IAIN STEWART: [INAUDIBLE] theory graphs calculating the effective theory graphs using the same infrared regulator, renormalizing each of them separately, and then subtracting them to figure out higher-order corrections to the Wilson coefficients. And as part of that discussion, we also talked about scheme dependence because when you do that procedure for normalization in the effective theory, you're making a choice for the scheme. We picked m s bar, but you could pick other choices.

And at the end of lecture, we talked about the fact that when you make different scheme choices, it can affect things. So it can affect the Wilson coefficients. It'll affect the matrix elements of operators. It'll affect your matching coefficients at one loop and your anomalous dimensions at two loops, but those scheme dependencies cancel out in observables.

So it's important to take the scheme into account and work in the same scheme. But if you consistently use the same scheme for everything, then you will be OK. Now, if you're just doing calculations on pen and paper, it's very easy that you just-- well, we love the m s bar scheme. Let's use that.

But remember that sometimes, there will be things in your result that you may want to get from elsewhere. For example, maybe there's some matrix elements of the operators. You want to get them from the lattice.

Well, if you look up results in a lattice, they're not using m s bar. There's no non-perturbative definition of m s bar. So they'll be using some other scheme. And those results will have to be converted to m s bar if you're going to put them together with your results. So that's just something to be aware of.

Before we leave this topic and go on to something else, I thought I'd say a few words about phenomenology. What is all this technology good for? So a nice example of that is beta s gamma. So beta s gamma-- it's a neutral current process. It doesn't happen in the Steiner model at tree level.

And therefore, what that means, if it doesn't happen at tree level, is that it's sensitive to loop corrections. And you could go-- since we talked about a channel with b to c, you bar d, but this is very analogous. We have a b meson in the initial-- b quark in the initial state.

So it's the same scales in the problem. Effectively, the b quark scale is the light scale. We want to get rid of the things that would mediate this decay that are inside the loop, which will be w bosons and top quarks in this example.

And so you draw this in the standard model. There's various diagrams that contribute. One is this one.

So here's a b quark changing into a strange quark through a top quark and a w. And if we integrate out the w in the top, then we get some local operator, much as we were talking about in our examples up here. It's just that we have different diagrams.

And we can also, from the low-energy point of view, enumerate the operators. For some reason, I started calling it Q instead of O. Let me give you an example of some of these operators.

So here's what you would call a magnetic dipole operator, couples directly to the photon, which is in the F mu nu. And it takes the b to an s bar. And that's an example of a higher dimension operator.

Remember, this is dimension 2 and then plus 3 there, so that's 5. There's a factor of the b quark mass that just comes in because of the chiral structure of the operator, which you can think of as there needs to be a mass insertion here. You need one factor of that mass in order for the diagram not to give 0.

If you really start to do beta s gamma, and you want to go through the whole story, then you have to actually think about other operators. And there's a whole basis of them. And one thing you could do is you could take the operator up there and just replace the photon by a gluon.

That won't give a tree-level contribution to beta s gamma, but these guys are charged under electromagnetism. So you could just have a direct coupling to the b quark or the strange quark, and then loop up the gluon, and get a correction from an operator like this one. And then there's also four quark operators, like the ones we talked about before, but with different flavors.

And if you enumerate all of them, there's nine more different ways of making four-quark operators. So you could build some basis using the equation of motion to simplify the operators as much as possible. And then you get down to these ones.

And you can go through the program that we talked about, of doing matching and renormalizations with evolution. And really what I want to talk about here is a little bit about phenomenology, because the interesting thing about this loop is that if there was nu physics-- since in the standard model at tree level it doesn't happen, if there's nu physics, then you can be sensitive to it here, because you're sensitive to heavy particles in this loop. That's why beta s gamma is always used to constrain nu physics.

So in our effective theory, we have just an operator that does this, which is this 07 gamma. And if you think about what C 7 gamma is at lowest order, let's just calculate that diagram over there. And the result from doing that will be some function of m w over m top, because those are the things that are appearing in the loop. And if you do that calculation, you get a number like that, if you stick values for the top m w.

And then you can start doing loop corrections. The first thing you might think about, which is actually not even suppressed by any factors of the coupling alpha s strong, would be to just loop up the Q1 operator, which I guess I don't know why I wrote it this way. This should be b.

You could just loop the Q1 operator, contract the u u var, and then you can get a beta s transition from this operator. So this is the u quark here, and then attach a photon. And there's no factor in that loop of the strong coupling, because this is an electromagnetic coupling, and this guy here is just this Q1 operator.

So this doesn't look like it's loop suppressed relative to that. And this is a little subtle, but this guy actually is 0. So you have to use a good scheme for gamma 5, but it turns out to be 0.

So the first type of loop corrections that you get, which are suppressed by a factor of alpha s, come from diagrams like-- well, there's various diagrams, but one of them is like this, where I take the same diagram there and I attach an extra gluon on top of it. This guy diverges. And this two-loop calculation is order alpha strong, and it gives the leading order anomalous dimension, what we were calling gamma 0.

It's not the only diagram. There's other diagrams, too. So we could go through that and do the similar type of thing, just with more diagrams than we had in our example. In particular, we could construct from the tree level matching in the one-loop anomalous dimension, the leading log result. Let me just write that down for you. Putting in numbers for the anomalous dimensions, at least sometimes putting in numbers for them.

So the eta factor here is the ratio of alphas. So this is similar to what we saw before in our example where we got a ratio of alphas raised to a power. And if you want to pick a mu for this process over here, beta s gamma, then the right mu to think about is m b. So we want to take mu to the m b.

And so if I plug in numbers-- and this is really what I wanted to emphasize-- if I plug in numbers here for these various things, this guy here gives a factor of 0.7. This guy here is this minus 0.2 that we talked about up there. This factor here is a bit small, 0.085. 0.96. That's right.

And then this piece here is a substantial correction. And if I wrote down all these numbers correctly, then the final result comes out to be, if I keep three digits, minus 0.3, which you can see is a fairly substantial change from minus 0.2. It's a 50% change.

So just taking into account the evolution gives a 50% correction. So if you didn't take it into account, and you just said, well, forget about effective theory. I just calculate this graph. That's the standard model, you'd think that there is nu physics.

We've certainly tested beta s gamma at better than 50%, more like the 10% level. So you really have to take into account these effects that we've been talking about, like this leading log evolution, if you want to look for nu physics, because you've got to get the right standard model result. 50% larger.

And actually, people go two orders beyond what I'm talking about here. They go to the next, the next leading log order, when they really do precision beta s gamma physics. So some of the state of the art calculations of multiloop diagrams have been done exactly for beta s gamma because these effects are so important for looking for nu physics.

And it's even worse when you get it put it in the branching ratio, because a 50% enhancement in a coupling when you put it in the branching ratio, you're squaring the amplitude. So that's a factor of 2.3. So these are really crucial corrections to take into account. So that's what this electroweak Hamiltonian is actually used for when you do phenomenology.

So that's what I wanted to say about the electroweak Hamiltonian, just to give you a flavor for it. There's lots more that you could do with it. We could talk about more phenomenology, but let me stop there, since the idea is to give you an introduction to the concepts and we've done that.

So now we'll move on to something else, which is a different concept, unless there's any questions. So all this business of schemes and stuff comes in when people are talking about beta s gamma and making this kind of model prediction. And there's actually schemes you can pick where you mess this up, but then, if you're careful, you get the same answer in the very end.

So the next topic that I want to talk about is an example of something that's bottom up effective field theory. We've talked about top down with this example of removing heavy particles. And the classic example of bottom up is chiral perturbation theory or chiral Lagrangians.

So our purposes here are not-- again, they're not a full exploration of this topic, which is a very large topic. So what are our goals? Bottom up effective theory example. We will also see in this example the utility of using something that's called the nonlinear realization of symmetry, non-linear symmetry representations, and the kind of connection to field redefinitions. Since field redefinitions is something we've been talking about, we'll talk about that.

Another thing that this is an example of is an example where loops are not suppressed by the coupling constant. Instead, they're actually suppressed by powers in the power expansion. So that's kind of totally different than what we saw when we were integrating all the heavy particles, where you just put a loop. You're down by some factor of alpha s, but you're at the same order in the power expansion and 1 over large scale.

This is going to be different. And therefore, in some ways this has a non-trivial power accounting-- more non-trivial, anyway, than what we were just talking about. So we'd like to give this as an example of non-trivial power accounting, and in fact, prove something that's called a "power accounting theorem."

What the power accounting theorem means is that ahead of time, you should be able to figure out from your effective theory what order various things are. If you draw a diagram, you should know even before you calculate it how many powers in the power accounting expansion you have. If you didn't know that, you wouldn't know which diagrams to compute and which ones not to compute. You'd just have to compute them all. That's, of course, way too much work.

So in order to formulate the effective theory-- especially since there's an infinite number of diagrams. So in order to formulate the theory, remember, you really have to have power accounting under control. And that means you need things like this in order to identify what's leading order. So we'll talk about that.

So I'm imagining that maybe 50% of the class has seen chiral perturbation theory before in some form. I know I teach it in Quantum Field Theory 3, so if you took QFT 3 with me, you saw a glimpse into it. The things we're going to emphasize here are a bit different, but I am going to assume that you have knowledge of it at some level, and I'll assign you reading if you don't.

The other thing I'm going to assume that you have some knowledge of is spontaneous symmetry breaking, because that's not the topic. That's not one of the things that I've listed here. That's not something that I really want to delve into, but of course, if we talk about the chiral Lagrangian, that's something that comes in, in particular, when we're talking about it from QCD.

So I'll remind you of some things that are hopefully familiar, and anything that's unfamiliar, you should do additional reading on. So I know that there are some people that are taking QFT 3 right now, so this may be something they haven't got to yet. So I will do some review, but for further reading, you should see QFT 3 or some other source. I've posted some readings on the website.

So if we start with QCD, massless QCD, we can divide it into a left-handed part and a right-handed part. And then we can talk about the transformation where we take the left-handed field and transform it by a separate amount than the right-handed field under a unitary transformation. And that's a symmetry of the theory.

Now, exactly what symmetry you have depends on how many components you're talking about in the psi field. If I say the psi field is light, then you might think of up and down, and that's one possibility. Or you could throw the strange in there, and say, well, the strange is light, too, and then you have up, down, and strange.

And that just the difference between SU2 and SU3, and both of these are viable choices. So let's make a little table. We have a group, which is the transformations given by the left and the right. And it's going to be broken spontaneously to some other group.

So it could be SU3. So there's saying that the left and the right are in SU3, each of them. And in QCD, that's broken to the vector subgroup. So in this case, the psi would be u d s.

And you get Goldstone bosons from that. And there's eight of them. And that's the pions, the kaon, and the eta. And keeping thing from our perspective what is the expansion parameter, and it's not so great, because the strange quark mass is not that light.

So you can think that the strange quark mass over lambda QCD is maybe something like 1/3-ish, but you're not really getting better than that. So there was 8 generations here, 8 generators here, broken to 8 here. And then you have 8 Goldstones.

You could do better, but make less predictions, if you just considered SU2 and just maybe up and down light. Then you just have the pions. And then you just have MU and MD over lambda QCD, which is more like 1/50, so a much better expansion. But then you can't do kaon physics with this, because the kaon is not part of the effective theory, and that can be the case.

So in order to construct a theory for the Goldstones, which are the light degrees of freedom-- so this is we've identified the light degrees of freedom-- you'd like to construct an effective theory for them. Those are bound states of the particles in your original theory, and you don't know, on pen and paper, how to calculate the matching. And that's characteristic of a bottom up effective theory-- that you don't do the matching, that you just start from the bottom up. So in this case, you don't do the matching because it's non-perturbative.

So what you do is you say, let's construct a type of field theory based on the fact that I know what the degrees of freedom are and I know something about symmetry. And in particular in this case, you know something about symmetry breaking. So one kind of logic is, then what you'll get is you'll get coefficients times operators again.

The operators will be built out of your pion/kaon/eta fields. And you'll be able to calculate matrix elements of these. But you'll get some coefficients and you don't know the value.

So these coefficients you could fit to data, because you haven't determined them from some high-level theory. You just could fit them to the data and do phenomenology. So it's kind of exactly the opposite from our high-energy point of view, from the theory with the electroweak Hamiltonian, where we thought the matrix elements were going to be non-perturbative.

Here are the matrix elements. They're something you calculate. And the coefficients are including the non-perturbative physics.

You could also get these from the lattice. So a lattice QCD calculation can tell you the values for the C's to use. And then you could just use the chiral Lagrangian to do phenomenology.

Now, because we have this bottom-up point of view, there's something that we don't know. And that is, we actually don't know precisely what theory we started with, because all we're encoding about that theory is the symmetry breaking pattern. And so if there's a bunch of upper-level theories that have the same symmetry breaking pattern, they all look the same. They'll look like they have the same chiral Lagrangian.

And the thing that distinguishes them is that they would have different coefficients. And you wouldn't know that unless you figured out what the coefficients are. So that's another way of just saying that the high-energy physics is being encoded in the coefficients. So the same chiral Lagrangian would show up for different high-level theories that have the same symmetry breaking pattern.

So this is just one example of chiral Lagrangians. And we're going to use it as an example, which is perhaps the most familiar, to illustrate our bullets. So one of our bullets was related to non-linear representations in field redefinition, so let's start with that one.

Problems set's due today. Problem set 2 is posted.

So let me talk about something called the "linear sigma model." And we'll use this as an example of something that has a symmetry breaking pattern that's similar to the one we want, same as the one we want. And we'll construct, from this, the chiral Lagrangian. And since it's the same symmetry breaking pattern, it's also a viable chiral Lagrangian directly for QCD.

So what is the linear sigma model? So I'll talk about fields that I call pi without a vector symbol on top. They have two components. One component that's in the diagonal, then one component-- these are matrices in the off diagonal, and different entries in the sigma 3.

So I want to think of this is a kind of full theory. So the full theory for the sigma model is the following-- kinetic term, mass term, interaction term. And I can couple it to something else like a fermion to make it more interesting, with Yukawa couplings.

And this theory has an SU2 left cross SU2 right symmetry, where I take psi left to L psi left, psi right to R psi right. And I also transform pi to L pi R dagger. And if you think of these L's and these R's, you should think of them as something with some generators.

So L would have something like that, with tau left generators, and then some parameters alpha L. If I worked infinitesimally in those parameters, then if I expanded this out, this would give a linear transformation to the sigma and the pi vector. That's what it's meant, that it's linear-- so a linear infinitesimal transformation to pi and sigma.

So this theory has spontaneous symmetry breaking. And if we write out the potential by taking the traces, and we can write in a way that makes that obvious by completing the square. And we can always throw away irrelevant constants when we do that.

So we get something like that. So the minimum is shifted. And so if we want to shift ourselves over to describe perturbations around that vacuum, then we can do that.

So let this guy have a VEV, which I'll call curly V, square root mu squared over lambda. Pi vector field has no VEV, because we want to maintain the vector symmetry. And we talk about a shifted field, which describes perturbations around the VEV when we do quantum field theory in the sigma twiddle.

And then we write our Lagrangian in terms of the sigma twiddle, just by making a change of variable. And that makes clear who's getting masses when we expand around that vacuum and who's remaining massless. And of course, the Goldstones are remaining massless, and the Goldstones, by clever choice of notation, are called Pi.

Let's see if I can squeeze it all in here. This is something they tell you in Professor 101 never to do, but I'm going to do it anyway. Since if you can't read something, you can always look at my notes when I them.

So I didn't write all the fermion notes down again, but I could do the field redefinition in those modes as well, the shift in those terms. And the vector subgroup, which remains unbroken, is no transformation for sigma and the Pi field has left equal to right, so we just get some matrix V on both sides.

And if we look at the Lagrangian that we have, then we can identify the masses of various things at tree level. And I didn't write down the fermion part, but the fermion gets mass 2 from the [INAUDIBLE] coupling, just like in the standard model. And the Pi, of course, remain massless.

And the idea of thinking about this is an effective theory is that we can take these to be large. And then there's a clear separation between low-energy degrees of freedom and heavy degrees of freedom. So if we want to describe this with an effective theory, we would like to describe the physics of the pions without worrying too much about the sigma and the psi.

So we could do that just by thinking about this Lagrangian, but it turns out that you can make field redefinitions and think about using different formulations which are entirely equivalent. And some of those are more useful than this linear sigma model. So we're thinking here of field redefinitions as an organizational tool. So you'll see what I mean when we go through this.

So let's consider some different choices. So there's something called the "square root representation." So I just make a field redefinition like that-- involves a square root.

And if I expand it, it starts a sigma twiddle, and then it goes a bunch of other terms. So it's certainly within the realm of the things that are allowed by our field redefinition theorem. And then I also talk about making a field redefinition for the Pi as well.

And again, this is Pi plus other terms. So in this square root representation, we're going over to these fields S and psi. So these are our new fields.

So we can make that change of variable and just write out our linear sigma model again. And it's the same theory, it's just field redefined. And it looks kind of Godawful, but it's more beautiful than it looks.

So that's something you don't want to do more than once. So that's one possible way that we could deal with the effective theory, is in terms of these variables, S and phi. Let me write down a couple other ways, and then we'll talk about which we might pick.

So there's something called the "exponential representation," also very common. These are very common in the literature, where instead of using S and phi, we use S and sigma. So we take our original fields, and we rewrite it as v plus S times sigma.

And then we think of sigma as the exponential, and hence the name, of our fundamental Goldstone field. So this is not the same Pi. It's some Pi prime, if you like. But I'm going to drop the prime. So you write everything in terms of S and sigma, and you keep in mind that inside the sigma is the Pi.

So the S part at the beginning is the same. And this guy's a little bit nicer to write down. So the terms that are pure S remain the same, so S cubed, S 4th.

Since there's no funny square roots in our field redefinition, things are a little simpler. So that's, again, an equivalent version of the sigma model, just using different fields. Everything I can calculate in the original sigma model, I can calculate these formulations as well.

And the final one is going to be different. And that's the non-linearity chiral Lagrangian. And what I'm going to do to get there is I'm just going to drop the things that have mass from my previous theory, which are S and psi.

So I take this exponential representation of the sigma model. These guys are massive. I think about-- well, I integrate them out explicitly. In this case, what we're doing here, which is not QCD, we can do that, just remove them.

That amounts to the lowest order, just dropping them. And then we have just the thing involving the pion, which is just very simple-- that. So this is a viable thing for low energies.

The first three actions that we wrote down are identical. They're just field redefined versions of each other. They're really equivalent theories.

And this final one, L chi, is equivalent for low-energy phenomenology of the pions. So the first three are actually equivalent to the last one if we restrict ourselves to low-energy interactions. So in order to see that, let's do an example where we calculate something.

Let's think about calculating Pi plus Pi 0 goes to Pi plus Pi 0, so scattering of Goldstone bosons. And q will be the momentum transfer. So in kind of an obvious notation, it's the difference between the final and the initial, or the initial and the final for the charge guy or the neutral guy.

And there's two types of diagrams that can come in. You could have a direct scattering diagram or you could have an exchange diagram. so this is Pi plus coupling to either the sigma or the S, depending on which representation we're using. And then the Pi 0 is down here.

And I want to take these, and I want to expand. And I'm going to expand in q squared over v squared. That's what I'm going to mean by "low energy". v was setting the scale of the masses.

So let's see how that works out in the different cases. If I do it in the linear case, and I have both of these types of contributions, this guy just gives me some symmetry factor, which is 2 minus 2i lambda. This guy as a propagator, square root looks very different.

And it turns out in the square root, that you can find out that this guy is order q to the 4th, so we won't even write it down. Exponential-- this guy is again ditto and ditto, and then I don't have room, but I'll put it up here. In the nonlinear, we don't even have that diagram because we just dropped it.

And if I take this line, which is the only one that looks different, I can combine these two terms together. And once I do that, then I realize that this is also giving the same answer. So just rearrange it, put in the relation between mass and coupling and VEV, which looks like that, and expand. And you get i q squared over v squared as well.

So if I stop at order q squared, all versions give the same thing. The linear and the square root were equivalent. The linear, the square root, and the exponential are just different versions of the same thing.

And all that's changed by making the field redefinition is how I think about these two diagrams. In the linear, the leading order term is not coming from just this diagram, it's also coming from that diagram. And it's actually coming from a cancellation between those diagrams.

In the square root and exponential representation, this diagram alone gives the leading order term and this diagram gives higher order terms, which is kind of what you want if you want to think about these heavy particles as something that is giving high-order corrections. So that can depend on exactly how you formulate it. And from the point of view of just keeping the low-energy degree of freedom, the nonlinear just gives you immediately the answer from very simple Lagrangian. Of course, it was just the exponential with something dropped.

So from the point of view of doing calculations, the linear version here is the one that you don't want to use, because there's cancellations between diagrams that you have to figure out. And those cancellations actually affect what you call "leading order," because leading order for this calculation is order q squared, and you don't see that until there's that cancellation that's taken effect. So it's very hard to formulate a power accounting for the linear theory for that reason. But if any of these nonlinear theories we could formulate a power accounting, then everything is nice and beautiful.

So what happens is that because of chiral symmetry, we have derivative couplings. And in the linear version, we don't see that until we cancel terms. So if you like, if you think about that as a property of the symmetry, you'd like to make it explicit, and the other representations do that.

The version that's most convenient is the non-linear guy, since it's the simplest. And we just think about it, forgetting about the heavy stuff. And it's what we really want for our bottom-up effective theory because it only has the low energy sigma field, which has a pion in it, and it has the derivative couplings.

If you look at how the symmetry works, which I didn't talk about when I wrote down all the different versions, S is for singlet. That's what's behind the name S, so it doesn't transform.

Sigma transforms on both sides with an L on the left and an R dagger on the right. And that comes from looking back, noting that S doesn't transform, and looking back at how the Pi field transformed. It's just exactly that way.

Now, if you write out the sigma, which is transforming linearly, as the exponential of i tau dot Pi, and if I put it in a convenient normalization, which is the VEV, or the v, then you can work out from this transformation how the Pi field transforms. And it transforms nonlinearly, and hence, that's the name for-- that's where the name comes from.

If you do the infinitesimal version of the transformation, and you work it out for the Pi a field, which is a-th component of the vector, a being 1, 2, or 3, part of the transformation is simply a shift. And then there are additional pieces, which are things that you keep, that are order pi squared and higher. And that's those pi squared and higher terms that they're saying it's nonlinear. This term here is what's telling you it better be derivatively coupled, because if it's derivatively coupled, if you have a derivative, that kills this constant, and that was something that was hidden in our linear version of the theory.

Now, we've constructed it here from this kind of point of view of integrating out. So I said QCD, we can integrate it out, but all we care about is the symmetry-breaking pattern. So we wrote down a theory, which is this linear sigma model, had the same symmetry-breaking pattern.

And we could remove the heavy fields in that, and then the low-energy theory, I claim, is the same one that you would use to describe QCD, because it's the same symmetry-breaking pattern. But that's kind of a roundabout way of getting to where we wanted to go. And you'd really like to just get right to the chiral Lagrangian from the start, and you can do that.

So rather than going through the linear sigma model, we could also just directly get to where we want to go. And here's how you do that. So go back to what the symmetry-breaking pattern was and figure out what is it that we're trying to do with the low-energy effective theory.

So we have G that's broken to H. The Goldstones are transformations in the coset, which is G/H. And we'd like to parameterize them. And what we're doing with this sigma field is we're parameterizing fluctuations to take us around the coset.

AUDIENCE: I'm a little confused, because if you don't expand sigma, that Lagrangian has still the full symmetry as you do left, as you do right. But it contains the v, [INAUDIBLE] parameter for the rest of the symmetry. That Lagrangian is presenting the unbroken phase or--

IAIN STEWART: So remember, I went over to the shifted fields, the sigma twiddle, and then I was expanding around the correct vacuum-- I mean, the one that's the lowest energy vacuum, not the unbroken. I went from sigma-- I originally had sigma-- sorry. Thank you for that.

So really, when I talked about the linear version, there was two versions of the linear version. There was the original version where I wrote it down in terms of sigma and Pi vector, and then I went over to sigma twiddle and Pi vector. And when I went over to sigma twiddle, it was just a shift where I shifted myself to the proper vacuum.

So I'm really doing the perturbation theory in the linear here about the proper vacuum. And all the other versions, I'm also doing about that proper vacuum, the lowest energy vacuum. So there was a step at the beginning, which is the classic step for spontaneous symmetry breaking, which is-- in some sense, I had to do that to get started when I went from the sigma to the sigma twiddle field.

But you should really think about it as the sigma twiddle field as being field in this linear case. That's a sigma twiddle there. Other questions?

So how would we do this construction of sigma if we didn't know about this linear sigma model way of getting there? So we have a generator, G, which is in L comma R. That's my notation for the combined SU2 left cross SU2 right, for example.

And that's going to be broken down to something that's in the vector sub-group, which I'll denote like this-- V comma V. And then an element of that we can call h. And if we want to parameterize the coset, you can think that you have a G left and a G right in a kind of obvious notation. And you want some kind of field to parameterize fluctuations, where we pull out the h. So I'll call that "cascade."

So let's just think about an example. So say we had something that looked like this, which is some set of generators that is in the original L comma R. And let me insert in here 1 in the form of gR, gR dagger, and then write it as a product of generators that looks like this.

So I just multiply the various-- this guy multiplies that guy, this guy multiplies that guy. So this guy here has the same entries in both, so that's in h. This is in the v comma v, which is h.

And this guy here, this gR dagger, is then the thing that's parameterizing the cascade field. So this is a matrix that's parameterizing the coset, and transforms in the way that we said. So that's the idea behind what the sigma field is doing.

Now, you can ask, well, that just seems like one choice. And I could think about parameterizing the coset in many different ways. And that's true.

And there's actually a nice formalism that takes that into account. So if you just talk about broken generators, which I can call X, then a very general definition of what this cascade field is is just an exponential involving those broken generators, and some fields to describe the fluctuations, and normalized in some way. And this is due to Callan, Coleman, Wess and Zamino.

So it's the CCWZ prescription for parameterizing a coset. And the point of their prescription is that you have choices here for how you represent the broken generators. We start out with left generators and right generators, and then you go over to the vector generators, but what ones are broken?

Well, you could take different linear combinations that are broken. And those would be equally viable choices. Doesn't have to be a group, the broken generators, so you can pick different choices of this X.

And if we pick X, which is the left generators, that's actually what gives us what we were talking about before. Because then we end up parameterizing the cosets by something as 1 in the second entry, because we don't have the right. We're just saying pick the left, the left are broken. One possible choice, left plus right is unbroken.

Then we end up with an entry here, which is sigma. That's our sigma. And also, if you work out the transformation, how the sigma transforms, you could reproduce-- you can also derive that sigma goes to L, sigma R dagger.

But that's just one choice. You could do other choices. So for example, you could pick X a to be tau left a minus tau right a. That's another possible choice. And that would lead to a different field, but an equally valid parameterization of the coset.

And this was actually also popular. It's something that people usually denote by C. I'm going to leave further discussion of that to your reading. There's a very nice review by Aneesh Manohar, where he goes through some of these things for CCSW, and does a nice job of talking about both of these in more detail than I've done here.

So further discussion of this point will be in your reading. You can also look back at the original paper, if you like. But Aneesh has distilled it nicely.

So any questions about that? There's a way of thinking-- I'll just keep talking until someone raises their-- there's a way of thinking about field redefinitions, even from this low-energy point of view. That's the key here. And it has to do with the freedom that you have and how you parameterize the coset.

So even if we're constructing it from the bottom up, there's different representations. In fact, Weinberg likes the square root version of parameterizing the coset. Maybe he's the only person. He has very good reasons for being allowed to stick with his original results, rather than the exponential representation, which everyone else seems to favor, including me.

So if we go back to talking about QCD, then it's common to call this curly V f over root 2. 1/2 is a decay constant. And so the conventions, then, are the following-- at least one possible choice of conventions.

There's some choice about where you put the 2's, and then everything else is fixed. So this is one possible common choice for the exponential representation. And then that gives you the chiral Lagrangian-- looks like that.

And if you expand this out, you get the standard kinetic term for the phi field. That's what this normalization factor does. And then if you keep expanding, you get some interaction terms like this one, et cetera. So there's four point interactions in that Lagrangian as well. And that's because we had a curved-- our coset is nontrivial.

So part of this story here when you talk about symmetry breaking has to be about the fact that symmetry is also broken explicitly in the standard model. And so you have to add-- the Goldstones are only pseudogoldstones. You have to add a mass. And the way you do that is by doing a spurion analysis.

Remind you how that goes. You write a mass term like this, and to left and right. The mass, you could say, is just the diagonal matrix of up and down for SU2. And you could pretend, rather than talking about things that are violating the symmetry like these guys, you let the mass transform and pretend that you actually have something that's invariant.

By pretending that the mass transforms, you're able-- if you pretend the mass transforms in the chiral theory-- of constructing things that violate the symmetry in the same way. You construct invariance in both theories with this kind of transformation law. And then when you go back and fix them, and say no, no, it doesn't transform, it's really a fixed thing, not a field that could transform-- it's fixed, the number-- then you violate the symmetry explicitly in the same way as up here.

It's a trick for how to build things that are covariant in a certain form. So you could add the variant operators of the chiral Lagrangian. This is a different v, some parameter, different than the curly one we were talking about earlier, which is now called f.

So this is the spurion story-- how to break symmetry explicitly in a different theory, the same way. And if you expand a quadratic order, you get mass terms for the pions. I won't go through that. But one kind of an important thing about it is that the mass is proportional to v times the quark mass, so v0, so the parameter in the Lagrangian, linear in the quark masses, quadratic in the Goldstone mass.

You can also a couple of currents. And you can do a similar type of analysis with spurion-type transformation analysis for the currents. And I'm not going to go through this in any gory detail, but just enough to do what we need to do here.

So here's a left-handed fermion current, standard model. Could be coupling to the W boson, for example. And you could think of getting J by taking a functional derivative with respect to a field that's left handed. And then I can do the same kind of thing, where I think about this L and let it transform, just like I was letting the M transform. So think about J as this capital J times minus L.

And if I then think about this L mu transforming, I can build a chiral Lagrangian, and then use this formula to construct the current in the chiral Lagrangian. So you couple a spurion current, L mu a tau a, and you, in this case, in order to couple it, you can let it transform like a left-handed gauge field. And then you get something that's invariant.

So the transformation that will make it invariant is to think of it as a left-hand gauge field. So L mu goes to L, L mu, L dagger. And then that gives you an invariant in the original theory. And then you build in a variant of the x theory.

And so if you do this, and you're replacing, in your chiral Lagrangian, your partial derivative by a covariate one, it acts on the left because that's where the sigma is transforming. And so it's an easy way of building that in. And you just get an i L times the sigma on the left.

So you take everywhere you have a derivative, you replace it by this combination. So we had two derivatives in our chiral Lagrangian. Expand it out, and we can figure out how to put it in the left-handed spurion. So the spurion is just kind of a general way of going from one to the other just by tracking symmetry breaking.

And you'll get some more practice with some of this on your problem set. So on your problem set, what I've asked you to do on problem set 2 is to do a one-loop calculation in this chiral theory. So you can really see that this is a quantum field theory, an effective field theory, not only a tree-level mnemonic theory, but really a field theory in its own right where you have to consider loops, contractions, Wick's theorem.

Everything is the same as a real quantum field theory should be. It's an effective field theory of certain degrees of freedom. So you do a one-loop calculation for the decay constant.

And there's two problems in the problem set. The first one is fairly straightforward. And then that's the second problem. That's all I'm asking you to do because it's fairly involved. It's like maybe two normal problems.

So here's our chiral Lagrangian. And we're going to count in this chiral Lagrangian, in terms of power counting, partial squared as being order v0 ab q. That means we're going to count p squared as being order M pi squared. That's going to be our accounting.

And so we're going to expand in these things as small and something else that is large. And we'll figure out what the large thing is in a minute. So we do the expansion like that, where something is downstairs that has mass dimensions and the things that are upstairs are p squared and then Pi squared.

So the type of expansion we have here is a combination of a derivative expansion, although it's a bit of a different derivative expansion than before, and an M q expansion, simultaneously driven expansion in these two things. If we look at L0, then it has things in it like a propagator-- dashed line for the scalar plan propagator. It has four point interactions.

And if you look at how those scale, we could write out what the Feynman rule is, but if you look at how they scale, they go like p squared over f squared, or M Pi squared over f squared. The p squared you could put on shell, and it would be M Pi over f squared. And you will actually need this Lagrangian and this Feynman rule for your homework, so let me tell you what it is. And I'll leave it to you to work out the Feynman rule.

So commutators, there's a factor of 6. And then there's also a term that would give four-point scattering from the M2 term in the Lagrangian. So that also gives an M Pi squared over f squared term, and that looks like that. So you could build up a higher point function, six-point function, et cetera from the leading-order Lagrangian that has all this in it. It has two parameters, f and v0.

So what about loops? And what about this? What is this scale lambda chi?

So the type of loops that you can have if you have a four-point interaction, and just draw a solid line since it's easier than drawing dashed lines-- think of all these as dashed. So there's cross diagrams as well as the original diagram. This is not the calculation I'm asking you to do in the homework.

If we thought about our example of Pi plus Pi 0 scattering, then we could contract up Pi plus and Pi 0 fields, and we could have a loop that looks like that. And in order for our theory to be unitary, we have to think that these loops make sense, because there's cutting rules, and all that story should go over for this quantum field theory. And it does.

And that means that if you like the imaginary part of this loop is needed, and you can't get the imaginary part without the real part, so we have to also think that the real part is a physically meaningful thing. And so we can go ahead and compute it, and figure out what's going on. So there's derivative couplings that go like f squared, 1 over p squared over f squared.

L is the loop momenta. Primes are the final. Unprimes are the initial. You can parameterize it however you like.

I'm using here dimensional regularization. I like to dimensional regularization, and so do you. Dimensional regularization here preserves chiral symmetry, which is something that we want to preserve. And I'll tell you a little later what would have happened if we'd done this with a cut-off.

So if you do this calculation, just calculate those guys, you get terms that go like p to the 4th. You get terms that go like p squared, M Pi squared, terms that go like M Pi to the 4th. You get a 4 Pi, because it's a loop, and then you get an f to the 4th, because there's four factors of f.

So what this behaves like, it's p squared or M pi squared over f squared, which was our tree-level, four-point function, times something that's causing suppression, which is p squared or M Pi squared over 4 Pi f squared. So the loop is actually down by a factor of p squared over 4 Pi f, all squared. So just by explicit calculation and not breaking the symmetry, we see that loops are suppressed by p squared over lambda chi squared, where lambda chi here is defined to be 4 Pi f.

So the 4 Pi is important because f is like 130 GeV, and you need this 4 Pi in order to have some range for your chiral expansion. So if you plug in numbers, depending on your conventions for f-- and there's a couple of common conventions. You could get numbers that are like 1.6 or 1.2 GeV, so some scale of order a GeV.

On general grounds, you could also ask the question, if I just think about it from the effective theory, what is a reasonable scale for the denominator, for this lambda chi? Remember, you're going to have to construct higher order operators as well. And so you need to figure out what's the right scale for the suppression of those higher order operators.

And if you did it on physical grounds, than what you'd expect the lambda chi to be is about the mass of the rho. Because that's the lowest energy hadron that you've left out of this chiral theory. You might expect lambda chi's of order M rho, which is sort of 770 MeV.

And so you can't really say whether it's 770 or 1.2, because the difference between those, although important numerically, is not within the realm of the kind of factor 2-ish meaning of these twiddle symbols. So these twiddle symbols mean "parametrically of order." Then there's two symbols-- there's much greater than or twiddle.

"Twiddle" means factor of 2-ish. "Much greater" means parametrically [INAUDIBLE] or much less than. And just with those two symbols, there's some freedom. These are different ways of doing an estimate for what lambda chi are, just saying that there is that freedom, and you have to do the full calculation in a particular channel to really figure out-- or a particular piece of phenomenology-- what the lambda is. Extract it from data if you like.

So that is likely a good place to stop. And next time, we'll talk a little bit more about these loops. And we'll talk about what happens to these loops in the power counting theorem that you have, that organizes all diagrams in this theory, and tells you what you have to do if you want to construct the Lagrangian order by order-- how you deal with when to put in the loops, when to put in the counter terms. We'll talk about those things next time.