

Effective Field Theory (8.851) Spring 2013

Homework 3

Problem 1) HQET Chromomagnetic Operator: Renormalization of $c_F(\mu)$

Consider the $\mathcal{O}(1/m_Q)$ magnetic moment HQET Lagrangian

$$\mathcal{L}_F^{(1)} = -c_F \frac{g}{4m_Q} \bar{Q}_v \sigma_{\mu\nu} G^{\mu\nu} Q_v. \quad (1)$$

Split this into a renormalized operator and a counterterm depending on various Z 's. Explain how you would compute the anomalous dimension of the renormalized coefficient $c_F(\mu, m_Q)$ and draw the corresponding diagrams (without cancelling any diagrams by specifying a particular gauge choice). Discuss whether the kinetic energy Lagrangian $\mathcal{L}_K^{(1)} = -\bar{Q}_v D_T^2 Q_v / (2m_Q)$ mixes with $\mathcal{L}_F^{(1)}$ under renormalization. Argue that the anomalous dimension for c_F vanishes in the abelian case, and therefore is proportional to the adjoint Casimir C_A . (Hint: For this last part there are various ways of proceeding, and I am not requiring that you compute any loop integrals. You might consider trying to find a gauge where it becomes obvious at the integrand level.)

Problem 2) Reparameterization Invariance for Heavy-to-Light Currents

Consider the $\mathcal{O}(1/m_Q)$ heavy-to-light vector currents in HQET

$$\begin{aligned} O_1 &= \bar{q} \gamma^\mu i \not{D} Q_v, & O_4 &= \bar{q} (-iv \cdot \overleftarrow{D}) \gamma^\mu Q_v, \\ O_2 &= \bar{q} v^\mu i \not{D} Q_v, & O_5 &= \bar{q} (-iv \cdot \overleftarrow{D}) v^\mu Q_v, \\ O_3 &= \bar{q} i D^\mu Q_v, & O_6 &= \bar{q} (-i \overleftarrow{D}^\mu) Q_v, \end{aligned} \quad (2)$$

with Wilson coefficients B_1 to B_6 . Using reparameterization invariance determine which of these coefficients are fixed by the coefficients C_1 and C_2 of the leading order vector heavy-to-light currents $\bar{q} \gamma^\mu Q_v$ and $\bar{q} v^\mu Q_v$.

Problem 3) Generalizing the R -RGE

In lecture we discussed the solution of the QCD beta function equation with $\beta[\alpha_s]$ specified at any order in perturbation theory. We then solved an R -evolution renormalization group equation for the MSR heavy quark mass $m(R)$ of the form

$$R \frac{d}{dR} m(R) = -R \gamma_R[\alpha_s(R)], \quad (3)$$

where the anomalous dimension $\gamma_R[\alpha_s(R)] = \sum_{k=0}^{\infty} \gamma_k^R [\alpha_s(R)/4\pi]^{k+1}$, and γ_k^R are numbers.

- a) The precise definition of the variable R has some arbitrariness to it (much as the meaning of μ differs between the MS and $\overline{\text{MS}}$ schemes). Let $R = \kappa R'$ where the parameter κ is an $\mathcal{O}(1)$ constant and derive an RGE equation in the R' variable which expands in $\alpha_s(R')$. Express your answer as results for $\gamma_n^{R'}$ in terms of γ_n^R and β_ℓ 's for $n' = 0, 1, 2$.

Different choices of κ correspond to different R -schemes, and κ variation can be used to estimate uncertainties in scheme-independent results.

- b) Consider the solution of the R-RGE in Eq. (3) given in lecture that expresses the evolution of mass from $m(R_0)$ to $m(R_1)$ as an integral over the anomalous dimension γ_R . Consider the LL result, and take the limit $R_0 \rightarrow 0$ setting $m(R_0) = m_{\text{pole}}$, and show that you can manipulate the integral into the form of an inverse Borel transform integral over u (the conjugate variable to $\beta_0 \alpha_s / 4\pi$). Identify the integrand as the Borel transform $B(u)$ and find the location of the pole closest to the origin on the positive u axis, and its residue. Is this pole at the expected location?

This problem shows that the R-RGE can be used as a probe for renormalons, much like the bubble-chain. In the case of the R-RGE we are probing renormalons with the full running coupling in a manner that can be improved order by order. (Beyond LL order it is not easy to find the appropriate change of variable, but it is straightforward to directly take the Borel transform of the all orders form of the RGE solution for $R_0 \rightarrow 0$. Higher order terms provide corrections to the residue of the Borel pole.)

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