MARKUS KLUTE: Welcome back to 8.701. So in this video, we'll talk about the nuclear shell model. We've already seen an interesting empirical model to describe nuclear binding energies-- the liquid drop model. But it comes short in the description of all aspects of the nucleus. So let's see what we can find here.

First of all, you probably remember shell models from atomic physics. And shell models are very successful in describing hydrogen, for example. The question is, can this also work for the nucleus? After all, the nucleus is a many-body system, compared to hydrogen, where you have a proton and an electron circling around. There's no analytic solutions, like the Schrodinger equation. There is no dominant center for a long-range force, like the proton has been the dominant center. And we have short-range forces with many pairs of interacting nucleons. And I can continue the list of difficulties.

On the other hand, the interactions kind of average out and result in a potential which depends only on the position, but not on the timing of the nucleus. And that leads us, then, to what we call a nuclear mean field. So on average, our proton and our neutron inside the nucleus sees a specific potential. And we can use that, then, parameterized as potential with a harmonic oscillator, and use that model, then, in order to describe our nucleus.

So this works, actually, surprisingly well. But before we go there, we'll look at experimental evidence for closed nuclear shells.

So again, here is our plot of the binding energy. And you see that there are those areas here that seem to be some sort of higher binding energies. And it turns out those happen at so-called magic numbers. Magic numbers are 2, 8, 20, 28, 50, and 126. So the question now is, how can we explain this? Where does this come from?

So again, the experimental evidence is numerous. We find that the number of stable isotopes or isotones is significantly higher for nuclei with a proton-- or neutron, or both-- numbers equal to one of those magic numbers. The nuclear capture cross-section, meaning the likelihood to capture a proton or a neutron, are high for nuclei where exactly one nucleon is missing from a magic number. But it's significantly lower for nuclei with number of nucleons equal to the magic number, meaning that there is this concept of a closed shell. We either just add a nucleon to close it or you have to pay a higher price.

The energy of excited states for nuclei with a proton or neutron number equal to the magic number are significantly higher than for other nuclei. And these are all experimental observations. And the excitation probabilities of the first excited states are low for nuclei with a proton-- or neutron, or both-- numbers equal to the magic numbers.

Quadruple moments-- we haven't discussed those at length, but you can think about them as deformations of the nuclei. They almost vanish for nuclei with proton or neutron numbers equal to the magic numbers. So those are more kind of sphere kind of objects.

Here's a plot which shows or points out the double magic numbers-- as in, those are a nucleus where both the proton number and the neutron number are laying on the magic number. So calcium here has two of those, with 20 protons and 20 neutrons, or 20 protons and 28 neutrons. And there's alphas. Those are specifically interesting object of research.

There was some historic confusion in this, and it came from the fact that while the experimental data pointed to nuclear magic numbers of 2, 8, 20, 28, 50, and 126, if you just think about a flat bottom potential, just a flat potential, you find magic numbers which are 2, 8, 20, 40, 70, and 112. And those are typically not in agreement. So therefore, it seemed like that this shell model kind of worked, but not really. We found agreement here, but then disagreement in the higher part of the magic numbers.

So something was missing. And so what was missing was the spin-orbit part of the discussion. We alluded to this in the nuclear force. What you have to do is, beyond three-dimensional harmonic oscillator, you have to add the spin-orbit coupling to the Hamiltonian. And when you do that, you change the orbit such that the magic numbers agree with the experimental data. So you see here, the potentials for proton, which has also the Coulomb repulsion added, and the nuclear potential, and then you see that the spin-orbit coupling slightly changes the potential.

All right. As a comparison here, the nuclear and atomic shell models, just for an example. And you see we call them shells because we see that the energy gaps between individual shells are quite large, much larger than within the shell. And the same-- this is for the atomic model. And for the nucleus, you see very similar. So it's not that extended, but still larger gaps in energy when you go from one state to the next.