[SQUEAKING]

[RUSTLING]

[CLICKING]

HONG LIU: Let us start. So last lecture, we discussed the Compton scattering. So with that example essentially we covered all the tree-level diagrams in QED. So tree-level diagrams means those not involving loop.

OK so far encountered diagrams, which are called the tree-level -- without loops. So these are called just tree-level diagrams.

And so now you should be able to actually treat all possible tree-level diagrams. You already have all the tools to treat all possible tree-level diagrams. But tree-level diagrams, actually there are many important things missing just in the tree-level diagrams. So now today we will talk about how to go beyond that, so we talk about quantum fluctuations.

So this is a very big subject and actually will consist of a big chunk of quantum field theory II. So here it's more just to give you a preview of the basic idea of the subject. So one feature of the tree-level diagram we have encountered.

So if you want to summarize the say features of tree-level diagrams, what would you say? Other than the looks. Other than looks like tree. Yes?

AUDIENCE: There's like no free momenta.

HONG LIU: Exactly. So the key is that all intermediate momentum say in all propagators, all intermediate momenta, they are determined from external ones. So when you treat the tree-level diagram, there's no momentum integration.

So this looks like a very simple mathematical feature, but actually it turns out there's a profound difference, both mathematically and physically. When you include the loops, there are profound differences. And also you can show that the-- So now let me just summarize.

So now when including the loops, there are many new features arises. And so let me just first give you some simple examples including loops. So let's just consider say for example one is just consider the electron propagator and it's just a line. But now you can consider adding a photon line here. Adding a photon line.

So this is the process and now you have a loop. It consists of this photon line and this electron line. And so you can consider this as a process that the electron propagates, and then can emit a photon, and then it absorbs itself. So this photon is purely virtual. It's purely a virtual photon.

But such kind of process actually affects the propagation of an electron. So you can view these photons as something fluctuate from the vacuum. So you can imagine these come from the vacuum fluctuations.

So this is one process. And another simple process, if you consider propagation of a photon. And then the photon can actually break up, say can compare create the electron and the positron pair. And then this pair can annihilate again and then to form another photon.

Again, this is a correction to just a simple propagation of a single photon. So initial state is still one photon and the final state is still one photon, but something happens in between. And again, this something that happens in between, you can interpret it as some kind of vacuum fluctuation. So both electron and positron here are completely virtual. There's no real electrons you can detect.

So the third simple process is this one, remember we have the electron, the photon interacting vertex. So this is i e gamma mu. So this is the electron phonon interaction vertex. And this vertex can also be corrected. So imagine you have a photon. So imagine the electron when it comes to and then first emit a photon and then get absorbed by the electron which comes out.

And so this will correct the vertex, and so this is called the vertex corrections. So these two correct the propagator and this one is the corrected vertex. So these are the simplest loop diagrams. These are the simplest loop diagrams. In each of them here, there's a single loop and then there's a free momentum in the loop. So you have to integrate over the momentum in this loop because it's no longer determined by external momenta. It's no longer determined by external.

So it turns out such kind of momentum integration, so if I call this momentum k, and then you were involving-- So let me first talk about the mathematical features. So mathematical new features for loop diagram.

So mathematically now you need to integrate over the internal momentum. So now you have some free momentum to integrate. So this may look like something just not very-- you may say this is not a big deal, but actually this turns out to be a very big deal because this momentum, this integration, just by definition, is you integrate over all possible momentum. So each k is integrated from minus infinity to plus infinity.

So in particular, this includes the region that k0 can go to infinity and the magnitude of k can also go to infinity. So we include those regions.

So you have this infinite range of integrals, then there's a possibility you get divergence. There is a possibility you get divergence. And in fact, the divergence does happen and happened almost everywhere. Whenever you look at loop diagrams, you always, essentially always, find the divergences.

So this leads to divergences. And such divergences are now more complicated. It's not like the simple divergences. We already considered, we've already seen where is divergent quantities, and so far those divergent quantities can be treated in a more or less simple way. So we add or subtract the infinite constant or you multiply by the infinite constant. And in the case of when you multiply by infinite constant, you can always take the ratio and then those infinite constants go away. Or you can just subtract the infinite constant, redefine your energy, redefine your charge, et cetera.

But in those diagrams, the divergences are much more complicated because they come from doing some integration of some complicated expression. And so those divergences cannot be simply removed where you just add the constant, or remove a constant, or just multiply by a factor.

And so they actually cannot be removed. In general, cannot be removed simply by say subtractions. Removed by simple subtractions or divisions. So in other words, the expression you get from doing such integration just don't make sense. So the results from these diagrams are not well-defined.

So the result is not well-defined. So normally when we encounter divergences, we always tell a student we encounter divergences of course, one obvious reason is that you did something wrong. You just made a mistake. But if you have done everything right, when you encounter divergences, actually that's an opportunity. Because each time you see a divergence, then that means there's something you don't understand about physics.

If you understand, by definition, in physics you do not have divergences, if you understand everything well. And so divergence is mathematics telling you there's some important physics you're missing.

So when you look at this divergence, you can say, oh, what physics are we missing here? So the physics missing here is very simple. The physics missing here is very simple. Because these integrals, because the divergent region corresponding to those quantities go to infinity, so those quantities go to the infinity because one is you have very high energy, very large momentum.

In other words, from very short distance physics. But very short, that comes from-- So divergences come from naive extrapolation. They say consider QED to arbitrarily high energy scales. Because we used the framework of QED but when we do this integration involving very large momentum.

But nobody actually tells us that a QED should be valid at not very high energies because the real experiment you cannot probe very high energy process or very short distance because it's the limit of our energy resolution. So there's no reason to trust actually QED should be valid for k 0 and k to infinity. Yes?

AUDIENCE: Is the divergence the result of the problem in the perturbative expansion? Or is it a problem with the original [INAUDIBLE]?

HONG LIU: Yeah, it's not a feature of the perturbative expansion.

AUDIENCE: So it's a legitimate problem.

HONG LIU: Yeah, it's a legitimate problem. So this divergence just comes from we extrapolate QED to arbitrary higher energy scales. So you say, OK, then what do we do? Then what do we do? So the simplest thing you can do you say, I only can measure suppose in the 1930s when people first encountered this problem. And you say, OK, and then let's put the cutoff. Here let's only include the energy regime which we can access.

But again, what cutoff do you put here? So it's not clear what kind of cutoff you put there. What is the validity of this theory? So ideally you should put a cutoff there to add the boundary of the validity of the QED. But before you actually can probe the scale which QED breaks down. Which scale should we put there? So we don't know.

So for many years, people thought just doing these kinds of diagrams just doesn't make physical sense because we don't know the short distance physics. We don't know the short distance physics. And indeed, this point of view was justified. So for some years people just gave up on these ideas. They just said, maybe this does not make sense.

But then after the Second World War in the late 40s, a few young physicists at the time, Schwinger, Feynman in the United States and Tomonaga in Japan, they actually say, OK, even though there are these kinds of divergences, and even though we don't understand the short distance physics, let's see whether we can just use brute force, try to design some mathematical way to get rid of those divergences, and see whether I can get sensible results.

And so that was their starting point, and that was their starting point. And it was a crazy starting point because of the physical reason for those divergences are very clear. But then it turned out that they actually managed to be successful. So in the late 1940s people found the method. So Feynman, Schwinger, and Tomonaga managed to find a way to remove the divergences and create a physically sensible answer from such diagrams.

And also, at the beginning, their work was very complicated and very few people actually understood them. And then Feynman used Feynman diagrams, so he invented the Feynman diagrams to do this. Because previously people do those calculations. They don't use Feynman diagrams and they use very complicated perturbation theory, as you do in the long relativistic physics. It's very complicated, and actually that's what the Schwinger and the Tomonaga they were doing.

So at the beginning, this thing was very complicated, but then Freeman Dyson soon afterwards, he understood all their work and actually greatly simplified the process. And actually, the current understanding essentially came from the simplification of the Freeman Dyson. And the Freeman Dyson actually operated a Feynman diagram fact -- Feynman diagram training -- essentially operated the Feynman diagram training at camp, at the institute for advanced study when he was a member. So people went through there, they all learned this Feynman diagram techniques from Freeman Dyson, and then they spread it out to all other places. That's how Feynman diagram techniques got spread out in the physical community.

And anyway, so Freeman Dyson, he simplified the process, and then people eventually understood. And there is a well-defined mathematical procedure you can actually get rid of those divergences and get a physically sensible answer. And so it was a funny story.

This is called the program of renormalization. The process of getting rid of the divergences. And this is a funny story. It's because, as I said, there's a very physical-- This is a very funny story for the following reason. As I mentioned, we do know where the divergences come from. Divergences come from our lack understanding of short distance physics. And normally you believe you can only cure the divergence if you understand that physics. If you don't understand that physics, how you cure that divergence?

So that's what most people thought at the time, including all these quantum mechanics big shots. All these old people, like Dirac, Pauli, all those giants of quantum mechanics. They all believed you have to do something radical to QED, otherwise QED cannot be made sense. You have to invent some completely new formalism to circumvent those divergences exactly.

But then Dyson later commented that those young people at the time they said we were conservative. We want you just to look at those calculations just want to make sense of those calculations. We don't think of those grand physics. I just want to make sense of those calculations, and then they succeeded.

So this is an example in which very old people, they want to do grand new physics, but young people wanted to do conservative physics, just two calculations, one calculation at a time without clear physical motivation. But those young people turned out to be correct, and so they came up with this program. Yes?

AUDIENCE: So [INAUDIBLE] resolve is more accurate than the tree-level diagram? And then how much more accurate?

HONG LIU: No, it's not about accuracy. It's about finding a sensible way to hide the divergences.

AUDIENCE: [INAUDIBLE]

HONG LIU: Yeah. Because today we will not be able to go into the detail. I'm going to only tell you the result. So that's why their result was actually radical in some way. Because they realized there is some way somehow those divergences, even they are there, they do not affect those observable physics you can measure. They don't affect those things. So you can find some mathematical trick to remove them.

But for many years, so after they succeeded, so this is late 1940s, and the Dyson was also late 1940s, early 1950. And then as I'm going to mention at the end of today's lecture, the first calculated quantity is this anomalous magnetic moment, which now requires doing a calculation of this one. Doing a calculation of this one and Schwinger did the first calculation. And then he calculated the correction of this to the magnetic moment of electron. And that agreed very well with experiments, so that was a triumph at the time. So the experiment verified their success.

But still, many people didn't believe this actually was something sensible. For many years, people thought this is just a mathematical trick to get some answer. And so many people still, for many years people were doubting this program of renormalization. And then only much later in late 1960s or 1970s, when Ken Wilson came up with this idea called renormalization group, and then people finally understood the physical reasoning and the physical basis behind this program of renormalization. So that was a revolutionary progress made by Wilson. So all this will be covered in detail in Quantum Field Theory II.

And so today let me just very quickly tell you what these three loop diagrams do. So first let me make some general remarks. So mathematically, you get divergences and then you have to find a way to get rid of the divergences. And physically, there are also some new features. New important conceptual features. So any questions on this?

So the first feature, I already mentioned. We include such loops. You essentially include the quantum fluctuations of the vacuum. So all this process, so that electron pair, that photon, and that photon they all some virtual particle just come from the vacuum, just from the vacuum fluctuations.

So you can actually show mathematically rigorously. That the tree-level diagrams. Any tree-level diagrams can be obtained from solving your equation of motion perturbatively.

So just take your nonlinear equation of motion, take the QED, you write down the nonlinear equation of motion, and then you have this coupling, e, and then you just solve that equation perturbatively order by order. And now all tree-level diagrams can be understood from that process. Essentially just coming from solving your equation of motion.

But then we know that the quantum nature comes from doing the path integral. Equation of motion captures some classical physics, but the genuine quantum physics come from doing the classic path integral, which you integrate over all possible configurations. And so that is not captured. So those features of path integral are not captured by the tree-level diagram.

But you may ask, does tree-level diagram actually capture any quantum physics? Of course it does. It does capture quantum physics because all this notion of the particles, even the notion of quantizing the particles, that's already included in the quantum effect. Just the tree-level diagram does not include the fluctuations from the vacuum. Good?

And so this is the first feature. So when you include the loop, you actually now finally include the quantum fluctuations. And the second feature, it's very important, is that the physical mass charge and the physical fields are not the same as that appearing in the Lagrangian.

So let me elaborate what this means. So remember, when we write down the Lagrangian for QED, you have i psi bar gamma mu partial mu minus i e A mu psi minus m psi minus one half one quarter F squared. So that's Lagrangian.

And then we say that this is the charge of the electron, and this is the mass of the electron, and this psi denotes the electron field. Means that when you acted on the vacuum, you can create the electron or anti electron. And similarly, A mu because A mu is massless, and when A mu acting on the vacuum, it can create the photon.

Turns out when you include the loop effect, the mass we actually measure. So by physical mass, means the mass of the electron we actually measure in the lab, and the charge we actually measure in the lab. They are no longer the same as the one appearing in the Lagrangian. So the y in the Lagrangian should be just viewed as some kind of parameter. And so and the geophysical mass and the charge are functions of those parameters. Now there's a more complicated relation between those parameters and your physical mass and the charge. Also, the physical field, which actually creates the electron, is no longer directly given by this psi. It's related by psi by some relation.

So this difference is also called renormalization. So here this true sense of renormalization, which is actually used for completely two different contexts. So this renormalization just means the removal of the divergences, and this renormalization means the parameters in your Lagrangian may not be the same as the physical mass and the charge you measure. So the thing you measure actually it's related to the parameters in the Lagrangian in a non-trivial way.

And this renormalization is in principle not due to the divergence. It's just due to the interactions. So heuristically the physical is as follows-- just imagine you have a free electron. So if you have a free electron, and then the mass of the electron, then it is what you measure. But now if you include the interactions, and now electron can interact with the photon in a complicated way, and that interaction then can change the mass of the electron. So that's why, in general for interacting theory, what appears in your Lagrangian, they don't have to be the quantity you actually measure. Because the interactions can change how you translate those quantities into the physical quantities you measure. So this renormalization just means the renormalization changes the value of your parameters.

So now let me just make this a little bit more explicit. So before I do that, do you have any questions? Yes.

AUDIENCE: [INAUDIBLE] looking at QED by itself [INAUDIBLE] any other theories, wouldn't we have to assume that this parameter m corresponds to a physical mass? [INAUDIBLE] how we say [INAUDIBLE]. But you're saying that QED on its own has its way of explaining how this parameter m relates to the physical m?

HONG LIU: Yeah, so when I write down this Lagrangian and then I have parameters here for the e and m, so if I ignore the interaction, then the m will be the mass of the particle you actually measure. But now when you include the interaction, then the interaction can change that mass. It's not related to the Higgs story. The Higgs explains where this m comes from. Hicks explains where this parameter comes from. So this is a renormalization on top of that. Good.

So now let me just elaborate what this means. So for this purpose now let me put a subscript B here on everything just to emphasize that they are no longer--

AUDIENCE: Can I ask a question?

HONG LIU: Yeah.

AUDIENCE: [INAUDIBLE]

HONG LIU: Yeah, it changes the energy you need to create the particle. So now I denoted everything with B, so those are called the bare parameters. So eB is called the bare charge, and mB called the bare mass, and this psi B and A mu B is called the bare fields. And when I write this F square means that the F square constructed out of A mu B.

So now let's just illustrate this thing using these three simple diagrams. So let's look at first that diagram, the propagation of the electron. The propagation of the electron.

So what you need to do is you calculate this diagram and then you find the correction to the propagator. So this diagram will induce a correction to the propagator of the electron. And from this correction, then you find that the-- So let's first write down the tree-level propagator.

So this is given by minus 1 i k slash minus m plus i epsilon. So this is just mB. So you can interpret the mass of the electron as the pole of this k. So when k is equal to mB and then that corresponding to the mass of the electron. And the mass of the electron corresponding to the pole of this propagator. Yes?

AUDIENCE: What does it mean [INAUDIBLE]?

HONG LIU: Yeah, what I mean is just when you invert it, this becomes k squared plus m squared minus i epsilon. It just means k squared equals this. So upstairs we have k slash plus m. It just means k squared plus m square. Just essentially the mass electron corresponding to the pole of the propagator.

So now when you include such corrections, then you find your propagator modified into the following thing-- Z divided by i k slash minus some other mass plus epsilon. And then now this m mass is no longer the same. So this physical mass, it's now just mB plus some correction. So this is a physical mass.

So this tells you that the mass of the electron when included such diagram actually gets shifted from your bare mass. So if we calculate it and then you find the contribution of this diagram is infinite and so that's why you need to include this renormalization. And then to calculate the finite quantity of this delta m.

So you find that the pole of the propagator changed. You also find actually the residue also changed. So here the residue just 1. Here the residue becomes some other constant Z. So that means that when you act psi B on the vacuum, you no longer create just the k-- here I'm just writing schematically-- you're actually creating square root of Z times k OK because of the prefactor.

So remember, our previous psi B is equal to a dagger etc., when you act on the 0 you will create a particle normalized in a certain way. But now with this Z factor, and now you create a lot of the factor now with a prefactor Z.

So we normally define the physical field as the one which psi, when you act on the vacuum, which just precisely created the state. And so now there's a relation between this bear field and the physical field. And now there's a relation between the bear field and physical field through this factor. So this tells you that the magnitude of the field itself also get renormalized.

Any questions on this? So the similar thing is happening for the photon case. So if you consider this process, so this corresponded to a modification to the photon propagator.

So this created a propagation to the photon propagator. So now you find that actually the photon mass is not changed. So even when you include such corrections, the photon is still massless. So remember, previously we say why you cannot add a mass to a photon? So remember why we cannot add a mass term to the Maxwell field? The guage invariance. So the fact that the mass cannot be generated by such correction just means that the gauge symmetry is maintained, so the photon remains massless.

But you do have similar renormalization of the strength of the Maxwell field. So now the bare field is related to your physical field through some factor normally called Z3. Yes?

AUDIENCE: [INAUDIBLE]

HONG LIU: Yeah, this just a constant. Yeah, this is just a constant.

So this is defined to be the residue of the pole of the propagator. Good? So finally, let me mention a little bit the last diagram.

So this last diagram, which I will redraw it here. So remember, if you just have the original one, then here is just i eB gamma mu. And now when you add such a correction, so this is the tree-level one and then at one loop level you can add such a thing. And when you add such a thing, then you renormalize the charge. Then the charge gets changed.

So now you find interestingly that the new electric charge is related. So this is the measured charge, physical charge. Through a complicated calculation you find this physical charge is related to the bare charge by the same factor, Z3, as in the case of the relation between the bare field and the renormalized field. You find that the same number that appears in the relation between the physical electron charge and then the bare charge.

So this is important for the following reason. So now you observe because of these two relation, that eB A mu B is actually equal to e A mu. So the product here is the same because the square root 3 factor cancels. And this is important. Because of this, that means your covariant derivative does not change.

So remember, the structure of this covariant derivative is needed for the gauge invariance. It's needed for the gauge invariance. So that this is maintained, again, is an implication that the gauge symmetry is maintained. In other words, you want to say that if we want to maintain the gauge symmetry, then eB and A mu B, they have to renormalize in the opposite way so that this is the same.

So this diagram has two effects. One effect is correct the electric charge. And another effect is generate the magnetic moment. Generate an anomalous magnetic moment. So this has two effects. The first effect is to renormalize, to change your charge, and the second effect is generate an anomalous magnetic moment.

So you may have learned in your quantum mechanics that e minus has a magnetic moment, which is normally written as following-- mu e equal to minus e divided by 2m g and the spin operator. Just because the electron has spin, spin half, and so it has a magnetic moment. And this spin half of course is normally written this spin operator. It's in the spin half space, then just a sigma matrix.

And this g factor, so this is a g factor which relates the g, just a number. g is just a number. So normally called g factor-- Yes.

AUDIENCE: [INAUDIBLE] diagrammatic expansion of the perturbation theory in powers of lambda. So you're saying you add a term and you change e with the --

HONG LIU: Yeah, the e can change itself Yeah. So you just shift the value of e right.

AUDIENCE: [INAUDIBLE].

HONG LIU: Yeah, change is also small. The change is a higher order. But this is a very good question. This is a very good question because, as I mentioned, naively if you look at those corrections, say this delta m and this change between e and eB, they are all divergent. So that's why they're all very complicated. These are all divergent, so that's why it's non-trivial to find a scheme to make sense of those equations. But we'll have all those divergences flying around. Other questions? Yes?

AUDIENCE: What is the difference between square root of Z and the square root of Z3?

HONG LIU: Sorry?

AUDIENCE: What is the difference between square root of Z and the square root of Z3?

HONG LIU: Oh, they're just different numbers. So this is one number, this is some other number. Just the electron field renormalized in a different way from the photon, but somehow the charge has to be renormalized in the same way.

AUDIENCE: [INAUDIBLE].

HONG LIU: Yeah, but that come out from-- It comes out from the gauge symmetry, from the requirements of gauge symmetry. Good?

So this g factor, so one of the triumphs of the Dirac equation, when Dirac first wrote it down, it said the Dirac equation predicts that g equals to 2. But in reality, in the late 30s and early 40s, the experiments were accurate enough. At the beginning, people thought that g equals to 2, so that was considered to be a triumph of the Dirac equation. But then later, when the experiment became more accurate, they find that g is actually slightly greater than 2.

Of course, this accuracy is later. At the beginning, in the 40s, they can already see that the g is no longer 2. There's some small corrections. So there was a big question at the time-- how do you calculate this correction? So that amounts to calculate this diagram. So that's what Schwinger did. And so he calculated this diagram, he managed to get rid of the divergence, and he managed to reproduce these small corrections.

So now let me just very quickly tell you tell you where this result comes from. Why Dirac equation predicts the g equal to 2. So this actually I had at some point was thinking as a homework exercise, but never got to put it in the homework, but it's an instructive exercise. And so now let me try to explain where it comes from.

So we have the Dirac equation. So now let's consider the Dirac equation. Of electron coupled to the electromagnetic field. So let's consider this equation.

So the claim-- from this equation, of course, we don't see anything about the magnetic moment. We don't see anything about the magnetic moment. So the claim is that if you take this equation and go to nonrelativistic limit, if you go to nonrelativistic limit, you will get the Schrodinger equation, you get the so-called Pauli equation. So you get the Schrodinger equation, and with a spin half particle and with a magnetic moment precisely given by g equal to 2.

So now let's try to see this. Do you have any questions? So this is just the Dirac equation itself, but now if you include that vertex correction, and then that will generate this additional term.

So now let's look at this nonrelativistic limit. So remember, the Dirac equation, as it was originally conceived, was already an equation of the Schrodinger type. So you have the form of the i H t psi equal to H psi. So you just multiply the i gamma 0 on both sides and then remove everything away. So just this side keeps only i partial t.

So through this procedure you have H is equal to m times beta plus alpha p plus eA then plus eA0. eA0 is 0 component of A and eA is the vector component of A. So the beta is just the minus gamma i gamma 0, and alpha i is equal to minus i gamma 0 gamma i, and the p goes to minus i, the standard gradient operator. So now the claim is that if I want to take the nonrelativistic limit, this equation becomes an ordinary Schrodinger equation, including the magnetic moment.

Is this clear what we want to do? So now let's choose the basis for the gamma matrices. So for this purpose, we go to the nonrelativistic limit, it's convenient to use this basis, which actually I wrote down at the beginning when we talked about the Dirac equations. Again, 1 is always 2 by 2 identity matrix. So from here, you find that the beta just equals to 1 0 0 minus 1 and the alpha just becomes 0 sigma sigma 0.

So to reduce-- Right now we just choose then we also need to write psi in terms of the two components. So each of phi and the chi has two components, altogether four components. So let's write psi in terms of two upper components, two lower components.

And now let's plug all this into that equation. All this into this equation. And then we get partial t phi equal to m phi plus sigma dot p plus eA chi then plus eA0 psi. And then for the equation for chi, you get a similar equation, but here it's minus m chi plus sigma dot p plus eA phi plus eA0 chi.

So you get these two equations. So now so far we haven't done anything. We just wrote this equation in a different form. So now let's consider the nonrelativistic limit. Now consider the nonrelativistic limit.

So in the nonrelativistic limit, this p is essentially the spatial momentum if you act on the wave. And so we take it to be mv. And of course, v to be much, much smaller than 1. And then this e A0, which has a unit of energy, so it should be of order mv squared. And this e A, which should be the same order as p, and then e A should also be of the same order as mv.

And now let's look at this nonrelativistic limit. So far I have not done anything. I just specified this. And now let's consider a transformation. So in the nonrelativistic limit we know that your total energy--

So remember, a typical wave function is always the time dependence, so you always have something like iEt, and in the nonrelativistic limit, E would be m plus one half mv squared, et cetera. So this is much, much larger than this term.

So we can just isolate this bigger piece. So we can try to isolate this big piece, so we will make a slight transformation. So we write phi equal to exponential minus i m t capital phi. And the chi equal to i m t x or capital chi. So now this will only contain this low energy part, so those field will only contain the low energy part that already isolated the big things in here. So the time dependence here will only include the low energy.

So we assume that as a t phi would be mv squared, so phi similarly this X. Similarly this X. So this is the fast oscillator part in time, this is the slow part in time.

So now we plug this in to these two equations. Plugged into those equations. Then we find i partial t. i partial t phi equal to sigma dot pi x plus eA0 phi and i partial tx equal to 2 minus mx. Let me just call it x for simplicity. So I also have introduced the notation that the pi is equal to p plus eA.

So now you see the difference between these two equations. So the mass terms here have opposite sign. When I isolate that piece, and then the mass term for this equation canceled, but in this equation they add together, this minus 2m. We get this minus 2m.

So now let's look at the magnitude of each term in this equation. So this is like mv squared X. So this is like m times X. So this is just something phi. We don't know the relative magnitude between X and the phi, and so this is also mv squared X. We said eA0 should be of order mv squared. So these three terms, they're all related to X, and this term dominates over these two. So you can forget about these two terms in the nonrelativistic limit.

So from here, if I call this equation 1 and this equation 2, and from 2 we get in the nonrelativistic limit, we just get 2mX plus sigma pi acting on phi equal to 0. So now we find that X, we can now solve X in terms of phi-- 1 over 2m sigma dot pi phi.

So now we can plug this back into this equation. So this is equation 3. If we plug 3 into 1 again, then we get Schrodinger equation for phi. So X is no longer independent field, it's just expressed in terms of phi. So now we get the equation for phi.

So now you see each term, they are of the same order of magnitude. So this is mv squared and this is mv. Upstairs pi of order mv. So this is m squared v squared divided by m, so this again is mv squared and eA0 is mv squared. So now everything is nonrelativistic and of the same order. So now let's look at the meaning of this term. Now let's look at the meaning of that term.

So that term you can write explicitly. Sigma i pi i, sigma j pi j, but remember pi i and pi j, they are operators, so you need to keep the ordering there.

So we can just write down, so we just have sigma i sigma j just gives me delta ij plus i epsilon ijk sigma k and then pi i pi j. And so the first term gives me the pi squared, and the second term I can just, using the antisymmetric in ij, I can write it as epsilon ijk sigma k and the pi i and the pi j. So I just write the commutator in terms of them.

So now it's easy to calculate pi i pi j. So this is just minus partial i plus eAi minus partial j -- minus i partial j -- plus eAj. So if you calculate the commutator, you just get minus e Fij. And now if you plug this Fij into here, so epsilon ijk times Fij, what do you get? Do you remember when epsilon ijk multiplied by Fij, what do you get? You get B.

So now this equation, when you plug this in here, and then this just becomes pi squared plus-- just make sure. e sigma dot B. Because Fij multiplied by the Fijk just gets Bk and Bk sigma k, just sigma dot B.

AUDIENCE: [INAUDIBLE]

HONG LIU: Yeah, it's one half epsilon ijk times Fij give you. So if you use one half epsilon ijk Fij. OK, good. We are there.

So now we are done, essentially. So now the Schrodinger equation, now we get the partial t phi equal to H phi. Now H is equal to p squared. So now you have pi squared. So pi squared, if I write it explicitly, is p plus eA squared divided by 2m, then plus e divided by 2m sigma dot B, come from this term. And then the last term is eA0.

So now if you recognize this part of the Hamiltonian is precisely what you get when you couple Schrodinger equation to your electromagnetic fields. You just add the p plus eA for the vector potential and here you get eA0. But now here you get the additional term, so this is called the Pauli term. When the particle has spin, has magnetic moment, then you have an additional term.

So this term can be interpreted. So remember, when a particle has a magnetic moment. So when a particle has a magnetic moment, then you have a piece corresponding to mu dot B. If you compare this with that term, then we conclude that the mu-- So in this case, compare that term. That term will be equal to e 2m sigma dot B.

So now we conclude, from here you can read what's the mu. mu just minus e divided by 2m times 2 times sigma divided by 2. So if you compare with this, you find that the g is exactly equal to 2. You find g exactly equal to 2. And so this way, you see actually the Dirac equation contains the information the electron has magnetic moment. It has a magnetic moment. Yes.

AUDIENCE: [INAUDIBLE] would we expect g equal to 2 for all fermions then?

HONG LIU: Yeah, yeah, yeah.

AUDIENCE: Even if we look at these perturbative calculations like the 1 plus 0.001, etc., that should apply also?

HONG LIU: No, no, no. Because the correction can be different for different formulas because different formulas, they have different interactions. And so that will change this vertex differently. For example, muons and electrons will be different. Yes.

AUDIENCE: [INAUDIBLE].

HONG LIU: Well, in the high speed you can no longer-- So when you have high speeds or this effect, they mix together. So relativistically, you can no longer isolate the contribution from the angular momentum from your orbital angular momentum, spin angular momentum. They all come together. So it's only in the long relativistic case you can isolate like this. So normally when we talk about magnetic moments, we always think it about in terms of nonrelativistic situation.

So now just some final words. Just erased that number. So Schwinger then did this calculation. So including this diagram, Schwinger did this calculation, and then he found that there's a small correction. So normally, its called -- say if I call it -- 1 plus F. And so this is the correction.

And then the correction, which Schwinger first calculated, is given by alpha just divided by 2pi. The reason it's alpha is because here there's a e here, here there's an e, so e squared is alpha. So all this complicated calculation you do, essentially you find this 1 over 2pi. Because this correction has to be proportional to alpha, and this number you do a very hard calculation, you find it's 1 over 2pi.

And then this actually agreed with experiment very well. So this is first calculated by Schwinger in 1948. So this was really a theoretical triumph at that time and agreed very well with experiments.

So then people then calculate to higher orders. So the next order must be proportional to alpha squared, so we can parameterize by alpha divided by 2pi squared, and then with some number a2.

You do an even more complicated calculation, much, much more complicated calculation, you find that this a2, again, it's a single number. And this contains seven diagrams. So this is one diagram, but when you go to the next order, actually involving seven diagrams. And that was calculated in 1957. Again, it was a very important thing to do at that time, because just to check whether QED, because the experimental accuracy was at the level actually you can compare this number. Actually, also there's some kind of story behind this number, which at the beginning they calculated wrong, et cetera.

So now you can also calculate the next one, a3. You can parameterize again by cube. And now this now includes 72 diagrams. And it took two physicists-- I won't write down their names-- it took them 25 years to calculate it. So actually they used it. So in 1996 they finally calculated this.

But now if you want to say, oh, let's try to calculate a4, this includes 891 diagrams just in QED. And actually, this is also calculated using computer. And also, you can calculate this for muon. At a certain level, you also need to include the contribution from other interactions, not just QED, weak interactions, et cetera.

Anyway, so the reason you want to calculate such a number to very high accuracy is because this F can be measured to very, very high accuracy experimentally. So actually, experimentally, this is one of the most accurately measured number in physics. They can measure this F to order of 10 to the -12.

I think at the theoretical level, you actually cannot calculate to this level accuracy, but still you can compare to very, very good agreement. So QED actually describes nature very, very well.

So I think that's all. I'm honored and very happy to have this opportunity to take you to this first part of the journey to quantum field theory, and I want to thank you all for making this journey very enjoyable and rewarding for myself, and I hope you have learned something from this class, and whatever your career path takes you, I hope you will later find this course useful. Thank you.

[APPLAUSE]