MARKUS KLUTE: Welcome back to 8.701. So in this lecture, I'd like you to have a first connection between particle physics and the Lagrangian formalism. In classical mechanics, you have seen that you can write down the Lagrangian using the kinetic and the potential energy of a particle, and from that derive equations of motions.

In quantum field theory, you can translate this idea and derive Lagrangian densities. It's beyond the scope of this class to do all the mechanics of this. We'll visit this topic later in the class when we introduce the Higgs mechanism, for example. And we'll be a little bit more systematic then.

I'm introducing the topic now because it allows you to answer one of the homework questions. So you can just follow this lecture and then you should be able to answer the first question of the second p-set.

All right, so you just have to trust me at this point that you can write the Lagrangian for a Dirac field, or Lagrangian density for a Dirac field this way. One exercise would be to use this and show that from this Lagrangian you can derive the Dirac equation for a spinor field.

But that's not what we're trying to do here. You're trying to see what's the effect is of having this Lagrange density being invariant or unchanged under global symmetry. So we are able to rotate our spinor field with a global phase. And we will see that the Lagrangian doesn't change and the consequence of this, which is if we can.

This is exercising Noether's theory. There's an overarching global symmetry. And out of the symmetry follows the conserved property-- in this case, the current.

All right, so we can express the symmetry with infinitesimal phase transformation, as shown here, for our fields and for our adjunct fields. For the field and the derivatives, then, you just have to do the math and we find those expressions, which we can then put back into our Lagrangian.

First of all, we write the change of our Lagrangian in this way. And as we just have seen the Lagrangian, its invariant under this transformation, and therefore the change is going to be 0. So then we use this information at this in the equations. We find this very complicated-looking set of equations.

OK, so now we get this. And then we can rewrite the terms. So this is already with a vision of what we would like to actually find later. So if we now look at the terms involving the derivative with respect to du mu of our spinor, we can express this equation as shown here.

And with that, we find the next equation. I only show this for the spinor not for the adjunct spinor. This looks exactly the same. But you, however, find in this part here, this looks like Euler-Lagrange equation. And this part needs to be 0. So we only have to worry about this part of the equation, and the same for the adjunct field.

So this then leaves this equation here where we have i epsilon, a derivative of this part of the equation. And something like this you have seen before in our continuity equation, something like this. It's our continuity equation. We discussed this in one of the last recitation session, which leads us then to conserve currents.

And let's go one step further. If we now identify this part as our current, we can then use the partial derivatives of our initial Lagrangian. Now we're just calculating those terms here. And we find that our current, our conserved current is given by the adjunct spinor, gamma mu spinor.

And so what we have just seen-- and this is conserved, so the derivative is 0-- have seen that we have a Lagrangian density, we have a global symmetry. And out of that, we find that the current is conserved.

So this is all I wanted to show here in the homework set now. We start from a different Lagrangian. So this is our Lagrangian for a massive spin half particle, which satisfies the Dirac equation. In the homework, we are looking at a scalar particle, a massive scalar particle. And the exercise, however, is very much the same.