[SQUEAKING]

[RUSTLING]

[CLICKING]

**HONG LIU:** OK, so let us start. So last time, at the end of the lecture, we discussed relativistic generalization of the cross section. So we want to derive a relativistic generalization of the cross section from some initial state to some final state. So initial state is denoted by alpha, the final state denoted by beta.

So we will, for most physical process, we will consider the two particles scattering. So this is just two particles. And they scatter. So this is our initial state, always only have two particles. But the final state can have arbitrary number of particles. You can have an arbitrary number of particles.

And then we define, so we require that d sigma alpha beta to be Lorentz invariant. But because the probability for-- that should not depend on say, your frame. We also want to define to be symmetric in 1 to 2.

So to make a connection with the nonrelativistic story, we can consider the rest frame, say-- say as a starting point, we can consider the rest frame of particle 2. You can always go to the rest frame of particle 2.

And then in that frame, then we can define this quantity to be the probability per unit time from alpha to beta dt, and then divided by the incident flux of particle 1. So this is the p alpha beta t is the probability-- let me just write it. So this is the probability per unit time for this process from alpha to beta.

And then, the dN1/dt dA would be the incident flux of particle 1. So this expression is a natural generalization to relativistic context, which in the rest frame of particle 2, in the rest frame of particle 2. And so, our strategy is to first work out this quantity in the rest frame of 2. And then, we use the Lorentz, the requirement of the Lorentz invariant and the symmetric in 1 and 2 to write it in the general frame. So that's our strategy.

And so first, let's talk about this quantity, this probability from alpha to beta. So we have the probability from alpha to beta. So by definition, it's defined as the following. So we want to look at the transition amplitude, which gives you the beta state at time plus infinity and from alpha at time minus infinity. So this is the amplitude. And then we take the square.

And then we divide by the normalization over the beta itself under the alpha itself. We divided by the normalization of the beta and alpha, the state themselves. So that is the whole thing, really defined the probability.

So the upstairs, we already defined before. So previously, we defined the beta plus, infinity alpha minus infinity. So this is by our previous definition is given by 2 pi to the power 4 delta, p alpha p beta. So the p alpha p beta is the total momentum of the initial and final state. And times M alpha beta.

M alpha beta is defined to be the scattering amplitude from alpha to beta, So this was essentially, our definition of the scattering amplitude. Yes?

**AUDIENCE:** T minus-- so the plus infinity, minus infinity, is that in space time or just time?

**HONG LIU:** Time, yeah. it means that the T equal to minus infinity. So this is the Heisenberg state. So this is the Heisenberg picture.

So this means that at t equal to minus infinity, we have a free particle state described by alpha. And the t equal to plus infinity, we have free particle state, given by beta. Other questions?

So now, let's take the square. Now, let's take the square.

So remember, so the reason we need to put in downstairs, you say, why do we need to put the downstairs? Because are they supposed to be normalized? But in practice when we do the calculation, or in practice when we talk about the free particles, we use the plane wave basis. But plane wave, remember, is not normalizable. It's only normalizable as a delta function.

So the normalization is not equal to 1, exactly. So we have to be careful. So that's why we need to include them here.

So yeah. Good? So now, let's look at this quantity squared. So let's just move it down.

So to save time, I just want to copy it. So this quantity squared then just equal to the right hand side square. So we have 2 pi 4 the delta function square. And then we have M alpha beta squared.

And now, let's look at the delta function square. So the delta function squared will have two pieces. So the delta function square-- let me just save time. So this square, then we can take one piece, one copy of it. We just write it as p alpha minus p beta. And then the other piece will be just delta 4, we can just evaluate at the 0.

Because we already set the p alpha equal to beta from the first delta function. So the second factor can be just become delta 0. So now we know how to-- so we have seen this before. So what is this object? Do you remember? Yes?

**AUDIENCE:** Space time volume.

**HONG LIU:** Yeah, it's the total space time volume. The reason is that the 2 pi delta 4 0. So this is the 0 in momentum space. So this is just equal to d4x exponential ikx evaluated at the k equal to 0, evaluated at the k equal to 0.

Because this is giving you 2 two pi delta k. And so you evaluate at k = 0. When you evaluate the k = 0, then of course, this is just the space time volume. So I take the spatial volume to V. And then this T, is the time period.

So physically, you can interpret the T as the duration at the time duration we actually doing the experiment. Because only that part is relevant for our experiment. So this capital T can be interpreted -- OK, good?

And now, we can just replace here, so now we can just replace here by V times T. We can just replace here, V times T. So now let's look at the downstairs. Now, let's look at downstairs.

The downstairs, so the alpha and the beta will be just say-- yeah, so the typical alpha, so the alpha state would be say, if your initial momentum is P1 and P2, it would be like this. For scalar particle, it just will be specified by P1 and P2. And if you have polarizations, then there will be polarizations. Here, I'll just give you an example.

So now, if you look at the normalization of this kind of states, remember the P1 and P2 are defined. So remember, if you have a single particle state of momentum p. So that is normalized to be 2 pi cubed 2p0 delta 3 p minus p-prime, because they're always on-shell. Initial and final state, they on-shell.

So p0 are functions of p. So that's how we normalize them. So p is normalized by square root omega k omega p, say ap acting on 0. And for example, for scalar particle, and then you look at the normalization just like this.

So now if you look at the p with itself, then you have p equal to p-prime. And then you have essentially, you just have 2p0 times V. The V is the spatial volume.

Because here you just have 2 pi cubed the delta 0. And delta 0 is in the spatial momentum. And spatial momentum for the same reason, just gave you the spatial volume.

And so now, we can just apply these two to alpha and beta. So the alpha, So if I take the alpha to have momentum P1, P2, you can see this particle of momentum p1 and p2 on beta, to consist of momentum say, k1 and kn. Suppose there are n particles, so n can be arbitrary integers.

And then from here, then the alpha with itself, would be just 2 E1 times the volume and 2 E2 times volume. E1 and E2 are the energy over the p1 and p2. So E1 is equal to P 1,0 and equal to P 2,0. And similarly, the beta would be to take the product j from 1 to n, 2kj, 0'th component times V.

And so example, just kj0 just equal to kj squared plus mj squared. So here, we allow the different particles to have different mass and similarly with p 1,0 and p2. OK so now with those preparations, now we can write down dpt. So now, the dp alpha to beta dt then essentially just equal to p alpha beta divided by t, the total duration of the physical process.

And so, the reason they just relate it. You say, how can you do this? This is a differential. And how can you just divide it by the t? Do you know a reason why we can just divide by this? Yes?

**AUDIENCE:** Well presumably, it's the probability was small or something. You would just expect it to go up linearly in time.

**HONG LIU:** Just because of-- yeah, that's a good statement. But the statement, I wanted you to say is that because the time translation symmetry. And so, you would expect that the probability per unit time should be independent of time. And so the probability, say for duration of time, and you can just multiply by the total time.

And indeed, in order to make this equation make sense. Of course, the p has to be small. Because otherwise, if you multiply-- yeah, this thing has to be small. Otherwise, when you multiply by T, of course it will be greater than 1 But just to divide by T, it's from the time translation symmetry.

So this is good because we know here, from the delta function, there's a factor of T. There's a factor of T. So if we combine everything together, and then we can just-- then the upstairs is given by, so we have the 2 pi power of 4 delta 4 p alpha p beta, the m alpha to beta squared times VT. And then divided by T, and then this T go away, so can just erase this T.

So this is the upstairs, and then divided by T, a capital T. And then the downstairs, we just copy these two expression. Downstairs, I just copy this expression and this expression. So this is 4 E1E2 times V square and then k from 1 to n 2kj0 times V. Yes?

**AUDIENCE:** So now taking T to be the total time of your experiment, are you taking V to be the volume of your experiment?

**HONG LIU:** Yeah.

**AUDIENCE:** Of your detector?

**HONG LIU:** No, it's not the detector. It's just, we are computing S matrix. So x matrix, you always assume you wait for a long time so that your initial state is free, and your final state is free.

So of course, it's much not. It's the range of your detector. You have to put the detector very far away in order to measure the particle.

You can imagine the V just essentially is the volume, which the experiment is happening. Other questions?

So, you see there's all this V flying around. So they look very unpleasant. But don't worry. Later, if you are doing the right thing, then all these unpleasant things will go away.

So that's the rule of physics. And in the intermediate step, you may see a lot of unpleasant things. But if you are doing the right thing and then all this unpleasant thing will go away, or you say, well, then I must be doing the right thing. So here, actually this probability is not the probability we actually measure. Because here, I assume that the final state-- here I'll assume the final state-- yeah, here, I assume the final states have precise momentum k1, kn.

So in reality, of course, we will not be able to make the precise measurements. So in real experiment, detectors have finite resolutions. So what we measure is actually when we say the particle say, have momentum k1, we actually means that the particle 1, it's within some dk 1 around k1.

So there's always some finite neighborhood of k1 which are allowed by our detector resolution. And similarly, in reality, it's particle 2 in dk2 around k2, et cetera. So the particle n, dkn around kn. So then, we actually need to integrate over all these resolutions.

So that means that we should multiply dP/T. This corresponding to sharp final momentum, and we should multiply by those uncertainties. So that means that, so if I take this equation to be star, this equation to be star, so the one actually, is measured by the experiment. When we say P alpha beta dt, from the real experiment, it's actually corresponding to the star times-- those uncertainties around the-- so j equal to 1 to n d3 kj 2 pi cubed.

So we need to multiply the number of states within each dk momentum space volume, So the number of the state, and the number of states is given by this times the volume. So do you remember where this volume come from? Yes?

**AUDIENCE:** Density of states.

**HONG LIU:** Exactly. So this gives you the density of states. It's because the-- so if I just remind you-- remember if you-- so the way to think about it, just imagine you put your system in the box, and then your energy level will be quantized in terms of 2 pi, say 2 pi cubed divided by the volume. So this is the number of states in momentum space, the density of states. And then you multiply it by the dk.

So this is actually the quantity we are interested in. This is the quantity we're interested in. Any questions on this?

So now, let's just plug that star into here. So now, the nice thing, now we can count the number of Vs. If we are doing the right thing, all the V should cancel except yeah, but not quite yet. So upstairs, we have one V. Here we have V squared. So let's first cancel this V so that we don't have to worry about that. So let's first cancel that V.

And now, we have downstairs, we have V for each j. And the upstairs, now we multiply this by this. And then for each j, have one V. So all this V will cancel that V. All this V will cancel that V.

So we're left with only this single V, this single V. So now, we can write it as the following. So I can write it this as M alpha beta squared. So I just copy this. And then I divide it by-- let's keep this-- divided by 4 E1E2 times V. And then I group everything else into what I call d mu. And d mu is everything else.

D mu is defined to be this delta function. And then, j from 1 to n, d3 kj 2 pi cubed 2kj0. So I just combine these two products, combine these two products.

So the nice thing, the reason I group all this together because now d mu now is Lorenz invariant. So this is a Lorentz invariant measure. Because you remember, this Lorenz invariant. And remember, this combination is also Lorentz invariant from your first day, essentially in QFT, in your first pset.

So now, this nice thing is that this now is a Lorentz invariant. So now, we have this nice expression. We have this nice expression. We have this, by definition, is Lorentz invariant.

And then we have this 4 E1E2, and times V, 4 E1 times V. And now, we want to write them into a Lorentz invariant way. But we haven't done it. This is only upstairs.

This by itself, is not Lorenz invariant. We have to divide it by the downstairs. So the downstairs is the flux. It's the flux.

So now, in the rest frame of particle 2, so lets calculate the flux of the particle. So the flux of the particle-- so we need to calculate the flux of particle 1. So that's the thing we need to divide. So this is the flux of the particle 1.

So the flux is the number of particle per unit time and per area, per unit time, per area. So this is the same as the number of the particle, number density of the particle 1 times the velocity of the particle 1. So remember, the flux is essentially the density times the velocity.

So this gives you the number of particles per volume. And this is giving you the distance traveled per unit time. And so together, they give you that.

So this is the velocity, so this is the density of particle 1. So this is all in the rest frame of particle 2. So now in this experiment, we have two particle scattering. So what do you think will be the density of 1.

So what do you think is the n1? Yes?

**AUDIENCE:** 1.

**HONG LIU:** Exactly. How many particles do we have? We only have one particle. So this is just given as 1/V. So this is just given by 1/V.

So now we can find the d sigma. So the d sigma alpha 2 beta, then is defined to be dp/dt now divided by the flux of 1, particle 1.

So now, this is just given by-- now the volume cancel. Because n1 is 1 over volume. And then, there's a volume here. And now finally, this volume cancel. So now, if we divide that by the flux and the volume cancel, and then we get m alpha beta squared d mu. And then divide it by E1E2, and v1, the velocity of the particle 1.

So v1, of course, can also be interpreted as a relative velocity between 1 and 2. Anyway, so this is the expression we find. So this is the expression in the rest frame of particle 2.

So by definition, we want this thing to be Lorentz invariant and the symmetric in 1 and 2. Yes?

**AUDIENCE:** I was thinking like regarding the cancellation of the 1 over V factor, is another way to interpret it like, you think of instead of a plain wave, like a narrow wave packet or something centered at k1. So you would get like a V from the wave packet sensitive states and that would cancel. Is that valid?

**HONG LIU:** Yeah, I think it's the similar idea. Because let me see-- yeah, because essentially, you get rid of that V. When you consider the wave packet, then you get rid of this V. And then, you also get rid of this V. So you just get rid of the V one upstairs one downstairs, yeah, that's right. Good.

So now, we want to write this in the Lorentz invariant form. So this is emphasized this is in the rest frame of 2. So now, I look for object, so now, I want to look for object, which I will call it sigma. So sigma is Lorentz invariant.

And let me write here, sigma is Lorentz invariant. And the symmetric in 1 and the 2 and in the rest frame of 2, then the sigma becomes this downstairs, become this E1E2 times V1. So this is not the manifest Lorentz invariant object. It's also not symmetric in 1 or 2.

But that we should be able to find the object sigma that by itself, is a Lorentz invariant, symmetric in 1 and 2 and in the rest frame of 2, reduced to this object. If I can find this object, then I'm done in finding the cross section. Yes?

**AUDIENCE:** Should there be a factor of 4 in the denominator?

**HONG LIU:** There should be a factor of 4-- yeah, there should be. Thank you. Yeah, yes, some other questions? Good.

So I want to look for this object sigma. And so you can do a little bit trial and error to find such a sigma. So I will just write down the answer for you. And to save you the trouble, I could have put it in your pset, but I decided not to do it.

So turned out the sigma is given by-- so when you write down the answer, it's very simple. You can almost guess it in a sense. So P1 dot P2 squared minus m1 squared times m2 squared. m1 and m2 are the mass of the particle 1 and 2. So this object satisfies these three conditions, this object satisfies these conditions.

Also, I will leave the exercise for yourself to check it. It's very easy to check it. So you just go to the rest frame of 2. And then you can check if it's reduced to that.

So now, we almost have our final answer for the cross section so the d sigma for these 2 to n scattering. So now, we just collect our final result. So we have the sigma alpha beta now, is equal to m alpha beta squared, d mu divided by 4 sigma, just d mu.

So this is just our final answer. And the sigma is given by that guy, a capital sigma just given by that guy. And so, this is manifested in Lorenz invariant. And the manifest symmetric on the 1 and 2, because sigma is obviously symmetric on the 1 and 2.

And this is a very beautiful formula. Even though we went through a lot of trouble, went through a lot of Vs and Ts. But in the end, we get a very beautiful answer.

So now, let's can see some kinematic regimes of this formula. So now, let's talk about some kinematics of this formula. So it's convenient.

So we have two particles, two initial particles. It's convenient to introduce the center of mass energy for the full system. So we can define something called s, small s. This is not big S. So big S is reserved for action.

So this is small s P1 plus P2 squared. So P1 plus P2 is that of the total momentum of the initial state. Of course, it's also the total momentum of your final state, just from the momentum conservation. And then if I look at P1 and P2 squared, and then this is just essentially the invariant mass for your full system.

So this s is-- the square root of s is the invariant mass. It's the effective mass of the whole system. Of this though, when I say the whole system, I just mean, the whole system of particle 1, particle 2, and also the final state.

So we can also write this sigma, turns out we can actually write sigma in terms of s, because the-- so because P1 dot P2 can be written as 1/2 half P1 plus P2 squared minus P1 squared minus P2 square. So this is just m squared. So P1 squared is minus m1 squared. P2 squared is minus m2 squared.

So this is just equal to minus 1/2 s-- sorry, it should be totaled. S minus m1 squared minus m2 square. so P1 and P2 squared is just equal to minus s. And then, the sigma just then, just can be expressed in terms of that.

So the sigma-- so let me just write down the final expression for the sigma in terms of s, just equal to square root s squared 2s m1 squared plus m2 squared plus m1 squared minus 2 squared squared. So this whole kinematic factor of sigma can be just expressed in terms of s. Any questions on this?

So now, another thing is that often, we-- even though this formula can be used in any frame, but sometimes, depending on your question, the expression is simpler in some frame than some other frame. So one of the very frequently used frame is so-called the center of mass frame.

It's called the center of mass frame. So in the center of mass frame, so the center of mass frame, essentially, it's in this frame that the total center of mass of the system does not move. So it means that the total momentum in the center of mass frame, total momentum-- let me see. The total momentum equal to p1 p2 is taken to be 0.

So the full system is not moving. And so, that means we can take -- so means that P1 equal to minus P2, so equal to, let's call it Pcm. It just means the center of mass momentum. And now, you can find the Pcm from just solving the-- you can also express the Pcm in terms of the mass, it's because E1 plus E2 equal to square root of s.

So this is the total. So in the center of mass frame, the total momentum is 0. Then that means that E1E2-- so here, there's no spatial part contribution. And here, you just have E1 plus E2 squared equal to s. So E1 plus E2 equal to square root s.

And then you can actually solve for p by plus m1 squared plus m2 squared equal to square root of s. So you can now, solve the center of mass in terms of s. So this is a simple equation, which you can solve.

So this is a middle school equation. But turns out the result is very simple. Turns out the Pcm, the magnitude when you solve this equation, you find that this is precisely equal to this sigma divided by square root s. So this is a very beautiful simple formula, given by sigma divided by square root of s.

Or in other words, the sigma can be written in terms of the center of mass, momentum and magnitude times the square root s, times square root s. So now, we can simplify that the expression center of mass frame. So in the center of mass frame, now we have in the center of mass frame, now we have the sigma is equal to-- so let me just save.

Yeah, just you should assume that the subscript alpha to beta, d mu, and then divided by 4, center of mass momentum, magnitude of center of mass of momentum times square root of s. So we have a very simple formula.

So the most of the time, for most questions we are interested in, as you will see in next lecture, so most question we are interesting actually corresponding to 2 to 2 scattering. So now let's specifier 2 to 2 scatterings. So the final state only also only contain two particles.

So in this case, we can simplify the d mu, further simplify d mu. So let's just write down the-- so in the 2 to 2 scattering, you essentially you have some particle come in, say p1 p2, and then you have two final particle k1, k2. And then they're intact.

So now let's write down this d mu explicitly for this case. Let's write down d mu explicitly for this case. So the d mu, just given by-- so P1 plus P2 minus k1 minus k2. And then times, d3 k1 divided by 2 pi cubed 2k 1,0. Let's just call it 2 1 k0 E1 prime and d3 k2 2 pi cubed, then 2 E2 prime.

So the E1 prime and E2 prime, they are just energy for k1 and k2. So E1 prime equal to k 1,0 E2 prime k2. For example, yeah, so I will deload to the mass for the two final particles, m1 prime and m2 prime. So m prime 2 prime will be the mass of the two final particles.

So before simplifying this a little bit further, let us first introduce some notations. So it's often convenient so many of you may have already seen this before, often convenient to introduce the following quantities-- so essentially, they characterize all the Lorentz invariant quantities we can build up from k P1 P2, k1 k2.

So you can have so-called t is defined to be minus p1 minus k1 square, which is also the same from the momentum conservation, p2 minus k2 square. So u minus p1 minus k2 square, the same, minus p2 minus k1 square. So let me just copy the s is p1 plus p2 squared.

So these are the quantities, which are obviously Lorentz invariant. And they are the Lorentz invariant. You can build up-- Lorentz invariant quantities, you can build up from say, p1 p2, k1 and k2. So any Lorentz invariant quantities, which you can build from those four momentum, can be expressed in terms of some combinations of s t u.

In fact, s to u themselves, are not independent of each other. There's only two independent Lorentz invariants. With momentum conservation, there are only two independent Lorentz invariants you can introduce. So s plus t and plus u is actually not independent of each other. You can show.

So this again, I give you the exercise for yourself. You can show that. So if you know any of the two, then you know the s, you know the other one. And anything can be expressed in terms of these variables Any questions? Yes?

**AUDIENCE:** Written down, definitions are useful, for instance, these. How do they correspond to different kinematics?

**HONG LIU:** What do you mean, how do they?

**AUDIENCE:** Feynman diagrams.

**HONG LIU:** Oh yeah, then you Feynman diagrams can be conveniently expressed in terms of them, as we will see in next lecture and in homework. So this quantity this quantity is Lorentz invariant. So that can be conveniently expressed in terms of those quantities.

So now, including the d mu, so now, let's further simplify d mu in the center of mass frame. So this quantity, we can try to simplify further. So we can-- so now, let's consider to do this in the center of mass frame.

So in the center of mass frame, for 2 to 2 scattering is particularly simple, because your incoming particle have the opposite momentum. So one is Pcm. And the y will be minus Pcm and the spatial momentum. And then the final particle, they must also have opposite momentum. Because the total momentum, total spatial momentum in the center of mass frame is 0.

So let's can call this k cm then this must be minus k cm OK, must be minus cm. So for simplicity, let's just call this k cm just k. So now, we can just -- now let's just with this in mind, and let's just look at this d mu. Look at this d mu.

So there are a few steps. So maybe we don't want to go through all the details of these steps. Let me. See I think, yeah, let me just outline some here.

So we'll try to only go through some of the steps, not to doing all the steps. So now in the center of mass frame, then you find this d mu can be written as-- so first, these two pi forth. And then you have two factor of 2 pi cubed. And then you cancel. So you have this 2 pi squared.

Then you have 4 E1 prime E2 prime. Then you have one set of functions. So you have k1, you have E1 prime plus E2 prime minus square root s times delta 3 k1 plus k2 and the d3 k1 and d3 k2.

So as we said, the k3, they have to be equal and opposite to each other. And then you can just evaluate one delta function. And then the other dk will remain. The other dk will remain.

So essentially, you just have-- essentially, we can just forget about this. And then you just have dk, just have dk, d3k. And this d3k, you can write it as dk, the magnitude of k and k square, and then the center of mass -- solid angle the angle in the center of mass frame.

So this k vector can be decomposed into the angular directions and the magnitude. And then, now both E1 and E2 are expressed in terms of k squared plus m1 prime squared and the E2 prime is equal to k squared plus m2 prime squared. So, now you can further evaluate this delta function. Now you can further evaluate this function.

Because the E1 prime and E2 prime, they're just the functions of magnitude of k squared. So you can evaluate this delta function. You have dk over the magnitude of k here. So you can do that, so I will not go into detail. I will not going to detail.

And so, when you do that, when you solve that delta function and then you find from here, then you find that the final answer is given by-- you find the final answer is given by very simple. You find given by k cm divided by 14 pi square square root s d omega center of mass.

And this k cm is defined to be the solution of this equation. So k cm is the solution to E1 prime plus E2 prime equal to square root s. So again, this is expressed in terms of the s. So through some, just technically, just evaluate this delta function. Yes?

**AUDIENCE:** So when we calculate this, are we integrating over all the cases? Is that why we can evaluate all these type of functions?

**HONG LIU:** So you only evaluate the k around the momentum shell. So you can evaluate the delta function, because we always have a finite range of k to integrate over. So it doesn't matter how wide the range of this. Because the non-zero value of the mu is always around that will satisfy the momentum conservation. Does this answer your question?

**AUDIENCE:** Thanks.

**HONG LIU:** Yeah, good. Other questions? Here, I didn't write-- I think, you may wonder here, I didn't write the integral sign. How can I just evaluate the delta function? The reason you can evaluate delta function is just no matter what is the range of dk you integrate over, you will always evaluate around those delta functions. And so you can always take care of them.

So now, if you combine this result and this result, and now, you can write a simple expression for d sigma in the center of mass frame. So if you combine them together, then we find the sigma, the omega center of mass frame, is equal to M alpha beta squared divided by 16, 64 pi square s and k cm divided by pcm.

And the k cm is the solution to this equation. And the pcm is the solution to this equation. So the pcm is the equation to this equation. So they are given by the same equation, just replace the m1 m2 by m1 prime and m2 prime.

So this is the final answer for the differential cross section for 2 to 2 scattering in the center of mass frame, in the center of mass frame. And so this is the expression, which we will use later for certain-- do you have any questions? OK, good.

So this concludes the discussion of the cross section. So it's finally over. It's finally over. But we have to go through this, because this is the kind of thing which we compare with experiments.

And in particular, if you calculate the total cross section, and then you can just integrate this over all the solid angle. So before now, talking about the physical process, there's one more thing we need to consider. So here, we consider the initial state to be two particles. Because we say we don't normally do scattering with more than two particles.

But there's another situation, which still often could happen. So this is the situation, which initial state only have one particle. So when your initial state only have one particle, what do you have?

**AUDIENCE:** Decay.

**HONG LIU:** Yeah, then that is corresponding to decay. Yeah, so we can still have the situation. If you have an unstable particle, then we can decay. And then it's very important for many, many physical situations, to calculate the decay rate.

So now we have p1, say, suppose this is the initial momentum. And decay into k1 plus kn, say final n particles. So now the initial state, only one particle. And the final state is k1 kn.

So beta remain the same, but alpha only one particle. And the decay rate is much simpler to define, it's just the dP alpha beta dt. And we don't need to divide it by flux, all those things.

So now let me explain a little bit. Again, this dP alpha beta, we always mean-- we mean that the probability of P1 decaying into n final particles. with again, particle 1 with range, with the decay 1 around k1 and the decay around k2, et cetera, and decay n around kn.

So when we write d alpha beta, you should imagine this. We already include that. So now, we can just repeat what we did before.

So then, this dP alpha beta dt, dP alpha beta, then just given by this thing squared, beta plus infinity square and alpha minus infinity squared, divided by beta and alpha. And times j from 1 to n, d3 kj 2 pi cubed times V.

So now, it's the same thing. Now, it's same thing. So the only thing you need to -- and the alpha one, everything else is the same. So everything else is the same.

So you just repeat our previous analysis, which I will not repeat. So the only thing different is just now the initial state is just to a single energy. So you just repeat the whole thing. And then you find-- then you find d Gamma alpha beta equal to dP alpha beta divided by t. And then you find that this is just given by 2E1 alpha beta square d mu.

D mu is defined in the same way. D mu is defined in the same way. So this is the final answer for the decay case. And the total decay rate would be you integrate just over all momentum.

So the total decay rate-- so the total decay rate is just gamma total rate. Then gamma is summing over all possible choices of beta. And then you integrate over all momentum.

And the lifetime of the particle, tau is just equal to 1 over gamma. So one thing, one difference with the cross section case-- so the cross action we mentioned before, say it's Lorentz invariant. But decay rate is not. And decay rate does depend on the frame of the particle, does depend on the frame of the particle.

So decay rate-- so this lifetime does depend on the frame of the particle and the rest frame of the particle, corresponding to just E equal to m1, just equal to mass of the particle. And so the rest frame decay rate is the smallest among all possible decay rate because of the time dilation.

In all other moving frame, because the time dilation, the decay rate become faster. So that's what you meant, that the particles are moving and they have a longer lifetime. They have a longer lifetime. So this makes perfect sense. So any questions on this? Yes?

**AUDIENCE:** Do you need a finite set?

**HONG LIU:** Yeah, in general, in general, for the real-- you never know, but beta is what we discovered. We observe what are the decay final product. But you can also predict, from your theory, what are the possible decay? But in real experiment, there always may be some particles we don't know. There may be some hidden interactions.

Other questions? OK, good. So that finally concludes this discussion of the cross section and the decay rate. And now, we can study some process. So we only have 10 minutes to do it.

So we will not really be able to do it, just maybe to start it. So in general, we will consider 2 to 2 scatterings. OK In general, we consider 2 to 2 scatterings. And so one remark to make-- and in that case, we have this formula. We have this very nice formula.

So for particles, with spin, say whether spin 1/2 or spin 1. Spin 1/2 would be electrons, protons, et cetera. And spin 1 would be photon. And the scattering amplitude then will depend on the polarization of the initial and final particles.

So in most experiments -- in most experiments, so the initial beam, they are unpolarized, it's not easy, sometimes to do the polarized beam. So initial state, initial beams are unpolarized-- unpolarized means they're just corresponding to a superposition of all possible kinds of polarizations, incoherent superposition.

And then the final state and the spin polarization, a final state normally, it cannot be detected. It's a difficult question to detect the polarization of a particle. And actually, it's not even-- say if you say, observe a new particle, it's not even easy to tell whether this particle is a boson or fermion.

So to measure the specific polarization is even harder. Anyway, so the polarization of final state are normally not detected. So in this case, when we compare with experiment, so when we calculate this kind of cross section compare with experiment, we should-- that means that we should to compare with experiment. We should sum over, should sum over polarizations of final state. and average over polarizations of initial state.

So for the final state, we need to sum over them. Because we need to sum over all possibilities. But for the initial state all different polarizations contribute. So we need to average over them. We need to average over them.

So for example, if we consider such a process, it's a very important process, say in QED is your annhilation of particles. So if you have a particle and antiparticle, then they can annihilate. When they annihilate, then they annihilate into photon.

And then the photon can split into some other particle and antiparticle. So this is the process of particle a come in. So this is a, a-bar, this is b and b-bar. So this is a production, the pair creation of b and b-bar from colliding and its antiparticle. So in real life, in real life, by colliding, say, for example, electron plus positron, then you can create many particles.

So this is one of the most important way to discover new particles. Many new particles are discovered this way. You just collide the electron and positron. And then you will be able to create new particles. For example, you can create muon and anti-muon. And then you can create quarks and antiquark, et cetera.

So in all these cases, both the initial and final particles, they are fermions. So you need to specify their polarization. So this one -- supposed we have p1 r1. So here we have p2. So since this is antiparticle, let's call it r2-bar. And the b would be called-- say this is k1 s1. Say this is k2 s2-bar.

And then for the unpolarized process, let me just write down one last formula. So you need to say, suppose the scattering amplitude is M squared, so M. And then we need to, for unpolarized process, so we need to consider-- we need to average initial spin, r1, and average over the final, the r2.

And then we need to sum over s1 and the sum over s2. And then, I have Musk squared. So essentially, this becomes 1 over 4. And you sum over all possible spin polarizations of the M squared.

So this is the one we can say compare with experiments. So next time, we will write this down explicitly for this kind of process. Yeah, so let's stop here.