normally we like to consider phi-- normally we will consider-- normally we are interested in the situation there is no external force, OK? The system develop on its own. So in that case, so if we forget about the external force, then the simplest situation will be phi equal to quadratic, OK? It's some polynomial, phi squared, OK? So that's the next simplest function of phi. And m squared has to be a constant, OK, again, from the translation symmetry.

So in this case, then you have the equation of motion, partial square phi equal to m square phi squared, m square phi equal to 0. So this is a very famous equation. So this is called the Klein-Gordon equation. And the next time you will see very quickly why this is a famous equation, OK? And yeah, yeah, so this would correspond to the simplest field theory we can consider. And then you can consider more complicated field theories with V(phi) to be some higher polynomial or more complicated functions.

But keep in mind that this square is special. Because it's square, this is a linear equation. But whenever you have something which is not linear or quadratic, and you will get a nonlinear equation. And then the story becomes complicated, OK? Nonlinear equations are always complicated. And anyway, so this would be the simplest theory we will consider, OK? OK, so let's stop here.