Day 1, M 2/5/2024

Topic 1: Intro to differential equations

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1 Agenda

- Welcome!
- Administrative stuff
- DEs are rate equations
- DEs model physical systems
- DEs are slope equations
- Separable equations
- Problems

2 Administrative stuff

Teachers: Jerry Orloff, Jon Bloom

You should have gotten links to our websites.
– Canvas has all these links
– Look at our main website

- Read syllabus for dates
- Read "Grading and class policies"
- Pset 1 is posted
- Pset checker is on MITx

Grade: 40% psets, 55% quizzes, 5% participation

Key: Do the reading before class. We'll expect this, but we won't expect full understanding.

Class = lecture + problem solving.

All slides and problems posted before class.

Attend office hours!

3 Derivatives

$\frac{dx}{dt}$ is the rate x changes with respect to t.

$\frac{dy}{dx}$ is the rate y changes with respect to x, i.e., the slope of the graph.

4 Differential equations (DEs)

- Derivative
- Equal sign

4.1 Examples

1. $\dfrac{dx}{dt} = ax$ (order 1).
2. $\dfrac{d^2x}{dt^2} + ax = 0$ (order 2).
3. $\left(\dfrac{dx}{dt}\right)^3 + \dfrac{d^2x}{dt^2} = x^2 \sin(6t)$ (order 2).
4. $y''' + 3y'' + 4y' + 5y = \sin(6t)$ (order 3).

In (1), x depends on t: t is the independent variable, x is the unknown function.

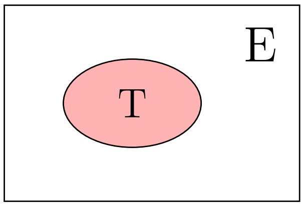
** Solving the DE means finding the unknown function $x(t)$ that satisfies the equation.

In (4), by contex, y depends on t, $y' = \dfrac{dy}{dt}$.

4.2 Some well known DEs

1. Newton's law of cooling.

$T(t)$ = temperature of a body at time t.
E = temperature of its environment



$$\text{Model:} \quad \frac{dT}{dt} = -k(T - E),$$

k = rate constant, dimension 1/time.

2. Gravity near the Earth's surface.

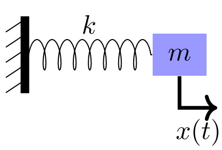
$x(t)$ = height of a mass above the ground.

$$\text{Model:} \quad \frac{d^2x}{dt^2} = -g, \qquad \text{g = -9.8 m/sec}^2.$$

3. Hooke's law.

Mass m on a spring with spring constant k.

$x(t)$ = displacement of m from equilibrium.



$$\text{Model:} \quad m\frac{d^2x}{dt^2} = -kx.$$

5 Separable equations

Example 1. Solve $\frac{dy}{dx} = x^2(y-2)$.

Solution: Independent variable = x, dependent variable y = unknown function.

Steps

1. Separate the variables: $\frac{dy}{y-2} = x^2\, dx$.
2. Integrate: $\ln|y-2| = \frac{x^3}{3} + C$. (Don't forget the C.)
3. Algebra: $|y-2| = e^c e^{x^3/3}$.
4. So,

$$\begin{aligned} &\text{if } y < 2, \quad y = -e^c e^{x^3/3} + 2, \quad (\text{note: } -e^c < 0) \\ &\text{if } y > 2, \quad y = e^c e^{x^3/3} + 2, \quad (\text{note: } e^c > 0) \\ &\text{if } y = 2, \quad y = 2, \text{ lost solution} \end{aligned}$$

It's easy to verify the lost solution is a solution. Lost because dividing by $y-2$ is dividing by 0,

Can summarize the solution: $\boxed{y(x) = \tilde{C}e^{x^3/3}}$, where $\tilde{C}$ can take any value.