[SQUEAKING]

[RUSTLING]

[CLICKING]

HONG LIU: So again, let us first recall say if we want to calculate the n-point function in the interacting theory. And this is given by this following ratio in the free theory. You look at this T, then you look at this. Again, take this product to be X, take the T X, exponential i S_I, then in the free theory, and then divide it just by time order i S_I, in the free theory.

So then what we do is we just expand these two exponentials in power series. We expand this exponential in power series. And so when you expand in power series, then, for example the nth order term in the upstairs, say upstairs, then have the following structure. You have i to the power n, n factorial. Then you have T X then S_I to the power to n S_I.

And the nth order for the downstairs, you can also expand the downstairs. So in the downstairs similarly then you have almost identical structure, except you don't have this X. So you have T, then you just have n S_I's. So there's n of them. So there are n of them.

And the last time we described how to compute say a typical terms in here. Let me call this equation star, equation star star. So last time we described the star and the star star can be computed using Wick's theorem. But in practice what we do is that we first draw Feynman diagrams and then we convert the Feynman diagrams into analytic expressions. Normally we go through this procedure, rather than directly do the Wick contraction. We mentioned the--

And the second line, of course, can be considered as a special case of the first line, with this x to be identity. This x to be identity. So any diagram contributing to star because the-- Sorry I should actually here I have n, let me call it m. I should call it m so that they don't-- because we already have n here. So mth order.

Any diagram contributing to the star has n external legs. So that's come just from this x, from this phi x1 and the phi xn. And then also with m vertices because each vertex comes from a power of S_I.

But for star star, diagrams contributing to star star have no external legs because there's no x here, there's no phi here. There's no external legs. So that's why they're all called vacuum diagrams. They're called the vacuum diagrams.

The downstairs can be understood as in the case which you just essentially take X equal to 1. So essentially we are calculating some kind of unnormalized transition. This is the overlap between the vacuum itself from minus infinity to plus infinity.

In Pset 6, just in the Pset we just posted last night, you can show that this quantity, which comes from summing over all possible vacuum diagrams, the diagrams without external legs-- Sorry, I should say have no external legs. This no is very important. Yeah, has no external legs and so they are called vacuum diagrams.

In Pset 6 you will show yourself. It's a pretty simple calculation, but it's an instructive one. You can show that this quantity can be used to calculate the vacuum energy of the interacting theory. Of course, this is divergent, but just as the vacuum energy we calculated before in free theory, this one can be used to calculate the vacuum energy of the interacting theory.

Of course, this quantity you will see its divergent, but this is the formal way to calculate it. And how to remove the divergences etc. is something we will not discuss in this class. It will be in QFT 2. Any questions on this? OK, good.

Let me say a few more words. When we compute two, there we say they can be computed using the Feynman diagram, so let me be a little bit more explicit. The computation of star or star star essentially involves two steps. Just elaborate that remark in the bracket.

Is that we first draw all possible -- all inequivalent Feynman diagrams. In this case, at each mth order you have an n points, n external points, and then you have m vertices. There you have say for star you just have n external points, say x1 x2 in coordinate space. You have n external points and then you have m vertices. And at each vertex you have-- let's call it y1, y2, ym-- and at each vertex you have four legs coming out. At each external point you have one line come out, and then you just need to find all possible ways to connect them. That is what we mean by joining all inequivalent diagrams. This -- you just draw it, it becomes mechanical. Yes?

AUDIENCE: Why do the vertices have to all have four legs?

HONG LIU: More specifically here we're talking about the S_I equals to minus lambda 4 factorial. We can see that this particular theory. I'm going to make remarks on other more general theories, but here it's more specific. Other questions?

So this is for star. So for star star, then you just don't have the x, you just have y's. You just have m y's and each y has four legs. You just connect all of them together. You connect all of them together.

In the star star, in the vacuum diagrams, it will be always closed because there's no external leg. Just because all the legs will be contracted, and so it will be a closed diagram. Sometimes it's also called the vacuum bubble. Sometimes we call them vacuum diagrams or vacuum bubbles. Here just know it's always closed diagrams.

And the second step is that now once you draw all the inequivalent diagrams, then you can just convert each diagram to an algebraic expression using Feynman rules. Then you just sum all of them together. Then you sum all of them together.

And then we talked about the Feynman rules last time. There's coordinate space rules, there's momentum space rules. You can do it in coordinate space, you can do it in momentum space. I will not repeat them here to save time.

Once we finish the second step, then we just have a bunch of analytic expressions and then you just need to do the integrals. So a bunch of integrals and you just need to do the integrals. Any questions? Good.

Some further remarks on 1. Further remarks on this first step. Here I consider a very special case. Here I consider a very special case corresponding to this S_I to consist of a single term, phi 4. In principle, of course, S_I can contain multiple terms. Can contain multiple terms. It can contain multiple terms, then you will have multiple types of vertices.

But the rule is similar. You just say suppose S_I contains two terms, then you just multiply them. Then you'll get many terms here. Just corresponding to different combinations of vertices for different types of vertices. But the rule is exactly the same, just now your diagrams are more complicated. But essentially otherwise it's identical.

Another thing to mention is in principle we can also have multiple fields. So here we can see only a single scalar field, phi, but you can also, in principle, consider more than one field. So if there are multiple fields, the story is, again, very much parallel, and the only difference is that each field, each propagator, is represented by a different line.

Other than that, these two steps again apply. Just now you actually have different type of lines. For example, if you consider a theory with two scalar fields, phi 1 phi 2 with S_I proportional to phi 1 squared phi 2 squared, and then in this case you will have a propagator for phi 1, then you also have a propagator for phi 2. Now I can use a dashed line. Then the interacting vertex then will have the following structure. You have two solid lines and the two dashed lines. Now that's your vertex.

Again, you just contract all possible lines according to your Feynman rules. Good?

Also one remark I want to emphasize is that the momentum space-- also just another point to stress-- this is obtained by Fourier transform Gn x1 xn times the delta function. This times this gives you the Fourier transform of that.

When you do the Fourier transform, p1 and pn, they just come from doing Fourier transform corresponding to momentum for x1, xn. So here the p1 and pn are just arbitrary. They don't have to satisfy any on-shell condition, they're just arbitrary momentum, and the only constraint is that this is non-vanishing only when sum of them is equal to 0, otherwise they can be arbitrary.

Good? So in practice, now we have a way to calculate the diagram for both upstairs and downstairs, and now we just do the expansion. We just do the expansion. Now let me summarize the example we did last time now just only using diagrams.

So for example, if we consider the two point function, let's consider the two point function. We have upstairs and downstairs. The upstairs, the lowest order you just have a single line. It just has a single line-- x1 to x2. To save effort we'll also not label x1, x2 from now on.

The next order is to bring down one power of this S_I. So you bring down one power of these four indices. Now there are two possibilities. One possibility is that you still have this, but then times this vertex contracts itself. And then another possibility is that you have something like this. And you contract with the vertex, so this is the plus lambda squared. This is to lambda order.

And downstairs you have 1 and then we just essentially have a single S_I. A single S_I, the only contraction is just given by this. And then plus order lambda squared. Plus order lambda squared.

So now each vertex carry-- so you should remember, each vertex carries a parameter lambda, a factor of lambda, which is considered to be small. Downstairs you have 1 plus order lambda, and in the downstairs then we have 1 plus this power series of lambda, and then we can do the Taylor expansion again to bring them to the upstairs. Remember we did before.

Now if you do that, then at leading order to lambda, then we still just have x1, x2 because this divided by that just x1, x2, and then you have this term. But now we can bring this term to the upstairs and then this sign changes to minus sign. This term can multiply that and then you get the term like that. Coming from bringing this term to upstairs and then you have this term.

And the rest, they're all order lambda squared. So you can also have this change to minus sign, multiply that, but that will be order lambda squared. And since here we already neglected lambda squared and here neglect is lambda squared, so we don't worry about it. And if do this and multiply this, also gives you lambda squared. Up to order lambda and that you have.

And then as we discussed before, these two cancel. So in the end you just have two diagrams, just one and then this one. Yes?

AUDIENCE: If we have two fields with a vertex with like one solid line, one dashed line. Here, you can read it as the two different particles interacting with each other or the same species interacting with each other if you read it like that, but you lose that if you--

HONG LIU: For here it doesn't matter. In this case, it doesn't matter how you draw them. You mean if I exchange them?

AUDIENCE: Yeah.

HONG LIU: Yeah, it doesn't matter.

AUDIENCE: But in that picture you can't read--

HONG LIU: Yeah, normally you cannot view up and up and down. Left and right here does not mean anything. The order is not important.

But actually, in QFT 2, you will encounter situations which these are matrices and then the order becomes important. But here they just fields because they commute. So no matter how you order them doesn't matter. They're just ordinary fields.

This order, we just have these two diagrams. This is much simpler than we did at the first time. Much simpler than we did the first time and the calculation is much easier. Let me just show you the cancellation in a slightly different way, almost identical, but I write slightly differently.

Alternatively, I can also write as this. I can also write G2. We look at these two terms. We can just take a common factor out. So the upstairs, the first two terms I can also write it as this times 1 plus this thing and then plus this. And downstairs I just have plus lambda squared. And downstairs I just have 1.

Now you can just look at these two. You can directly see, actually, these two can be canceled. You don't have to worry about this one because when these multiplied by that, you get higher orders. Now you just directly also get from here.

The reason I'm doing these in these two different ways is that this cancellation is actually not an accident. Such cancellation, this is not an accident. This is not accident. This actually happens to all orders. This happens to all orders, so you can actually show.

If I call the upstairs, now let's call the upstairs to be Un and this to be U0. Un is just the upstairs U0 is the downstairs. The Un, by definition, corresponding to sum over all Feynman diagrams with n external legs. And U0, by definition, is sum of all Feynman diagrams with zero external legs. With zero external legs.

So then you can show in general which the Gn is just equal to, as we wrote there, just Un over U0. You can show that this defined to be Un over U0. You find order, you can show to all orders this downstairs can always, always cancel. The statement is that this is equal to sum over the ratio of all diagrams of n external legs, but without any vacuum bubbles.

Now let me just explain what this means that without any vacuum bubbles. OK, so this diagram is a diagram which contains just a straight propagator times a bubble. This diagram contains a vacuum bubble. It contains a vacuum diagram, and this diagram does not. This diagram is connected to itself.

Similarly, when you go to next order, as we discussed last time, this diagram, so go to next order times, so this is also a diagram with vacuum bubble because you have this part, which does not have any external legs. This will not arise. When you calculate the ratio, then this kind of diagram we automatically don't include. Just don't need to worry about it.

Now when you do the ratio you can just forget about those diagrams and directly write down those diagrams without vacuum bubbles. That just a greatly simplifies your life. OK One second. So this statement is simply-- If you think a little bit, then you will see that this cancellation is generic-- actually will work through all orders-- and this thinking process is actually quite instructive, so this is left as a homework.

You can read the part in Peskin which tries to explain this cancellation, but that discussion is actually not great. It's not great, that discussion, but you can still read it and get the main idea. And so in homework I ask you to show this yourself. It's not difficult once you get the idea. It's actually pretty simple to see and will be illuminating. Yes?

AUDIENCE: I'm pretty confused on this cancellation. So if you're pulling out an order lambda term, won't your order lambda squared would now be order lambda?

HONG LIU: No, you pull the whole thing together. Those terms don't matter at this order. Whatever here, when you cancel them, you cancel the whole thing, this term doesn't matter. If you care only of all the lambda squared terms, and then you can just directly cancel these two. You can easily convince yourself. You don't have to do a diagram. Just write something-- a 1 plus lambda plus some other lambda and 1 plus-- Anyway, you can just convince yourself that this always happens. Yes.

AUDIENCE: So this cancellation, does it just come out of the math or does it is there like a physical interpretation?

HONG LIU: Yeah, there is a physical interpretation. Essentially you just say when you look at-- So this kind of thing essentially comes from the normalization of your state. This thing is just pure vacuum process and just comes from normalization of the states. And when you normalize your states, they just should not contribute to your correlation functions. Just roughly it's like that. Other questions? OK, good.

Now let's just put this in practice. Let's put this in practice, and let me just write down the most general, all the diagrams for G2.

To lambda squared order. Now let's look at G2 to lambda squared order. We have already found the lambda order. Lambda squared order, we already drew a bunch of diagrams last time, so you only need to keep those which don't include these vacuum bubbles.

Let me just write them down. Including this, including that, and also including the one I forgot last time-- this one. That's it. And if I remember, last time we drew many more diagrams, but you don't need to worry about the others, such as diagrams like this.

Good. Let's do another example for the 4 point function. Now it's become much easier because you have much fewer diagrams to consider. You have much fewer diagrams to consider. So at the 4 point function level, then you first have your free theory terms.

So the free theory returns you just have that. It just contracted with itself. Now we have four external points. This is coming from the free theory contribution at the 0th order. And then the first order, at order lambda, you just have this, so that all the four different phi's from the vertex are contracted with each of the external phi's. This is the first order.

And then you can also have a diagram like this. And you can also have-- I will not show all the diagrams. And you can draw more diagrams like this, but these are not vacuum bubbles. And you can also have diagrams like this.

The reason I chose them is because you are going to do some of them in your Pset.

Yeah, there's another diagram like that, et cetera. Let me just do two more. There's still some more diagrams you can draw to lambda squared order.

But again, it's mechanical. With a little bit patience, there's no mystery here. Now let me make some remarks here.

Let me make some remarks here. First, so the diagram, you can see separate into two types. So the full Gn separated into two types. One type is called connected diagrams. Connected diagrams, just essentially by its name, this corresponds to diagrams which all the external legs they are connected within a single diagram. This is a connected diagram or this is a connected diagram, but this is not, this is connected diagram, this is connected diagram, this connected diagram, this is not. This is also connected diagram.

And you can also have disconnected diagrams. Disconnected diagrams corresponding to diagrams that's here, so your external legs separated into sub-diagrams. They belong to product of sub-diagrams.

And later you will see, later we will argue, this is actually not interesting. Only connected diagrams are interesting, but in a few minutes. For the moment, let's just still define them.

And emphasize again, the external pi general. Each external line, which you can assign a momentum pi from the Fourier transform. For example here, you have p1, p2, p3, p4 in momentum space. Each one you can assign a momentum and note momentum general. It can be in principle, it can be anything.

Good. Any questions on this? Yes.

AUDIENCE: Is there a trick to know that you've got all of them, like do you count them?

HONG LIU: No, there's no trick to make sure you counted all of them. The only trick is patience. Just try to eliminate all possibilities. It's a finite number of them, you can always do it. Yes.

AUDIENCE: Can you use metrics, like count the number of diagrams.

HONG LIU: Sorry?

AUDIENCE: Can you count the number of diagrams with a certain number of objects and external points?

HONG LIU: No, I think there's no general formula to tell you at each order how many diagrams you should have. There's no such kind of magic formula. There's only one formula. It's not the formula. There's only one trend which you can show, that when you go to mth order, there are how many roughly how many diagrams at mth order? Yes.

AUDIENCE: m factorial.

HONG LIU: That's right, m factorial, so it's quite a lot of them. Once you go to say fourth or fifth order, it becomes a lot. You can be sure that we will not ask you to eliminate to fourth order in your Pset.

Actually, the growth of the number of Feynman diagrams implies something deep. It's a side remark. It implies that this perturbation theory in lambda, it's actually not convergent because the number of diagrams grow too fast. And so if at mth order becomes m factorial, then that means the mth order contribution also grows as m factorial, and then that means this power series is actually not convergent. It's only asymptotic series. But still, for many physical purposes, it's enough.

Good. Any other questions? Because this concludes our discussion of calculating such kind of Feynman correlation functions for interacting theory. Yes.

AUDIENCE: You just said that it's not convergent, but it is asymptotic.

HONG LIU: Yeah, it's an asymptotic series.

AUDIENCE: Sorry, what's the distinction between those?

HONG LIU: Yeah, asymptotic series is the kind of series which you-- Just the higher order terms will be small or will be smaller and smaller for a while and then grow again. And so if you don't calculate to high of an order, they're actually quite reliable. You can bound the errors. This is the heuristic way to say it. It's just when you look at the first few order terms, it's a reliable approximation to your true answer.

AUDIENCE: I guess we started out by setting the interaction term to be phi to the 4th, but the action side. But in general, you're going to have some series of terms added to your action. Is that how you would do a general interaction?

HONG LIU: No, depends on your specific theory. Maybe that's just your theory.

AUDIENCE: How do you apply this to a physical problem? How do you know what--

HONG LIU: Yeah, in physical problem-- In one second we will tell you how you come from here to calculate the scattering amplitude. And when you find the scattering amplitude, then you can measure the experiment, then you try to deduce back the action.

AUDIENCE: And in general what changes is just how many edges your vertex has? Is that the phi to the n?

HONG LIU: Yeah, that's right, that's right.

AUDIENCE: What if it's, I don't know, 1 over like something [INAUDIBLE], is that--

HONG LIU: It can happen. As a toy model, you can always write whatever theory you want, but in nature somehow the series we discovered so is all polynomial. OK, good?

Earlier we mentioned that there is this LSZ theorem, which tells you that the scattering amplitude can be obtained from this time-ordered correlation function. Now let's go back to this LSZ story again to tell you actually how to obtain scattering amplitude from correlation functions.

The basic idea is the following. The basic idea is following-- is that you take your n-point function with n external momentum p1 to pn. So as I emphasized here, each pi is general. But now suppose you take all momentum, all pi, to be on shell. Now you consider them to satisfy pi squared equal to minus m squared.

In order to get the amplitude, you need to do this step. You take all the external momentum. This is very reasonable to expect because when we're scattering particles, remember we say the initial particle can be considered as a free particle, final particle can also be considered as a free particle. If they are free particles, then their momentum satisfies this on-shell condition. In order to relate this to scattering amplitude, then we need to take external momentum to be on-shell.

Now if you look at this, you say there are two possible choices for this equation because this equation has two solutions. So for each pi, pi0 can either be plus minus omega pi. We are using the same notation we used before, so this Pi vector is the spatial part, and then we have omega Pi. This omega pi is just the--ok?

So you have two possible solutions.

Now the rule is the following-- let's take p1, take m of them. We take the negative root. So alpha would be 1 over 2m. Then for the rest, say Pm plus 1, let's consider the following situation. And then you take the positive root.

Ok? So let's consider the following situation.

Let's imagine we take such an n-point function in momentum space, and then we take all the momentum on-shell, but for some of them we take the negative root and some of them we take the positive root, and then there's a theorem. So this is called LSZ theorem, which is 3 person-- Lehmann, Symanzik, and Zimmerman.

This theorem is says, under this limit. Under this limit. Under this on-shell limit. So you will see why we emphasize that we call it on-shell limit. So imagine you don't actually put them to be exactly a momentum. You take the momentum to approach that. We approach that, we just emphasize.

So in this limit then you find your momentum space correlation function. Can you find your momentum space correlation function approach to the following quantity in this limit.

First, we write down the expression and then we will explain what this means.

And then we just do the downstairs.

Do the downstairs.

Plus infinity.

So the statement is the following, and here Z is just some constant. We don't need to worry about it. Z is just some constant. This is square root Z divided by this factor for each external momentum, and this is the scattering amplitude for m initial particles. Oh, sorry, I forgot the minus sign. It should be minus p1, minus p2, to minus pm.

This is the scattering amplitude for particles with initial momentum minus p1 to minus pm into particles with final momentum m plus 1 to pn. Essentially, whether you choose the negative root or the positive root decides whether you are in the initial state or in the final state. Whether you are in the final state in the scattering amplitude.

Let me just write here. Let me just copy this again. This scattering amplitude-- Yeah, let me just write here this pm, pm plus 1, pm plus infinity minus p1 minus pm minus infinity. This is the scattering amplitude. This is the scattering amplitude. This is an amplitude for the particle with momentum p1 pm at t equal to minus infinity transitioning to the rest particles at this momentum at t plus infinity.

To save the location, again, remember previously we defined--

If you remember before, we defined this object. This object should also preserve, because of time translation symmetry, this object will also preserve the energy and momentum. This object is also proportional, so you can extract out the delta function and then times a quantity, which we called the scattering amplitude for, say alpha to beta. Alpha denotes the initial state, so this is the initial state alpha, and this is the final state beta.

Now if you plug this expression into here, then the delta function will just cancel on both sides. And then you find Gn p1 pn is just equal to, approach this product of the delta functions, approach this factor, and then M alpha to beta. This is a statement of the LSZ theorem. This essentially tells you that such a scattering amplitude just can be obtained from this, and you multiply those things, and then just equal to that.

Now you observe, up to this factor of Z, these are precisely just propagators for the external legs. Now you conclude, in other words, that means that M alpha beta then corresponding to you just take the G, then you get rid of all the external propagators because they're just corresponding to the external propagators for each external leg. Just get rid of the external legs.

Here I'm just telling you the theorem and we will use it, but of course, the derivation is a little bit complicated. Let me just write it in words. To obtain the scattering amplitude M to alpha beta, you first calculate.

Suppose the total number of initial particles and the final particles are n and then you just take Gn for n particles. And then take pi on-shell, and the initial momentum take the negative root, and in the final momentum you take a positive root of that equation. And then the last step you just truncate all the external propagators.

And then you truncated all the external legs. So that just comes from you need to obtain M. You need to divide this side by this factor then up to this factor of Z. This factor Z will only give you a constant, which let's not worry about it for now. Yes?

AUDIENCE: [INAUDIBLE]. it seems like you just put it in ad hoc.

HONG LIU: Which is putting ad hoc?

AUDIENCE: Why are you just setting some of them to be negative?

HONG LIU: Oh, yeah. That I will explain. That I will explain. First, let me just state the rule. First, you understand the rule, then I try to make you more comfortable with the rule. Yes? OK. Yeah, here is just the rule. Here I'm just explaining here is the rule. This is a statement you can prove mathematically, so this is a mathematical statement you can prove. And then following from this statement, here is the rule to obtain the M alpha beta from the G.

If I want to write it in one sentence-- scattering amplitude equals to sum over all truncated diagrams with external momentum on-shell. If I summarize it into one sentence, it's just this.

I would say the only thing unintuitive here is this sign choice. Everything else I think is intuitive. Everything else is intuitive. You take this Green function, which contains essentially any external legs. And now you just imagine this internal legs corresponding to initial and final particles, and then it's very reasonable we make them on-shell because they are physical particles. Then the only thing which may be unintuitive is that why for the initial momentum we need to take the negative root, but for the final one we need to take a positive root.

Then the final thing here is just if you imagine when you scatter particles and how they propagate to the scattering does not matter. The propagator just essentially tells you this particle how they propagate to the scattering point. This should not matter for the scattering process. We will explain this at the end, but before we do that, let's just look at one example. Do you have any questions? Yes.

AUDIENCE: I just to make sure I understand when we say truncate we mean divide n and divide Gn and by this product.

HONG LIU: Yeah, you will see explicitly we just don't include all the external propagators. Let me give you an example. It will be clear. Let me just give you an example.

Let's consider four particles scattering. Let's consider. Following the convention here is the minus p1 p2, so you have minus p1 and minus p2, and then you have p3 and p4.

Let's suppose we want to calculate such kind of scattering amplitude to initial particles with momentum minus p1 and minus p2. When you take this to be the negative root, then the minus p1 will actually have positive energy. The minus p1 will actually have a positive energy, so all of them have positive energy. All of them have positive energy.

If you want to calculate in this case, then you just take the G4 p1 p4, and then you just take p1 and p2 to take to be negative root. Just to save the time to take to the negative root. And then p3 p4 to be the positive root. And then you put the t here, then you truncate your external propagator. Then you truncate your external propagator.

What this means, let's look at the leading order contribution to this. The leading order contribution to that, we already have the diagram there. The leading order, in the free theory, those processes don't contribute because those things, there's no interaction here. Particle just propagates. So the lowest diagram is this one. The lowest diagram is this one, so we just consider that diagram, so that's just leading order given by that diagram. Then you have minus p1 minus p2 and then p3 and p4.

Now according to our Feynman rule, this vertex just gives you a factor of minus i lambda. So if we want to calculate this Green function, then we need to include all these four propagators. But here it tells us to truncate, but all the propagators here are external, so we don't include them. So the answer here is simple-- just minus i lambda. The leading order contribution to these 4 particles scattering is just minus i lambda. And then you have all the lambda squared contributions. Yes?

AUDIENCE: Why didn't we consider the other, the one where one is just propagating and the other has a loop?

HONG LIU: I'm going to mention that good. That's a good question. I'm going to mention that. Again, that one does not correspond to the scattering. Essentially it's just corresponding to one particle propagated because there's no interaction between the particles. They factorize. This one involves the two particles then, there's an interaction, you go to two other particles. Here just one particle straight goes to the other particle, so this particle by definition has to be the same as this one, this has to be the same, so there's no scattering. I will mention this point separately a little bit later.

Another possibility if we want to consider the decay of a particle into three other particles. The decay of a particle into three other particles. In this case then, again, we take G4, so now we take minus p1, and then p2, p3, p4, and now we take p1, p4. Now we take p1 goes to minus root, p1 0 is the minus omega, and then p3, 2, 3, 4 take it to be the positive root.

And then that's corresponding to this process. And again, the leading term, the nontrivial term is just this. The leading nontrivial term is this, so now this is minus p1, so now this is p2, p3, p4. Again, the leading order, this is just minus i lambda because you don't need to include the external legs. This goes on to a single particle decay into three other particles.

AUDIENCE: In the phi 4 theory, there's no conservation law or anything for preventing this, right?

HONG LIU: No, no. So now let me make some remarks.

Oh, yeah. I erased this, but yeah, I'm fine.

So the remarks. You already asked, when we calculate this to the leading order, we only included the connected diagrams. The disconnected diagrams in general factorize. It means that when you have disconnected diagram, it means that your process factorizes into subscattering process of lower orders.

As we were doing here, there's one other diagram which corresponds to just this one. Just corresponding to this, you have p1 minus p1, minus p2, p3, p4. You can also have a diagram like this.

But in this case, just corresponding to p1 separately go to p3 and p2 separately goes to p4, so nothing really happens. So in general, you have m particle scattering. You have m particle scattering. So with some number of initial particles and some number of final particles. When you have a disconnected diagram, it means that you have a situation like this. You have one diagram with some subset of particles go to some subset of particles, and with a lot of diagrams some set of particles, some set of particles.

This is a three to three scattering, and then essentially this factorizes into two to two scattering, and then one particle just goes to another particle. This can already be understood in terms of lower order process. We don't include them. Essentially, they are already understood. When we define the scattering amplitude, we don't include them.

We are only interested in process involving all participating particles. We are actually running out of time. This is first remark.

Second remark. Since we truncate all the external legs, we truncate all the external propagators. This kind of diagram, you have a loop on one of the external propagators, so this diagram doesn't contribute. Because this one, after you have truncated the external legs, it's no different from this one, it's no different from that one, so you don't have to count this one separately. So this one is just corresponding to when the particle propagates to infinity somehow there's some other process to happen to this particular particle. When you truncate the external legs, they don't include them.

So that means, with these two, it means that the scattering amplitude-- Essentially can forget anything happening to the external legs. You can have complicated things happen external legs, but they don't matter.

To summarize, the scattering amplitude, when you compute the scattering amplitude, you just sum over-- following this definition-- you sum over all truncated. It means that you truncate all the external legs. Connected because we want all particles to participate. And then the external particle on-shell.

This is a simplification compared to calculating the general correlation functions. I have two other remarks to make, but we don't have time. One remark contains the choice of this sign-- why initial momentum corresponding to negative root and why this corresponding to positive root, and a lot of remarks related to this factor of Z.

We will talk about them at the beginning of next lecture. And then after that, then we are done with the general formulation of interacting theory. And now we will be equipped with the power. Essentially, you can calculate any interacting theory, any amplitude, using perturbation theory.

But in order to calculate anything useful, we still have to first learn how to treat fermions, how to treat photons, and then that's what we will do next. So we will, starting next lecture, talk about how to deal with fermions, how to introduce fermions, and after that we'll talk about how to introduce photons, and then we can talk about QED, how photons interact with electrons, et cetera. OK, good.