# MITOCW | 15. Soft-Collinear Effective Theory (SCET) Introduction



**IAIN STEWART:** All right, that's roughly where we were. So last time, we started talking about SCET. We said it's going to be a theory that can describe energetic hadrons and energetic jets. Our first example was discussing about a process with an energetic hadron, which is this pink pion.

So the pion has a large momentum, large energy. It's much bigger than lambda QCD. It's much bigger than in m pi. And it moves basically along a light cone direction.

So that was a motivation for us to use light cone coordinates. So we introduce an n and an n bar. And with that n and n bar, which satisfy n squared 0, n bar squared 0, and n dot n bar equals 2 as a normalization convention, we can decompose any momentum P mu in terms of components along n, components along n bar, and then the remaining two components which we call the perpendicular components.

So we can also write the metric out in these coordinates. And this kind of makes explicit that you have this off-diagonal nature to the basis, that you have n mu with n bar mu.

So unlike Cartesian coordinates, where the component along a direction is just given by dotting that vector into the vector you start with, here the component along n is given by dotting n bar into the vector. And that's reflected in the metric here in the sense that you have these terms n within n bar. So you can do this with any tensor if you have an epsilon.

You can find an epsilon perp tensor, for example, by taking epsilon and putting in an n bar and an n. And then this would be a two-component tensor that behaves in the perpendicular direction as an antisymmetric tensor. And this g perp mu nu would be effectively living in the little subspace of the perp coordinates.

And again, it's a metric tensor there. And this would be the antisymmetric tensor there. OK, so these are the coordinates we're going to use.

So n here had a physical motivation, as you saw from my picture. The pion was moving in the n direction. And bar was just a vector that we decided that we needed in order to define things.

So if you have some vector where n squared is 0 and you want to make a decomposition of coordinates, then you're required to introduce a complimentary vector, which is this n bar to make the decomposition for the reasons I said.

The simplest choice that you could make, if you made this choice for n, so if we choose n to be 1, 0, 0, minus 1, as I did, then the simplest thing you could do for n bar is to pick n bar to be 1, 0, 0, plus 1.

So then that would be a lightlike vector. When you dot it into this vector, you get 2. And it satisfies all the criteria that we would need.

If I choose these two vectors, then I have to find what perp is because perp is the space that's orthogonal to these vectors. So that's these two coordinates, OK? So perp in general is defined such that n dot P perp is 0 and n bar dot P perp is 0.

It's the orthogonal two directions to the ones that are picked out by an n bar. And so you need to know what n and n bar are in order to define what perp is. So this is one choice. You could make other choices. And we'll come back to this later on.

So just by way of example, if I have the same choice for n, but I choose n bar to be 3, 2, 2, 1, that would also be a choice that's equally good. I need this to be minus 1.

I'll do this, make my choice work. I guess, well, OK, if we want the same sign, then I have to do this. OK, so 9 minus 8, this thing still squares to 0. You dot it, you get 3 minus 1 is 2, OK?

So it still satisfies the criteria that we had here. And it points in some other weird direction. So the point is that this is an auxiliary vector, and there's some freedom in what you pick for it.

And once you've picked this, if you pick these two, you would have a different definition of perp. OK. But it's an equally valid possible choice. And we'll actually exploit this freedom later on. But for now, we'll mostly focus on picking the simplest choice.

OK, what we're actually interested in describing in these processes is not just the plan, but what goes on inside the plan. What is the quark level process? So we're interested in the constituents. That's where the dynamics are.

Is there any questions before I keep going? No. So in this process, B to D pi, if you think about it in the rest frame of the B meson, which is the most natural frame, then the B meson we've already learned how to describe that. We can describe that with HQET, same with the D meson.

And we know that the things that are inside the B and the D meson are one heavy quark and then a bunch of soft stuff. So I'll call these guys soft because the dynamical part is soft. And so we can use HQET for them as we did before.

And that means we're describing gluons and quarks that are inside these hadrons where the forward momentum are of order lambda QCD. The pion, on the other hand, is what we would collinear.

So as I already described, the pion's energy is much greater than its mass. Its highly boosted. If you were to talk about it in the rest frame, then like the B and the D meson, then the constituents of the pion would have momentum of order lambda QCD.

But if you were to talk about the pion in the rest frame, you'd have to talk about the B and the D in the boosted frame. So let's stick with describing the B and the D in the rest frame or close to the rest frame. The B meson is in its rest frame. In the D meson is slow.

And in that case, we're stuck with the pion being energetic. So in rest frame, our pion would be-- it would also have quarks and gluons our P mu is order lambda QCD.

And we can actually just take that result, once we know that, and boost it to another frame. So let's just boost along z hat by some kappa that's much greater than 1 as the boost. And the way that light cone coordinates boost is very simple. If you're along the axis of the light cone coordinates, it's multiplicative.

So P minus gets enhanced by some amount. P plus gets suppressed by the same amount. That's one nice thing about these coordinates. And of course, P perp doesn't change because it's perpendicular to the boost.

So now, we can get our pion, which is moving, which should be pink. And if we ask about its constituents, we just boost the components of this for vector. So we ask about how they scale.

And we look at the different components. The plus, minus, and perp scale differently now, so we have to break it up by that. And if we boost it by this amount Q or lambda over Q, Q over lambda, then that's the scaling for this boosted pion.

OK. So now, it's got a component in the minus direction. n bar dot P is order Q. That's what we saw before when we decomposed the P, that it was basically Q times a lightlike vector.

But that was the pion. Now, we're talking about the constituents inside the pion. Constituents inside the pion fill it out. They fill it out in the perpendicular by an amount lambda QCD that's perpendicular to the direction of its motion. And then the plus momentum got correspondingly smaller as the minus momentum have got bigger, so we have that scaling.

And so the relative scaling here is what actually defines something being collinear. So the relative scaling of this vector here is that the P minus is much bigger than the P perp is much bigger than the P plus. And that's what we mean by collinear.

It's collimated in some direction, and that direction is the direction of the large momentum. You always have to be careful when you say things like that because the component along the direction is the opposite lightlike vector, but I think you'll always know what I mean. OK, so in the n mu direction, we have a large component P minus.

And that defines this thing is collimated in a particular direction. It's perpendicular fluctuations to that direction are small. And so all the degrees of freedom that are in this boosted pion have that type of scaling.

So what we're describing, or what we want to describe if we have a field theory for this, is we want to describe, if you like, fluctuations about the pion momentum, which, ignoring the pion mass, we could just take it to be like this. And the size of the fluctuations we need to treat are things that can fluctuate by amounts of this size.

So the field theory is going to have to describe fluctuations about some kind of canonical scaling. And the field theory for this pion is going to have to be describing collinear fluctuations that are of this type. Just like the HQET had to describe soft fluctuations, P mu's of order lambda QCD ignorant of the heavy quark mass, here it's a little bit more complicated. But that's the kind of thing we want the field theory to do.

Any questions about that?

So the way that we write this is we say that P plus, P minus, P perp has a particular scaling that we call lambda squared 1 lambda where lambda is some small parameter. And if we have a momentum that scales that way, we call it collinear.

So this is generic any case with any lambda. And our allowed here was just lambda QCD over Q. But if we encounter another physical problem where the lambda was different, we would also call that collinear.

All right, so what's a nice way of picturing this, what we're doing here? Because it's a little bit different than you're used to with an effective field theory. Usually, with an effective field theory, what you're doing is you're separating modes by their invariant mass.

You have things with large invariant mass, small invariant mass. If you think about massive particles, well, that's just the invariant mass squared. So if you're separating massive particles from massless particles or less massive particles, you're really separating things along an invariant mass curve. Just an invariant mass variable is used for the separation.

And that doesn't quite suffice here. Because as you saw, the pion in the B and the D meson, they both had P squared of order lambda QCD. What separates the pion from the B or the D meson is this morphyne structure.

So SCET is actually an example of an effective field theory that requires at least more than one variable to describe where the degrees of freedom live. So we can draw a picture for what we've been talking about here in two variables. Let's just pick P minus and P plus.

And essentially, what's going on in this space is you can think that there's degrees of freedom that live in this space at different locations. So out here, if I draw a hyperbola like this, then remember that P squared was P plus times P minus minus P perp squared. But let's ignore P perp squared for this picture.

So if I draw a curve of constant P squared in this plane, then it's I hyperbola. So these are curves of constant P squared. And this one here has P squared of order Q squared, which might be Mb squared or some hard scale.

So this any degrees of freedom that live on this curve, or in particularly these ones, would be what we would call hard degrees of freedom. And those are something that we want to integrate out of the effective theory. And the other degrees of freedom that we've been talking about have smaller invariant mass.

So this hyperbola down here has P squared of order lambda QCD squared. But there's two different degrees of freedom that live on this curve. One of them has a large P minus. That's the collinear one, so it should be pink.

And then the soft one lives down there. So P minus here is scaling, if you like, lambda to the 0. And here, for this soft mode, which also exists in this case, this is actually going to be lambda.

So you can contrast that type of picture with a more usual picture where you would just have one line. And you'd say there's some modes up here and some modes down there. And you'd integrate out these modes. And you keep those modes.

This is a little different because you want to integrate out these modes. You want to keep both of those modes, but they live in a little bit of a different place. And that's actually going to be important to formulating the effective theory.

So the way that you should think about this, physically the way you should think about it, is that these modes are kind of localized in that region. This is the right physical picture, which requires another variable besides just invariant mass in order to specify that, right?

The reason we don't have to draw a third direction for P perp is because it was just redundant information. P perp squared is always of P plus P minus if you're talking about fluctuations that are near the mass shell. For a massless mode, that mass shell is P squared equals 0.

And so P perp would just be providing redundant information to our picture, and we just can leave it out. Now, the boundaries of the regions between soft and collinear here seems like an interesting thing to worry about. And that is, indeed, true.

You have to think about how you want to set up this effective theory. And of course, as I have been emphasizing earlier in the course, the easiest way to think about momentum degrees of freedom is with a Wilsonian picture. That makes physically what's going on very clear.

So the simplest thing would be to introduce a Wilsonian cut-off and set this up as a Wilsonian effective field theory. And then we would just take these regions. And I would literally carve them out in the way that I drew.

I would carve out some cut-off between them. And I would decide who's in the soft region, who's in the collinear region based on those hard cut-offs. But we don't want to do that, actually, because it would mess up all sorts of symmetries.

In particular, it would mess up gauge cemetery which is an important thing when you're talking about gauge theory. So we're going to use dimensional regularization, as we have for other problems.

And that actually will still leave us with this picture, which I drew as a cartoon. You'll still be correct. Think about the modes live in those places in dimensional regularization.

What's a little bit harder is how to think about the cut-off. And we'll treat that in some detail later on. So it's still the correct picture, but treating the region overlaps with dimensional regularization is a little more tricky.

But at least we can do it in a way that preserves the gauge invariance. So that's going to be our mode of operation.

This theory has a name. It goes by the name SCET2. That's why I called it SCET2. We'll come back in a moment to what SCET1 is.

So I can say that the degrees of freedom in this theory, the one that we've been talking about, are some collinear degree of freedom that's associated to some direction, some collinear degrees of freedom as well as some soft degrees of freedom. And when you have effective theories that are like this one, where the soft and collinear degrees of freedom live on the same mass hyperbola, they're called SCET2 theories.

And these are really the kind of theories that you get when you're talking about energetic hadron production. So any questions so far?

OK, so if that's energetic hadrons, then SCET1 will be energetic jets. And that's what we'll talk about next. So let's do another example which has jets in it. And we'll see what the similarities are to this SCET2 set up in terms of just identifying still what the right degrees of freedom are.

So let's look at e plus e minus to two jets. So e plus e minus collide. They produce a virtual photon, say. The virtual photon produces a quark-antiquark pair. The quark-antiquark pair starts to radiate. And we get jets, two of them.

So again, there's a kind of natural frame to describe this scattering. And that's the center of mass frame of the e plus e minus. Most e plus e minus colliders are built in that frame.

And if you're in the center of mass frame and you call the for momentum of this photon Q, then it just has an energy component. So Q here is not the same as the Q in our previous example, but we're always going to identify the hard scale as Q. So here, the hard scale is the scale of the energy of the collision in the center of mass frame.

And if you ask, what does the event look like in the center of mass frame, then these two jets which are going out have to balance each other. And so you have two back to back jets. So we draw it like this, one jet going this way, one jet going that way.

Our original e plus e minus might have come in from some other direction. So maybe e plus e minus were coming in from here and here. And then we have these two jets going out this way.

Since they're jets, that means they're collimated sprays of radiation. So they're not featureless. They have some size to them like this.

And in this process, we can quickly identify that there's two relevant directions. It's back to back, but let's define this direction of this jet to be n1, some lightlike vector that points in that direction, and this one to be n2. So generically, we could say you n is some 1, n hat.

And if we choose this to be the z-axis, then this must be 1, 0, 0 1, just like we had before. But we can always pick a lightlike vector that points along some direction n hat. So when I draw a picture like this, really what I mean when I say n1 points in this direction is that the hat part of it points in that direction in through space.

OK, so just like before, what happens with the jet is, as I said last time actually, we have a large energy flow in this direction and a smaller perpendicular flow. So if we measure perpendicular momentum, which we can think of as perpendicular to that axis, free axis, then that's the perpendicular flow inside the jet. And so if we want to talk about constituents inside the jet, then we'll be interested in smaller perpendicular flow than flow in the forward direction. And that's what makes it into a collimated jet. You have question?

**AUDIENCE:** [INAUDIBLE]

**IAIN STEWART:** Sure.

**AUDIENCE:** So you're not using anything about the QCD [INAUDIBLE] to tell you the things is [INAUDIBLE]. You're just saying we know that we have to [INAUDIBLE] topic.

**IAIN STEWART:** Yeah, that's the attitude, right. So I mean, we'll see why. You know, you can ask the question, why do we get jets in the first place? We haven't talked about that. And we could talk about that.

I'm taking the attitude here, this is what we observe. How do we design an effective field theory for it? And we'll see then, from that effective theory, we can go back and understand why it is that we get these objects and why actually, when you look at cross-sections, that this is the leading order description of what happens. We'll come to that later. Good question. Any other questions? All right.

Yeah, in the hadron case, you just say this process exists. And I didn't explain to you why the process could exist. But in the jet case, there's more dynamics going into the fact that the process exists, the fact that QCD likes to radiate collinear to a direction, which has to do with the infrared structure of the theory. And we'll come back to that.

I mean, the short answer, of course, is that things like to radiate in that direction because there's large logarithms that enhance splittings that are collinear to the direction of motion. And you also get smaller coupling consonants when you do that rather than a wide angle emission.

So we're saying here, what we're saying, is we measure e plus e minus to two jets. If I had an extra wide angle emission, that would be e plus e minus to three jets. So I ruled that out just right at the beginning.

OK, so what you can do with this picture in order to define what's going on is you can say, well they're to back-to-back jets. Let's draw a hemisphere. So this is supposed to be kind of out of the board point at you.

Let's draw a hemisphere between these two. And then we can call one side a and the other side b. And we can talk about momenta that are flowing in the a hemisphere and the b hemisphere. And you see that we have one jet in each hemisphere.

So we have a jet of hadrons in hemisphere a. And we have another one in hemisphere b. So in some ways, we car talk about a and b independently.

So let's start off by talking about a, which is what I called the n1 collinear jet. And if we ask about constituents in the jet, then the perpendicular momentum will be of some size, which let me just call it delta. And that'll be much smaller than P minus, which is of size Q.

So the energy that we pump in through the photon has to leave. And the only place that it can leave is that half of it has to kind of leave this direction. Half of it has to leave that direction by energy conservation.

And so we have a large energy flow in the P minus component. And then we have a much smaller amount in the P perp. And that's what I already said.

And given those two facts, you can ask, what about the constituents of the jet? And again, they're collinear because you have a hierarchy. So the plus minus and perpendicular momentum of constituents would scale in that way. And that is Q lambda squared 1 lambda, just as before with a different lambda.

So here, lambda is how much spread and perp do we have over Q. This delta doesn't have to be lambda QCD. It could be something much bigger.

Another way of thinking about physically what this delta is is to calculate something called the jet mass. So you could define the mass of the jet as the sum of the four vectors in hemisphere a of all the particles squared. And if you ask about how big that is, in these coordinates that we're using, it's P plus times P minus minus P perp squared.

If we align things so that this thing is really aligned with the jet, there won't be any P perp squared. But that wouldn't change us scaling our given anyway. So basically, if you ask about what this jet mass is, it's scaling like a P plus times a P minus. And it's scaling like delta squared.

So the jet mass is something of order delta squared. And that's much less than Q squared. So another way of characterizing that you have a jet is to measure the invariant mass of all the particles in this hemisphere. And if that invariant mass is small relative to the hard scale, then it's collimated, OK?

So MJ squared Q squared much less than 1 also means collimated. So we could talk about it either in terms of perpendicular spread, or we could talk about it as an invariant mass. OK. So this is much less than 1.

Now, with this delta, we didn't specify what it is. So what are possible values of delta? Well, let's first talk about what it's not.

If delta was of order Q, then obviously we'd break the kind of scaling that we have here that this should be much less than 1. And what happens in that case is we don't have dijets.

So if either the mass MJ squared of the particles becomes of order Q squared or the perpendicular spread becomes of order Q, then we don't have dijets anymore. And basically, in this case, you would be talking about inclusive sum over our jets

And that would be something that you would actually describe in a different way in the effective field theory because you wouldn't then have collinear degrees of freedom for this jet. It's really picking out the dijet process that means that these collinear degrees of freedom are relevant. If you did just e plus e minus to hadrons, that's something you could do with an operator product expansion without ever talking about SCET.

And that would be valid if you were really doing an inclusive sum over jets in all directions without any restrictions that tell you it's a dijet. And then the effect of power counting in your OPE would be such that delta is of order Q. So this is actually the OPE region. Let me call it the OPE region of Peskin or any other field theory book.

Another thing we could do is we could take delta to be very small. We could take delta all the way down to lambda QCD. And that's also not a jet.

If delta is of order lambda QCD, what happens with the spray of radiation is that it gets bound into a hadron. It just can't separate. Confinement grabs it, and you get an energetic hadron, not a jet.

So if the jets get too narrow, in particular this narrow, then the constituents are bound into a hadron. And that might be something you want to talk about, but it wouldn't be talking about e plus e minus, the dijets. You'd be talking about e plus e minus to pi plus pi minus or something.

And then you'd actually use this other SCET that we were talking about a moment ago, not the one for jets. OK, so anything kind of in between these two regions much greater than lambda QCD, much less than Q, then we can talk about it as being jets.

OK. So we figure out what region we're interested in by figuring out what region we're not interested in. So that was one jet. And kind of by symmetry we can talk about the other one.

So there's this one that I called n2. The simplest way of invoking symmetry is to take n1 to point along n, which is our, say 0, 0, minus 1. And then just take n2 to be n bar, which is 1, 0, 0, plus 1.

And then the description of the n2 jet is the same as the description of the n1 jet. It's just you switch pluses and minuses. Remember that what defines plus and minus depends on the choice of this n bar.

A priori, that choice of n bar has nothing to do with n2. And this is just a different physical vector. Both n1 and n2 are physical.

And bar is an auxiliary vector. But if I just happened to choose that n2 is equal to that auxiliary vector, then it makes things simple. Because we just have a relation between the two sets of degrees of freedom, this swapping pluses and minuses.

OK. So that's actually not the end of the story of the degrees of freedom here. And that is because, even if we make restrictions to getting these dijets, we can still have soft radiation that is between the jets. And so this one's a little less intuitive perhaps.

So I'll call these guys ultra soft modes. And I'll label them by US for Ultra Soft. And one way of thinking about physically what they're doing is that they're allowing you to communicate between the jets.

There can be radiation that's radiated from one jet that interferes with the other jet. This is a homogeneous type of radiation, so it has the same scaling in the plus, minus, and perp. So this is for these ultra soft modes.

If I compare this to the scaling that we have for, say, the n collinear modes-- so let me call that Pn-- that was delta squared over Q Q delta. And if I make the mirror for the other jet, then I would switch these two delta squared over Q delta. So when I say communicate, what I mean is that these guys can talk to both of these guys without interfering with their scaling.

So if they're not going to interfere with this guy's scaling, they better have plus momentum that's the same size. If it was any bigger, then we'd have a problem because they'd interfere with this scaling when they tried to communicate. And then likewise for the minus, they better have scaling of the same size. And then perp is fixed.

So the word communicate here in the way I'm defining it means sharing momenta of a common size-- well, means sharing momenta and not taking the other particle off-shell.

OK. So we can draw a picture for this one, too. Like we have our SCET2 picture, we could draw an SCET1 picture. And that helps. Pictures always help to make words more palatable, more absorbable.

So same type of picture where you have P minus and P plus, we also can think about things in terms of hyperbola of constant invariant mass. We now have some collinear modes for the n1 direction. There's going to be some purple collinear modes for the n2 directions.

There's going to be these soft modes. And then we could also have some hard modes that we want to integrate out. So these are the ultra soft modes.

So the reason that I call them ultra soft rather than soft is because, by soft, we meant something that sat on the same hyperbola. And here it sits on a lower hyperbola, so it should be softer. So we call it ultra soft.

This is the hyperbola where P squared is of order delta squared. This is the hyperbola where P squared is of order delta 4 over Q squared. If you square any one of these guys, you get delta 4 over Q squared. And up here, is P squared over Q squared. And this kind of thing is called SCET1.

So what does it mean to communicate? Well, it means that there's two momenta that are the same size. So the fact that I tried to line this up as best I could, partially succeeding, is what I mean by communicate. These things are the same size in the plus. These things are the same size in the minus.

And if we want to put in kind of how big these things are, we would say that. So the scaling of the ultra soft, unlike the scaling the soft, the ultra soft in all component scales like lambda squared.

So in terms of scaling parameters, we would say that it goes like this. And here, we're not talking about soft. It would have lambda in all components.

So this is the effective theory that turns out to be the right one for jets. And whenever you have this kind of situation where you've got collinear modes that are living on a higher hyperbola than the soft modes, which you call ultra soft modes, that's an SCET1 type theory. And these two actually cover a wide range of phenomenology, these two particular cases, SCET1 and SCET2.

OK, questions? We'll talk a little bit more about this picture.

**AUDIENCE:** Do you think of the soft modes as being smaller than or equal to lambda squared?

**IAIN STEWART:** Yeah. So you can ask, now, how should I think about drawing the blobs around these things, right? And you still should think about these things with blobs like that. But then it becomes a question which we didn't have, which was kind of more obvious actually in the SCET1 case.

And that is kind of how these things overlap with the axes. And we'll come back and talk about that later. But I think, for now, just think of them as being localized in that way.

In the case of the softs in the collinear we had before, you can think of them as uniformly kind of coming down into the infrared. Here, it's not like that because this guy is more infrared to begin with. And that will have some impact later on.

OK. So what are the important features here? Well, we see this idea that I mentioned would occur that we have multiple modes for the infrared. And you can try to get away with not doing that, but then you would have a lot of trouble with your power counting.

And since power counting, as I've convinced you hopefully by now, is just as important as other things when you're designing an effective theory, you really don't want to mess with that. And so you're forced into a situation where you start talking about having multiple fields for the same degrees of freedom because the scaling of the momentum in different regions of the space you're interested in is just different. So the derivatives that are corresponding to those momenta are going to scale differently.

And if that's the case, you're going to need to have multiple fields to describe those different regions. We'll also see the power counting is different. So you can ask, what are you integrating out?

And we're still taking the attitude that we integrate out modes that are off-shell and so above a given hyperbola. So that part of our story of effective field theory is really the same. It's just that, when we describe the low energy modes, we need more than one of them.

So what we mean by off-shell is the same as it always has meant. Off-shell modes get integrated out. Those are modes with large P squared, like the hard modes.

So one natural question when you're talking about these effective field theories is, how do I know I have all the modes? And that's a good question. And there's been examples in the literature where people wrote papers where they didn't know all the modes and modes were missing. So it's not even an academic question. It's really a pitfall in some sense that you can fall into.

So I take the following attitude towards that question. You should attack it from all sides. So one is that you should really physically think about what the modes are doing and have a physical description of why those modes are something relevant for your effective field theory.

After all, your effective field theory is supposed to be a description of nature and infrared physics. So there should be something physical associated to what those models are doing. That's the physics side.

Calculationally, there's various ways that you could approach this. So one thing you can do, which is sort of an order by order thing, is you can just calculate results at one loop and make sure you match up infrared divergences. Because if you don't match up the infrared divergences, then you're missing some infrared degree of freedom in the effective field theory.

OK. So that's one way you can check for the modes. So if you can either do physics, or you can calculate. And when you calculate, there's also something called the method of regions, which is a nice way of thinking about trying to discover modes, which basically says that, any full theory calculation I can do, I can do that calculation by dividing up the integrand into regions.

And if I'm using dimensional regularization, then the full theory answer is just the sum over regions. So you can just calculate using some EFT, as I described to you, and check it. Or could do a different way.

You could say, calculate the full theory result with something called the method of regions. And that's just a way of calculating the full theory of result, but it's a way of telling you what regions are important. There's all-other theorems in QCD about what regions can give infrared divergences. And that's another thing you can use.

And I won't talk about this last one or actually about this one. So these are different ways that you could look for what are the relevant degrees of freedom. And actually, when we started SCET, we didn't separate between ultra soft and soft. We had both of them at the same time.

And the reason was we knew that in some examples we needed soft and some examples we needed ultra soft. And we were thinking of it as one theory, not SCET1 and SCET2. And then at some point, we realized that all the examples we were doing, as we did more and more examples, they broke into these two categories, one for jets, one for hadrons. And so we were finding that, for example, if you add a soft mode to this picture, it just ends up being totally irrelevant. And you can just absorb it or remove it. You don't need it. And likewise, if you tried to put an ultra soft mode into the example of energetic hadrons, you'd find you don't need it.

So you could put too many degrees of freedom in as well. It's not just that you could have too few. You could put more than you need, and then you would see from your calculations that you actually had more than you need. The reason I'm spending time on this is because this is, in some sense, the tricky part. Once we know what the modes are, we'll just get the effective field theory and go.

**AUDIENCE:** Is your second point here the method or regions [INAUDIBLE], is that any different than the check it of the first point?

**IAIN STEWART:** Yeah. Because, here, what I'm saying is hypothesize some effective field theory.

**AUDIENCE:** But check it means full theory.

**IAIN STEWART:** And then check it against the full theory.

**AUDIENCE:** [INAUDIBLE]

**IAIN STEWART:** And so the direction here is that you write down something, and you do calculations in two theories. And you make sure they agree. Here, it's a little bit different because there may not be a one-to-one correspondence of the modes of the method of regions.

Method of regions is not technically exactly what the effective theory is. So this can give you hints about what the effective theory is. Then you basically go back and do this, but it sort of helps you because you have most of the calculations to do this. Yeah. So technically, they're a little bit different, but they're pretty related.

OK. So in the example we had here, it was a little more complicated than our B to D pi example because we had three things in the infrared. And you could ask the question, is there something with just one jet? The reason we had three things is because we had two jets. We had the purple jet and the pink jet.

And we can come up with a process with only one jet. So let me give you a third example. If we have something recoiling against the jet that's electromagnetic, like a photon, then we could just have one jet. So one way of doing that is to look at a process like b to s gamma, where you say that really what you want is B to one jet plus a photon.

And that's a region of phase space in b to s gamma, where the picture over there just has one jet. And so it's the same picture, but now we just have our pink jet.

So the right modes for this picture for this process in the region where you just have one jet would be like this where this is hard and you integrate it out. And you just have two things in the infrared, a Cn and an ultra soft. In this case, you also have to ask the question, what is the hydronic process? Well, it has to B meson.

What are the things inside the B meson that are binding it together? And you would make those the ultra soft modes. So in this process here, it's very natural to take P squared of this lowest line to be lambda QCD squared. And then these guys are binding the B meson there, the soft modes of the B meson.

This collinear hyperbola for the jet lives somewhere between. And this here, in this case, would be something of order the B quark mass squared, OK? So that's a little bit different than over here where there was kind of a no natural-- this thing was set by kind of what we chose to do with the jets was setting this. Here, there's kind of a natural scale where you know that there's going to be some degrees of freedom.

And so it's very natural, in this case, to take that as an input. And then you could actually figure out, given that what the jets should be and the jets would have, lambda QCD times Mb, which is in the middle. So that's kind of a natural scaling for the b to s gamma process, OK?

So just to give you an idea how, if I have a jet, it's going to look something like this, it might not look exactly like that. It depends on how many jets you have. And of course, these pictures actually get more complicated if you try to start drawing them when you have three jets because then the plane is no longer enough. All right, so any more questions?

OK. So when we did HQET, the first thing that we did is we started to expand. Before we designed the effective Lagrangian, we just said, well, what happens if I expand the full theory? And I'm going to take the same attitude here.

Let's just write down some full theory objects and expand them in the limits that we've been talking about. And then we'll see what kind of effective theory we want based on the results from those expansions. So let's start with spinners in a collinear limit.

So let me start with some massless QCD spinners in the Dirac representation. We could use some other representation, but let's just use Dirac.

So we have spinners for the quarks. We have spinners for the antiquarks, V, where this curly V and this curly U are two-component objects. I'll make sure it looks curly enough.

So what we can do here is we can expand. And you see that what happens when you expand is that you can think about the P3 vector being larger than P1 and P2. So let's just let our n be 1, 0, 0, 1 and our n bar be 1, 0, 0, minus 1. So they're back to back with each other.

Whether I put the plus or minus 1 there or there doesn't really matter. Let's expand in n bar dot P, which in this case is P0 plus P3 being much greater than P perp, just P1 and P2. And then that's much greater than n dot P, which is P0 minus P3.

And what that means is that you can approximate sigma dot P over P0 from these massless particles. It's just sigma 3 because you pick out the P3. That's the big component.

The P1 and P2, you can drop. Then it kicks out the sigma 3. And then P3 is also the same size as P0, so you're just getting sigma 3.

So what you get from this then would be guys that look like this. Just so, I put it in the two possibilities for the curly U are four-component spinners that look like this for U and then likewise for V of P.

We can work out what we get. We get that. So this is actually a little different than HQET. In HQET, what you would have found is that the antiquarks would have been just left out. And the quarks would've been there in the theory.

Here, both of them survive. And actually two degrees of freedom survive for both the particles and the antiparticles. So it's not like we're integrating out of something like the antiparticle, like in HQET. By this expansion, we still have all four of these degrees of freedom if you count degrees of freedom by whether you have particles, antiparticles, and spin states.

Nevertheless, there is a simplification that occurs. And that is the fact that the spinners that you have have a projection relation. So if you look in this basis that we're talking about here, what n slash is if you write down what the gamma matrices are in the Dirac representation, then n slash is this.

And another useful thing is n slash n bar slash over 4, which you can work out as just this. And these spinners here, which I need a name for-- so let's call this Un and call this Vn.

They satisfy n slash Un is n slash Vn is 0. And they also satisfy n slash n bar slash Un is Un for both of them.

So what we can do with that is the following. We can take the identity in this 4 by 4 space. And we can actually write it as n slash n bar slash over 4 plus n bar slash n slash over 4.

And that's because, remember, that gamma mu gamma nu plus 2 g mu nu and n dot n bar is 2. So this is just one way of using those. This is the anticommutator dotted into n and n bar. So I can write it out that way.

And this kind of formula here is the formula that is for projection operators, right? So as you act with the operator, you get that guy back again. So you could act twice with that operator.

And so what you can do with this one is you could let one act on psi of QCD. And if you did that, you'd get n slash n bar slash over 4 psi plus n bar slash n slash over 4 psi. And you could define these two pieces as being two different components of the full theory field that I'll call Cn and psi n.

And what happens at high energies, because of the type of thing we were doing over there, is that we only like to produce Cn's. We don't like to produce psi n bars. So if you look at some high energy process, this sort of thing I was doing at the spinners basically boils down to one sentence.

And that is that we produce or annihilate the components, the guys that live in this Cn, not the so-called small components, which live in this other guy. This language of calling them the small components is something that goes back to the early days of QCD actually. We may say that word a few times, but the history won't be so important to us.

OK, so, so much for the spinners. There is some simplification in the spinners because we do like to produce certain combinations. But it didn't really teach us too much beyond that.

And we didn't see that we lost a degree of freedom like we did in HQET. But nevertheless, there was some simplification. Let's do the same thing for the propagator of the quarks.

Take the propagator of the full theory, expand in this limit. So first of all, propagators always involve P squared plus i0. In our decomposition, that's in n bar dot P n dot P plus Minkowski P perp squared plus i0.

And if you look at the size of these things, they're all the same size. This guy is lambda 0, and this guy is lambda squared. And this guy is just lambda squared by itself.

So the two guys are the same size, although they become the same size for different regions. This is lambda 1 squared if you like. So I don't drop anything in that propagator.

And you see that actually that's not entirely true. And it does depend on what type of things you're interacting with. But if I just have P's that are collinear, as I've drawn here, then there's nothing to drop.

So if you look at fermions that are collinear and you look at i P slash over P squared plus i0, you can decompose P slash out in terms of n slash and n bar slash, write it out in terms of the coordinates we're using. And then in the numerator, you keep the full denominator. But in the numerator, there is one momentum component that's larger than the others. And that is the n bar dot P piece, which is order one.

So there is a simplification of the P slash. And it's related in some ways to the simplification of the spinners. If you take two of these guys and you do the sum over spin, you'll actually get n slash over 2 n bar dot P.

**AUDIENCE:** Is it n slash on the side of the 0?

**IAIN STEWART:** Yeah.

**AUDIENCE:** [INAUDIBLE]

**IAIN STEWART:** So the right way of thinking about the numerator here is that, if you do the sum over spins of sort of two of these spinners, like this, that's giving you the numerator. And if you look at sort of the overlap of this, if you look at any amplitude, you also have to take into account, but I haven't written yet-- which is this. And that's going to go like n bar slash.

So what happens is you have n slash, n bar slash. And that gives you the projector, which then overlaps order 1 with the spinner. Yeah. But that's a good question.

Later-- but that's a good point.

OK. So we can ask about, if we have some propagator for a fermion, then what does it look like? And you can take that first order term, and we can write it in a way that kind of is reminiscent of something more like NRQCD or this nucleon theory or even HQET. It's just another way of writing the same formula that is sometimes useful.

We just divide through by the n bar dot P. And then it's n dot P plus P perp squared over n bar dot P. And then there's the i0, but the sign of the i0 will depend on the sign of the n bar dot P.

The fact that this could be both plus i0 or minus i0 is the same thing as saying that there's antiparticles and particles in the theory. If it was just plus i0, as it was in HQET, we only had the particles. Here, it could be either sign. We have both. And so this thing, again, has both particles and antiparticles, which we saw when we were doing the spinners as well.

And if we want to think about particles and antiparticles separately, then we could have a definite sign for the i0's. But if we want to think about them in a combined propagator, then we have to write it this way. All right, so once we know what the propagator is, then we can also figure out what the power counting of the fields are because the propagator tells us what the kinetic term should look like of the Lagrangian.

So let me show you how that works. So where does the propagator come from? The propagator comes from the time order product of two fields. And in this case, it comes from the time order product of a field for this Cn component that we were talking about.

And if we just take the free kinetic term, which is the Lagrangian that we'd give that propagator, that's enough to determine the power counting for fields, as is always the case in any effective field theory. So we haven't figured out what the Lagrangian is, but we know something about what it's going to look like.

So let me write down enough of that to determine for you what the power counting would look like. So when I read it this way, as I wrote it, where it's linear in this n dot P derivative, you know that what that's going to correspond to in the Lagrangian is some n dot partial. And I have to have an n bar slash here because that always comes along with an n decomposing the metric.

If you ask about the power counting here, well, d4x has all 4 components of k. And x is the inverse of k. So the way that you assign a power counting for x is that you say the phase should be of order 1.

So the scaling for x is the opposite of k. So that fixes that d4x should be lambda to the minus 4. So x plus times k minus is of order 1, et cetera. And that tells you how many powers to associate with the d4x.

We know how many powers to associate with this partial because that was our momentum. That's lambda squared. And actually, the other terms here will also be lambda squared.

And then we just say, well, that's let the power counting in this field be arbitrary lambda to the a. And so if we do that, then we get an overall scaling for this Lagrangian that's lambda to the 2a minus 2. 2 to the power of the lambda minus 4 cancel by the partial n dot partial and we get that.

Now, the way that we do effective field theory is we look at the lowest order term, and we count everything relative to that. So what we do is we say, we want the lowest order Lagrangian to scale like lambda to the 0. And you can think of that roughly in a power counting sense as normalizing the free kinetic term or normalizing the kinetic term in general.

And then, once you do that, then you fix what the scaling of the field is, Cn to the order lambda. And that's different than the mass dimension. So I said that we were going to be doing a power counting that's different than the mass dimension. If we looked at the mass dimension of this field, Cn would have mass dimension that's 3/2, whereas it has, if you like, a lambda power counting dimension which is 1.

So if I look for the powers of lambda, that's 1. And that was one of the things I told you was going to happen is that, in this effective theory, we wouldn't just be counting mass dimension. We'd be counting something else.

OK. So that's collinear quarks. We can do a similar thing for collinear gluons. And we may not get to the end of that discussion today, but let's start it.

The momenta for a collinear gluon scales the same as the momenta for a collinear quark. That means collinear doesn't distinguish between quarks and gluons. So P squared, the full P squared, is still something that we're going to leave together.

And let's just consider looking at the propagator in a general covariant gauge and asking kind of the same type of thing that we did over here about the scaling of the field.

So the propagator for two collinear gluons and the general covariant gauge-- time order product vacuum matrix element of two fields. Ignore the subscript ends right now. I'm just writing down a full theory result.

We called C something else, so we better not make that gauge parameter. So let me call the gauge parameter tau. That's the gauge parameter of the general covariant gauge. And this is a full three result.

The thing that makes it collinear is if we say that k has a collinear scaling. So as above, k squared is k plus k minus plus k perp squared. That's of order lambda squared, and there's no expansion in there. And there's two k squareds here. There's one there, and there's one there.

And if you start looking at g mu nu minus k mu k nu over k squared, you also find that the terms there are the same size. So there's actually-- let's see how that works.

So let's do an example of that. So g perp you knew was just 1. So that's obviously something that has no scaling. And if you compare that to k perp mu k perp nu over k squared, k perp scale like lambda. k squared scales like lambda squared.

So this is lambda squared over lambda squared. So that's also lambda to the 0. So both the g perp mu nu and the k perp nu term are the same size. Yeah.

**AUDIENCE:** If it happened that it didn't work out, could you choose the gauge parameter have a [INAUDIBLE] with lambda?

**IAIN STEWART:** You could, but then you'd be restricted to the classic gauges that you would be able to use in the effective theory. And then you have to ask what gauge invariance would mean. If I don't make any restrictions on tau, I don't assign a power counting to it, that means my effective theory should allow all these gauges. And I actually want that. That's a good question.

You can do the same thing looking at, for example, g plus g minus. That's also 1, so lambda 0. Then you get k plus k minus over k squared. That's also lambda squared over lambda squared.

So this is what I was saying, that the two terms are the same size. If you dot in n mu n nu, that kills the g mu nu. g plus plus is 0. And then you just get k plus squared over k squared, which is lambda to the 1/4 over lambda squared, which is lambda squared.

So when the g mu nu is not 1, then the k mu k nu term can still determine how big something is. And that's what would happen for these off-diagonal terms. So if we go through the same type of exercise that we did for the fermion, d4x scales like 1 over k to the fourth. So that's lambda to the minus 4, just as before.

And actually, if we were to do the following, if we were to write this as-- I should have done that up there. If we were to write it as minus i over k to the fourth k squared g mu nu minus tau k mu k nu, the k to the fourth just matches up with the d4x. So those take care of each other.

And then the fields here have to match up with the rest. So the scaling of this should be the scaling of the A mu A nu. And that basically means that A mu in A nu scale like a momenta.

So A mu n for this collinear gluon scales like k mu, scales like lambda squared 1 lambda. Let me write that one more time.

And that's also a nice thing because it also means you could form a covariate derivative that's homogeneous by combining together, if I write it this way, k mu and g A mu. I can get a covariant derivative where, for each component of k, I also have a component of the gauge field that's the same size. So you could have argued it, originally just from gauge invariance, that you want to sort of have fields that are of the same size as your momenta, but we did it a little bit differently here.

So it's nice that that comes out. This all hangs together. So next time, we'll talk about the fact that what does it mean that the gauge field has components that are scaling in a different way. And particularly, there's a component of the gauge field that's order 1. There's no power suppression for that component. And we'll talk about what implications that has. Yeah.

**AUDIENCE:** So what about in the non-variant case, like n bar of A equals 0?

**IAIN STEWART:** Yeah. So you could go through this argument in that gauge as well, and it will work.

**AUDIENCE:** But you have n bar to the A equals 0, so--

**IAIN STEWART:** Yeah, that's fine. That's just a restriction on the-- you can still assign a scaling to it. And a special choice if it is 0.

Scaling and values of things are not the same thing, right? Like, you could have a field that scales like 1 in the power company, but happens to be 0. And that's OK. Yeah.

**AUDIENCE:** I had a question about SCET1 and SCET2.

**IAIN STEWART:** Sure.

**AUDIENCE:** So in the example adding one jab, and one hadron, like the [INAUDIBLE], why do I need to have different degrees of freedom to describe [INAUDIBLE]?

**IAIN STEWART:** Yeah. It's really because of the way the modes sat on those hyperbolas that, in the case of the hadrons, you had the soft modes and collinear modes on the same hyperbola. And that's going to change how the effective theory looks.

**AUDIENCE:** Yeah. But from a physically point of view, why the hadrons-- [INAUDIBLE] the hadrons behaving in a different way than [INAUDIBLE]?

**IAIN STEWART:** Yeah. So one way of thinking about it is, if you just had the collinear modes alone and you didn't have the soft modes, then it would be very similar. It would just be that the hyperbola moved down. And you think maybe those two theories are the same. There's examples of that actually.

There's one example that we'll cover where you don't need soft modes. And then you can't really tell whether you're using SCET1 or SCET2. But if you have a process that has both soft and collinear modes, then it's the kind of way that those modes talk to each other that distinguishes the cases of hadrons and jets in the cases--

**AUDIENCE:** For example, for the case of the jet, [INAUDIBLE] soft modes like as a need for the--

**IAIN STEWART:** Right. You could try, yes. And it turns out that those modes aren't relevant. So you have to be a little bit faithful from what I've told you so far.

**AUDIENCE:** But there is no physical picture for why it can understand [INAUDIBLE].

**IAIN STEWART:** You can understand it because what would happen with those soft modes is that they would take the collinear modes far off-shell. If you just had one of those soft modes interacting with the collinear mode, you end up not with the collinear mode back again, but something that's further off-shell that you actually want to integrate out. So that thing that has the scaling that allows you to just have collinear mode in something which we call ultra soft in and still have collinear, that's the ultra soft mode.

So if you want to communicate with the jet without disturbing it and blowing it apart, then you really need the ultra soft mode. Yeah. But these are all good questions, and we'll talk much more about it.