[SQUEAKING]

[RUSTLING]

[CLICKING]

HONG LIU: OK. So let us start. So first, I'll remind you the master formula we derived before for path integral. We said, if we consider time-ordered anything, OK, so let me just call it x will be, say, some product of operators. And then this can be written as, in the path integral form, D phi. And then so x, it just goes into here. And then you have is, OK? And then divided by D phi just without x just is, OK?

So this is the master formula we derived before, which applies to any theory, OK, in principle, any theory and any x you want, OK? And so things inside this x should always be time ordered, OK? So this T means time order.

So now we can apply this to interacting theory. So for interacting theory, we have S. We write S. So the action to be the free theory action plus the interaction part, OK? And the interaction part, in turn, can be written as say the integration over the interaction part of the Lagrangian density.

And it's also the same as the minus the interacting part of the Hamiltonian integrated over time, OK? So the reason for this simple relation is because we assume the interacting part does not contain time derivatives. So essentially, interacting particles binding to a potential in your Lagrangian density. And then in the Hamiltonian, they just differ by a minus sign. OK.

So now we can just apply this to this case, OK? So let's see. So we can just write this object. So now we can write it as-- also come here. So our goal is that we assume the Si is small compared to S0 because we cannot solve this theory exactly.

And so what we are going to do is we want to treat Si to be small and expand in the cautioning Si. OK, so example, of Si is this lambda phi 4, which we wrote down before. OK. So for this purpose, then we can just write this more explicitly as D phi. Then you have x. Then you have i S0. Then exponential i Si. OK? Then divide it by phi exponential i S0, i Si. OK?

And now so we can now treat this as the integrand for the-- yeah, so this is the integration for the free theory. OK? So now we can integrate this just as a integrand for the free theory, as some, say, x prime. So x times the exponential i Si as another x for the free theory and similarly here, OK?

So we can write this as follows. So we can write this as now in free theory. So upstairs should corresponding to the 0. Yeah. Yeah, let me just not worry about it at the moment.

So now you can write it as the time-ordered x exponential i Si 0. Now this is in free theory. So I put a subscript 0 here. Means in free theory. OK?

So in writing from here to here, so upstairs, we interpreted as a correlation function in the free theory. So here, we should also divide by-- yeah, in order to write in terms of the expectation value in the free-- yeah, so 0 here is the free theory vacuum.

So in writing the upstairs in this form, we should divide this by a path integral just with the exponential as 0. But since upstairs and downstairs we need to divide by the same path integral, they just cancel, OK? So I can just directly write down as the ratio of these two correlation functions. OK?

And so now this is a very nice form, OK? Because so this is-- yeah, let me call this equation star. So this is the star is the exact expression. OK? Which writes correlation function of an interacting theory in terms of those of free theory. OK?

So now these are just some correlation functions in the free theory. So both upstairs and downstairs, they can, in principle, evaluated as we discussed before in free theory. Yes?

AUDIENCE: So I'm a little confused. What is there to time-order for the phase for the e to the i -- with the interaction?

HONG LIU: Right. Right. As we discussed before, whenever you see time-ordered exponential, you should always expand the exponential in power series. And then you just have a bunch of factors. And you just time order them. Yeah. Other questions? OK.

So yeah. And this is precisely the expression, which derived in Peskin. OK, so this is the-- so this equation is the 4.31 of Peskin and Schroeder, which they derived using the interaction picture, OK?

So we see that for the path integral, to derive this equation using path integral is just trivial. Essentially, you just split the exponential for the full theory into that of the free theory and the interacting part. And then you automatically derive this equation. OK? So once you have understood how path integral works and then derive this equation becomes trivial.

And so this is actually general. Later, the same expression, we will be able to apply it for, say, when we include fermions and also photons, et cetera, OK? And now we will evaluate this guy using perturbation theory, OK?

So we will expand star in power series of Si. OK, we treat Si as small. And yeah, just expand it in power series. So more explicitly, say, we consider the Gn, so this Gn for the full theory.

So let me call this Gn, just say this is some n-point function. OK, so this is the Gn. And then we can just, more explicity, we can just write Gn as the 0th order. You just have T X 0, OK, upstairs. And the next order would be i dt say 0 T X H I.

So I have expanded to the first order. In Si, when I expand Si, then become dt times Hi with a minus sign. So I just write it in terms like that, OK? And so remember here, you need to time order them together. Because both of them are in the same integrand, OK? So we need to time order them together, and et cetera. OK, so this is upstairs.

And downstairs, you just expand this guy. So essentially, you have 1 minus i dt. Then you can time-order the Hamiltonian, the interacting part of the Hamiltonian. And then plus higher order. OK?

So you just now essentially do the Taylor series expansion. And yeah. And the first-- so you always assume the interacting part is small. So you can actually expand the downstairs also.

So this will be reduced to, say, the first order. So the 0th order just gives you the free theory n-point function. OK, so I denote now with a subscript 0 means the free theory correlation function. OK? And then yeah, and then you can add the rest, et cetera. OK. OK, so the higher order perturbations. OK. Good? Any questions on this?

Yeah. Actually, for later purpose, let me just write one more term just explicitly. Let me just write it. So if I bring this up, so when I bring this up and then this becomes i 0 T X 0, then times dt 0 T H I.

Also, so this is this term we bring up. And then multiply that term. That will give you that. And then there's another first order term is given by i dt 0 T X H I 0. And then so these are the full first-order terms. OK, so these are the full first-order order terms, which is the correction to the free theory n-point functions.

And we need to evaluate those quantities. And all those correlation functions in the free theory can be just evaluated using the Wick theorem because we know how to-- because all correlation functions in free theory just can be evaluated using Wick theorem. Just use the Wick theorem over and over. OK. So any questions on this?

So now we will develop techniques to evaluate such kind of series, OK? Still, even though this is not too difficult to do now, but still when you do it in practice, it's pretty tedious. So still worth making effort to actually simplify the process, OK, simplify the process. And the technique to simplify is called the Feynman diagrams. OK.

So now we will consider, so far, those equations apply for any theory, any kind of interaction, OK? So now let's work on for a specific case. So let's consider Li equal to minus lambda divided by 4 factorial phi to the power of 4, OK? So it means that H I would be integrated over-- so H I, yeah, so yeah anyway, yeah, the others you can just directly right from here, OK?

So let's consider this case. And so to be specific-- and now also, let's consider-- just in the simplest correlation function-- let's just consider the two-point function. Let's just consider the two-point function, G2. So two-point is also the Feynman function.

But now this is the Feynman function in the interacting theory. So now we have two-- so imagine we have two coordinates-- and then is equal to -- now. So let's consider this.

So now let's consider the Feynman function in the full theory. And now we can just now apply this equation. So let us try to calculate to the next order. So the 0-th order that's given by the free theory, two-point function. So by translation symmetry, it's always a function of x1 minus x2. So this does not depend on whether it's in the free theory or interacting theory.

And now let's just plug in here. Let's just plug in here. So H I is integration of this over a three-dimensional spatial. And then combined, this become just four-dimensional spacetime integrals. So then you just get the-- so this term becomes plus i lambda divided by 4 factorial.

So we will assume this lambda is small so that we can expand this lambda. And this term, just again, is the free theory correlation function. So we just have GF-- again, 0-- x1 minus x2.

And then times-- my board is a little bit-- so we have to go through here-- times-- then you have d4 x then 0, T, and phi to the power 4 x here. So this is multiplied with that. So that's the second term. And the third term is minus i lambda to the 4 factorial. You have d4 x. You have 0, T. Then you have to combine them together.

So this is phi x1, phi x2, and phi to the power 4 x-- and this x is integrated over-- And then 0. And so this is all time ordered. And then the rest is all times lambda squared, plus lambda squared.

Good. Any questions on this? So yeah, so let me just make a quick remark. So in the end, so just note can be-- so yeah. So here, even though we only write down the first-order term, but you can continue.

So all terms in the expansion can be evaluated by repeated apply the Wick theorem. So any such kind of functions, they all factorize into two-point functions. And the two-point functions are essentially just given by GF 0, just the GF 0, free theory two-point functions.

So the final result always have the form-- final result always have the form sum over products of GF 0's-- and plus some integration, so sum of the-- plus possible integrations. So, in general, the structure is pretty simple. They all just reduce to the product of those two-point functions.

So now, as an example, let's just evaluate these two pieces, evaluate these two pieces. So let's continue. So we find that the GF x1 minus x2-- yeah, so GF is the same as G2. So for two-point function, this is just the Feynman function for the interacting theory.

So the leading term is just GF 0. And then now let's evaluate this term. Now evaluate this. So this is all phi contracted on the same point. So there are four phi's. We could just contract them within themselves. So we essentially get to the-- so you have GF.

And then the next term is i, 4 factorial-- i lambda 4 factorial. And then you have GF 0. So x1,2-- so let me use x1,2 to denote x1 minus x2 just to save a little bit effort. And then we have this integration over d4 x. And then let's evaluate this.

So this just reduce to two Feynman propagators. But both of them have 0. So essentially, you just have GF 0, 0 is x minus x. So we just get that, x minus x. So let me just write one of them.

But we are actually missing some factors because we have four phi's. So when you contract the four phi's, there's three different ways to group them. so We should have a factor of 3. There's three different ways to group four phi's into two groups. Then we have a factor of 3.

OK, so now let's look at this term. Now let's look at this term. So this term, now have now we have to contract six phi's, so phi x1, phi x2, and then there are four phi's here. So we need to contract six phi's.

So we can have two possible patterns. Inside this, two phi's contract with themselves. These two phi's-- this phi x1, phi x2 contract with themselves. And then four phi's will have to contract among themselves. Or phi x1, if phi x1 contract with one of the phi here, then phi x2 also have to contract with one of the phi here. So you have two possible patterns for contraction.

So the first one is important, is just first contract phi x1 and phi x2. So again, so we have minus i lambda divided by 4 factorial. So when we contract x1, x2, then we just get GF 0 x1,2. And then we contract phi 4 themselves. So that's what we did before. It's the same thing as above.

So we just get a factor of 3 d4 x and then GF 0. We evaluate at 0 squared-- just the same as this. But then the other term, phi contract with this-- so each of these, x1, x2 contract with here.

So in this case-- so let's imagine this contract with them. Then there are four possibilities. So after this contract with one of them, this contract with that, then I have three possibilities. So you have 4 times three combinatorial factors.

So you have i lambda 4 factorial. Then you have 4 times 3. Then you have d4 x. Then here, we have GF 0 x minus x1-- so x1 minus x. It doesn't matter-- and GF 0 x2 minus x. And then the last one just contract with self, just again, GF 0, 0-- times-- and order lambda squared.

So these are the explicit expressions. So these are the explicit expressions. So now you can actually-- notice that these two terms actually are the same, right? So these two terms actually are identical. And they just differ by a sign, so actually, they cancel. So these cancel.

So we just left with that and then this term. We just with this term. So this procedure can actually be represented diagrammatically, be represented diagrammatically.

And I will just introduce a single line, just introduce a single line for GF-- say, x and y-- for GF 0 x minus y. So we just represent the line like that. So if I use that-- and then we can write them in the diagrammatic form, then the first term is just x1, x2.

Another second term would be 3 multiply 4 factorial. So this is i lambda divided by 8. And then you have x1 times x1, x2 then times-- this contracted with itself. And so you have two propagator. One is contracted like that, and one is contracted like that. And this point is x.

So you have x, but they all contract with itself. So it's x minus x for this one and x minus x for that one. And then similarly, of course, this is identical to that, is i lambda divided by 8. So this is times, so x1, x2 x.

And then the last one-- so this one corresponding to-- so now this is 12 divided by 24. This is minus i 2 lambda-- i lambda divided by 2. So x1 is to x, x and x2 to x. And then you have something like this, x1, x2.

So this here is x. So x1 x, x2 x, and then x to itself. And then you have all the lambda squared.

So this-- again, these two terms-- as we already said, these two terms cancel. And then you just get these two terms, just get these two terms. And we see that each factor of H I or each factor of S I you bring down-- so each factor of H I or each factor of-- you bring down corresponding to a interacting vertex like this, corresponding to interaction vertex like this.

You have four legs or phi's. And so we call this interaction vertex.

So in practice, it's actually much easier to just draw the diagram first, and then from the diagram, to write down the analytic expression. Because the diagram is much more intuitive. Essentially, what's happening here, whatever terms here, you just find all possible way to contract them.

And then diagrammatically, we can just do it diagrammatically through all possible way to contract them. That's much easier to do by drawing a diagram than by doing the counting. So in practice, much easier to draw the diagram first. You just draw all possible diagrams.

And then from diagram, write down the expressions. So those diagrams are just called the Feynman diagrams. So we just draw possible diagrams. And then from the diagrams, we write down the expressions. That's called the Feynman rules. I gave you a diagram. Then there's a rule which convert them into such kind of expression. We can convert them into such kind of expression.

So now let's give you an example. So now let's give you an example. So now let's look at the example in the second order. Let's look at the example in the second order.

So let's consider order lambda squared example. So the lambda squared, there are many terms. So one of the term is just to say you now have H I squared here. You also have downstairs, et cetera. There are many terms. But one of the term-- yeah, let me just say here.

So one of the term just x and then you have H I squared here. This is one of the term. They also have other terms. So let's now look at just that term as example.

So at the lambda squared example, there's a term like this. 1 factorial-- 1 factorial, when you expand this to the second order, then this-- yeah, a 1/2. And then you have minus i lambda 4 factorial squared because now this coefficient also expand to second order.

And then you have d4 x d4 y x1 x2 phi 4 x phi 4 y here. So now one of the term is like this. So later, we don't even need to write down such a term. We just immediately write down all the diagram. Just here, to help you to understand the process, let's just say let's look at this particular term.

So now let's try to write down all possible contractions by just using diagrams, just by using diagrams. So now we have x1, x2, and the four phi-- four x and four y's. So essentially, you have something like this. You have x1. You have x2. So these are the two external points.

And then now you have four phi's coming from x and four phi's come from y. So we need to connect, find all possible ways to connect. We find all possible ways to connect. So that's the goal. So yeah.

So here are the all possible ways-- so here are the all possible ways. So first-- there are actually five in inequivalent way. So let me just write them down.

So the first one is you just connect one of them to x and one of them to x-- yeah, one of the x-- yeah, let me just draw them down. OK. So one of them is the following. So you take one of the x-- so there are five of them. Let me write equal sign here.

So each one of them, let me give a label for later use. So here, you can have, say, x1 and x2. Then they can connect to x. And then x can connect to y. So this is one of them.

And the second one possibility is you just x1 connect to x and x2 to connect to y. And now there are three-- between x and the y, we have to connect. So we have something like that. So this is the second.

So the third-- and the third way, if you connect x1 to x2 itself and then times-- and then you have to connect x and y. So there are four of them. Just connect x and y like this. You can connect them like this.

Or you connect x1, x2 itself. And then you, again, have to connect with x and y. Or you can connect. Then the y-- and that's just between x and y. You can also have that. So some of x is contracted y, and some of x is contracted with itself.

Or you have the last possibility. So you have x1, x2. But then the x just contract with itself. And the y also contract with itself. So any questions on this?

So these are essentially all possible contractions you can have between them. We just draw the diagrams. But now we also have to calculate-- so we also have to find the combinatorial factors between them. We also have to find the combinatorial factors between them.

So now let's try to work out the combinatorial factors. So let me just give you some examples. And then the rest, I will just write down the answer because it just take too much time to do all the examples one by one.

So let's do this one as an example. So yeah, let me just erase here. We need space here.

So here, actually, there are two possible diagrams. I can have x1 and x2 connected to x. I can also have x1, x2 connected to y. Because x and y are symmetric. So I only draw one of them. So that means there's a factor of 2. I can either connect to x or connect to y.

So I have 1/2. So let me also copy this, lambda 4 factorial squared. And then I have a 2 factor. x1, x2 can either connect to x or connect to y. So x1 have four possibilities to connect to x, and x2 have three possibilities to connect to x. And then the remaining phi x then have four possibilities to connect to y.

And the other remaining phi x have three possibility to connect to y. So essentially, that's your prefactors, combinatorial factors. And so if you worked it out, so that give you minus lambda squared 1/2 squared. Yes?

AUDIENCE: Why were there four-- so after we x1 and x2 to x, why were there four remaining?

HONG LIU: Yeah. It's because now you have two phi remaining, two phi x remaining. So two phi x remaining have to connect with y. So the first phi, we have four possibility to connect y because there are four y's there. So that's this factor of 4. And then the other phi x have three possibilities to connect with y. And so that's the factor of 3. And once you do that, then this y just have to connect with itself. Yes.

AUDIENCE: Do we have a diagram x1 to x, to x to y, y to x2, and x loop and y loop?

HONG LIU: [LAUGHS] Which one? Say it again.

AUDIENCE: Wait, so you should just draw it on the board.

AUDIENCE: x1 to x.

HONG LIU: x1 to x.

AUDIENCE: And then one loop of x, x to y. Loop at y. And y to x2.

HONG LIU: And y to x2.

AUDIENCE: Oh, we didn't include the--

HONG LIU: Yeah, let me see. Yes. I should also have a diagram like this. Yeah, it's possible. I think I just forgot it. Let me see. Do I have something like this? No, actually-- yeah, I think forgot it. Good, good, good. This diagram is here. OK, so this is b prime.

[LAUGHTER]

Thank you. [LAUGHS] Thank you. So we can also have that. Yeah. Here, it's really for the illustration. I don't-- actually, I should claim at the beginning, I didn't aim for completeness and just for illustration. Because if we try to be too complete, then that may take too much time.

OK, good, good. This is also a possibility. Yes, good job. And so now let's look at this one. So now let's look at this one.

So again, let's copy this 1/2 minus i divided by 4 factorial squared-- and then the combinatorial factors. Again, I can have the freedom-- so x1 either connect to x or connect to y. That two diagram would be the same. And so I have two possibilities. Here, I only draw one of them because I just flipped them because x and y are symmetric. So I have a factor of 2.

And then the x1 will have four opportunities to connect with four of x. And x2 have four possibilities to connect four of y because x2 connect with y. So I have two factors, 4 and 4.

And so after that-- so phi x have three left and phi y have three left. And then take one of the phi, then have three possibilities you can connect to y. And the other one have two. And then that's it. That's it.

So that all the possibilities. And if you work it out-- so this gives you lambda squared divided by 3 factorial. And then-- so as an exercise, you should work out this one yourself. And yeah, which I didn't work out, but I did work out this one.

So this one I also give you an exercise, so this is minus 1/2 lambda squared 4 factorial squared times 4 times 3 times 2 equal to minus 1/2 lambda squared divided by 4 factorial. And so this one is the-- OK, minus 1/2 lambda squared 4 factorial squared times 2 times 3 times 2 times 3.

So that gives you minus lambda squared divided by 16. So this final one is minus 1/2 lambda squared divided by 4 factorial squared times 3 squared-- and then given by minus lambda squared divided by 2 1/8 squared.

Good? So yeah. So I write them down just for you to check yourself later. So I let you do this as an exercise for yourself, to do this exercise later yourself.

So here, we can already observe some patterns So here, we can observe some patterns. And so you can do this for each individual diagrams. But if you try to count this way, if you try to count it this way, as we are doing here, even though it's not difficult to do, but it's still tedious. It's still tedious.

Each diagram have to count the factor of 4, 3, et cetera. So it'd be nice if you find a better trick. So now there's a better way to do it, so the better way to do it, so better trick.

So it's by notice the following-- so when you go to the lambda n-th order, so there's always-- there's an n factorial coming from expanding the exponential. So because we are expanding the exponential and the-- so there's always a factorial come from the expanding of-- so now at n-th order-- yeah, I think I erased my vertex.

So at the n-th order, you have an n factorial from exponential, but you also have n vertices. Because each power, each lambda comes with a factor of these vertices. So now when you permute all these n, so when you permute the n interacting vertices-- and you also get the factor of n factorial because they're all symmetric.

They're all symmetric, doing the contraction. And if you have one way to contract, and then you can have another way to contract by permuting all these different vertices. So then these two factors cancel. These two functions are also modulo.

Modulo symmetries in permuting vertices-- if there are symmetries in permuting vertices, then, of course, you don't get n factorial. Because some of the permutations give you the same diagram. And so the modulo symmetries in permuting vertices-- then you have this n factorial. So this is the first observation.

So the second observation is that for each vertex which you have minus i lambda divided by 4 factorial phi 4-- so if each phi in this phi 4 contracts differently, contracts differently-- so all these four phi's are symmetric with each other. There's not any phi which is special.

But if each phi contracts differently, then again, you permute the phi's, it should be the same-- should lead to a-- that diagram should also be included. And then you can permute phi. Permutation of 4 phi's then leads to 4 factorial.

So, again, modulo-- so this 4 factorial again cancels with this 4 factorial. So the two 4 factorial cancel modulo symmetries in permuting phi's coming from the same vertex.

Again, so if these two phi's are contracted the same-- of course, when you permute them you don't generate new diagram. You don't generate new ways. So now we have two different permutations. One is to permute the vertices, and the second is to permute the phi's within each vertex.

So that means both of these n factorial and these 4 factorial, they cancel. So we can forget-- so this means forget about the n factorial factor from exponential.

And we also forget about this 1 over 4 factorial. And then treat each vertex minus i lambda, just coming from a factor of minus i lambda.

But then this way, we over-count. So this way, we over-count. And then we need to divide-- so we need to divide. And then divide by symmetry factors from permuting vertices and the legs.

Yeah. OK. So now let's go back to this diagram. So now let's go back to this diagram. So in this diagram, there's no symmetry between x and y. Because x and y, they're not symmetric. So we don't need to divide-- so there's no symmetry factor associated with permuting the vertices.

But for the vertex come from the x, these two legs are symmetric. So there's a factor of 2 permute them. And for this thing come from y-- again, there are two of them-- are symmetric. so That's why we divided by 2 symmetric factors, so this 1/2.

And similarly, from here, again, the x and the y, they are asymmetric. Because one is contracted with x1 and one is contracted with x2. So there's no symmetry factors from commuting from vertices.

But between x and y, there are three legs which are symmetric. When we permute, this three legs, and then we have three factorials. So we have three factorials. And similarly, I leave as exercise for you to do the other diagrams, to do the other diagrams.

So now we are ready to just write down our rule. So now we have drawn the diagrams. And now we can use the diagram to write down expressions.

So the rules of writing down expression from diagrams are called the Feynman rules, the Feynman rules. So here is the Feynman rules. So here is the Feynman rules

So these are all very intuitive. So for each external point-- so here, we have some external points. So each external point, you associate it with a line. So here is the, say, for example x1.

And then you just associate with a factor of 1. But there's always a line coming out of the external point because you have to contract with that one.

And then for each propagator, for the propagator, we already wrote down. You just have-- say, you have two endpoint, then that's corresponding to GF 0 minus y. So each such propagator, you can just write down a factor corresponding to GF y.

And then for each vertex-- I think I can erase them. So for each vertex-- so at point x-- so you associate with lambda and then integration over x.

And then the last step, you divide by symmetry factors, symmetry factors for diagram. So you just draw all possible diagrams using this kind of rule. So for each propagator, you have a line, and then you have vertices.

So now, from each diagram, by applying this rule, we can also write down the analytic expression. We can write down analytic expression. Any questions on this? So exercise for yourself to write down analytic expression for each of them.

So yeah, let me give you an example. So for b, as an exercise, you should try to do yourself for each of them. Let me just give you one example for b. And then you have first the symmetric factors, lambda squared divided by 3 factorial.

And then you have d4 x d4 y come from these two vertices. Yeah, I should just write it like this. Sorry. Yeah, let me just strictly follow this rule. OK.

So we have two vertices. So each vertex, we have i lambda squared. Then you have d4 x and d4 y. And then for each propagator, we associate these. And then we have GF 0 x minus y, then cube.

And then finally, we divide by symmetry factors because there's three permutations of them. So we divide by 3. So this is the analytic expression corresponding to that diagram.

So later, you don't even have to write down such expression in the top line I wrote down there. You just, at each order, you have certain number of external points. And then you just have a number of vertices come from n-th order. You just have n vertices. So you just try to connect all possible lines between them. And then that's all possible contributions.

Good? So this is the Feynman rule in coordinate space. It's also convenient to go to the momentum space. So it's also convenient to go to momentum space.

So to go to momentum space, then we can define the Fourier transform. So let's define the Fourier transform. So suppose we have an n-point function. Suppose we have an n-point function Gn.

Suppose we have an n-point function x1, xn. So this is just the standard Fourier transform. So because of the translation symmetry-- so we expect all the piece must be conserved.

Just from the conservation, I will derive it. But you should also be able to expect on the general grounds from the translation symmetry. So we expect that there must be a factor of the momentum conservation coming from here. And then we call that coefficient Gn P.

So our convention is that for momentum space correlation functions, we defined without this factor, without this factor. Yes?

AUDIENCE: For the Feynman rules, shouldn't it also restrict to connected diagrams?

HONG LIU: Oh, yeah. Yeah, we will talk about that later. Yeah, we'll talk about that later. Yeah. So here, I'm just talk about the general rule. And then we will talk about the more fine points, how to enumerate all possible diagrams.

OK. So when we define the momentum space correlation functions, we extract out this factor. So the reason for to extract this out is from the momentum conservation. So to see this, to see that there is a factor like this, it's very easy.

So because from the translation symmetry-- so the same argument that the two-point function, the x1, x2 should only be a function of x1 minus x2. So for the n-point function from the translation symmetry-- so this, you can just take one point to be the reference point.

You can subtract, say, all coordinate by the value of xn. And then the last argument becomes 0. So you can choose one point that's a reference point and subtract any other point from it. And that should be equivalent. Because it doesn't matter where you choose the reference point. So here, let's choose the reference point to be xn. So this should be equal to that.

So now we do a Fourier transform. So now do a Fourier transform. So you can easily convince yourself-- so when you do a Fourier transform of this object, when you plug this into here, when you plug this into here, and then you get a delta function.

So this as a simple exercise for yourself to do. And so if you plug in here, you find that there's always a delta function. So now I record that the momentum space, the Feynman propagator for the free theory in the momentum space, if you say k, then it's given by minus i k squared plus m squared minus i epsilon.

So now you have a line, but now it's labeled by momentum k. Line labeled by momentum k. So this k denotes the flow of the direction of the momentum, this arrow.

Good. So now, with this rule, you can easily write down what is the-- now you can just translate right. OK. So this is the one point and another point.

And the third point-- so let me just-- in order to go to momentum space, let's just mention several points. So yeah, anyway, here is one point and then here is another point.

And also, now let's look at the vertex. So at each vertex, we have integration. So suppose we have a vertex like this. So here is x And this is contracted with y1, y2, y3, y4. So let's imagine-- so we have a vertex like this, these four phi's contracted with four different y's.

And then at such a vertex, then we will have expression like this, GF 0 x minus y1 GF 0 x minus y2 and GF 0 x minus y3 GF 0 x minus y4.

And now again-- so now when you plug in the expression-- so now if you plug in the expression-- so I will not write all the lines here. I think you will just see the basic idea. So this one, if you write down-- let's write each of them in terms of momentum space.

So this, if you write in terms of momentum space, then you get a d 4 k1 exponential i k1 times x minus y1, then GF 0 k1-- and similarly for that and for that, for that. And now these have exponential i k1. So these have the exponential i k2. So these have exponential i k3 x. And these have exponential i k4 x.

And there's no other x dependence. And then you have a d4x here. So what do you get then is just get a delta function. So you just get a delta function.

So when you do that, when you go to momentum space, then you find that this should always be proportional. So that means momentum is conserved at each vertex.

So if I draw-- so k1 here, k2, k3-- so I use the same convention for all of them. It doesn't matter whether you draw it outgoing or ingoing. But we draw the same convention for all of them.

And then they should have to be-- then they should be proportional to a factor of this. And here, it doesn't matter what is yi. So y1, y2, y3, y4, they can either be some external point or some other internal point. Yes?

AUDIENCE: With Gn of t object, the n-point function in space, can you get that by writing expectation values of the momentum field operators, like pi k1, pi x1, pi x2? Or is that--

HONG LIU: Yeah. Yeah, you can. Yeah, you can.

AUDIENCE: And how would you do time ordering there? Because then--

HONG LIU: Yeah, you-- so this is strictly defined in terms of the-- yeah. So no, you cannot write that way in the sense that-- not as a whole momentum. Yeah. Yeah, no, you can not that way. Yeah.

AUDIENCE: So you to find--

HONG LIU: You really have to find the final expression and then do the-- yeah. Yeah, for the Wightman function, you can, not for the time ordered. For the Wightman function, you can. Yeah.

OK, good. So now we can write down the momentum space Feynman rule. And we will not have time to do an example. Let me just write them down, and then we will look at the example next time.

So yeah. So here is the rule to compute this Gn p1 pn. We use the following rule. So first, for each external point-- and now you just can associate, say, a momentum-- respect to that momentum.

And then you can associate with a factor of 1. And then you draw a line. You still draw a line. But now you associate a momentum with this line. Because when you Fourier transform this line, you will have momentum.

And for each propagator, the propagator is given by this. So now you have a momentum. And then this is given by the momentum space. p squared plus m squared minus epsilon.

And then now for each vertex, you have a factor of i lambda. And then you impose a momentum conservation. at each vertex.

So now, remember, when we do a Fourier transform, each propagator, each line, will have a momentum integration because each one of them is integrated over momentum. So each one of them is associated with a momentum integration. And now, some of those momentum integration can be get rid of by imposing the delta function at the vertex.

So you can get rid of some of them by momentum conservation at the vertex using the delta functions. But now you will have, say, a number of them left. So then you need to integrate over each undetermined momentum.

So each undetermined momentum you need to integrate with this factor. And then the same thing, you need to divide it by symmetry factor. So symmetry factor is the same in coordinate or in momentum space.

So this is the way which you can write down the momentum space expression for each diagram, momentum space expression for each step. Actually, maybe we still have some time. So we have a couple minutes. So we can just do this example. We can just do this example.

So this example, let's see what it looks like in momentum space. So in momentum space, we just-- but I erase the diagram.

So this diagram looks like this in coordinate space of x1, x2 and x and y. Now when we go to momentum space, the diagram is the same. Just now, we label the momentum.

So we have an external line here, a terminal point here. So here, I have a momentum, p1. Here, I have a momentum, p2. So now let's just call, say, this k1, k2. You can assign momentum direction as you want. Say k3.

So now we need to impose a momentum conservation. So momentum flow like that. So that means that the p1 should be equal to k1 plus k2 plus k3-- from the moment conservation at this vertex And from momentum conservation at this vertex, they all go away from this vertex.

And then in this case, then the p2 will be minus k1 plus k2 plus k3. So that means, actually, p2 should be equal to minus p1. So this makes perfect sense because here preserves momentum, preserves momentum.

So for momentum conservation, if you have p1 here, here, you should also have p1. So p2 I drew with a negative sign. And so we have negative p1. Yeah, so using this simplified notation, then we can draw a diagram like this.

So here is p. So let's call this k1, k2. And then this k3 can be solved in terms of the k1, k2. So this is just the p minus k1, k2. So this is the p minus k1, k2. So now we can just write down the momentum space expression.

So we have minus i lambda. We have two vertex. We have 2 minus lambda squared. Then we have two undetermined momentum, k1, k2. Because this p should be-- external momentum should be fixed. So we have k1 and k2. And then we just write down each propagator. So we just write down each propagator.

So we just have minus i k1 squared plus m squared minus i epsilon-- corresponding to this one. And then I have minus i k1, k2 squared plus m squared minus epsilon times minus i p minus k1 k2 squared plus m squared minus epsilon.

And then these depend on the k. And then we have two factors, so this times these two external factors. So this is minus-- they have the same momentum. So we just have p squared plus m squared minus epsilon squared.

And then we divide by 3 factorial corresponding to the symmetric factors. OK, that's it. So this is the expression for this diagram in momentum space. OK, so let's stop here.