

Problems Day 23, R 3/7/2024

Topic 10: Graphical methods: Direction fields, integral curves

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Problem 1. Use isoclines to sketch a direction field for $y' = x^2 + y^2$. Add some integral curves

Problem 2. For the DE, $y' = -xy$:

Draw the nullclines. The function $y(x) \equiv 0$ is clearly a solution. Discuss how this relates to the nullclines.

The nullclines divide the xy -plane into 4 regions. Put a big \oplus or \ominus in each region to indicate whether the slope of integral curves is positive or negative.

Sketch in some integral curves.

Problem 3. Open the Isoclines mathlet: <https://www.mathlets.org/mathlets/isoclines/>

Choose the DE $y' = y^2 - ax^2$.

(a) Set $a = -1$. Use the m -slider to draw some isoclines. Click in the graph window to draw some integral curves.

–Notice how the slope elements on an isocline are all alike.

–Notice how the integral curves follow the slope field.

–Compare the picture to your answer in Problem 1.

(b) Clear the graph window and set $a = 1$.

Use the m -slider to draw the nullcline.

Which portions of the nullcline are upper fences? Lower fences?

Add the isocline with $m = -3$. Identify any funnels. Check your answer by adding some solution curves.

(c) Add the isocline $m = 3$. Identify any new funnels. Check your answer by adding some solution curves.

(d) Still with $a = 1$. Clear the graph. Draw the nullcline and the solution with $y(2.5) = 0$. Without drawing it, for the solution with $y(1) = 0$, estimate the value of $y(4)$. Check your answer by drawing the solution.

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